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REMARK ON A FIXED POINT THEOREM OF CHANG

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Abstract

The purpose of this paper is make some remarks on a fixed point theorem of Chang.

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In [1] Chang has mentioned that the following question namely "Which Banach spaces have the Fixed point property (F.P.P.) for generalized non-expansive mappings ?" has been open for long time. Chang has attempted to answer this question in [1].

Definition 1. *Let K be a nonempty subset of a normed linear space X. Then $T : K \to K$ is said to be generalized nonexpansive, if for every subset F of K with at lest two points and $T(F) \subset F$*

$$\sup_{y\in F} ||Tx - Ty|| \leq \sup_{y\in F} ||x - y||,\ x \in F.$$

The following result is the main theorem of [1].

Theorem 1 *([1], Theorem 3). Every Banch space having normal structure has the (F.P.P.) for generalized nonexpansive mapping.*

It is proved that if K is a nonempty compact convex subset of a Banach space and $T : K \to K$ is a generalized nonexpansive mapping in the sense od Definition 1 and further if K has normal structure then T has a fixed point.

The following more general theorem than that of the above Theorem 1 of Chang has been proved by Pai and Veeramani in 1982.

Theorem 2. *([2], Corrolary 2.2) Let K be a nonempty weakly compact convex subset of a Banach space X. Assume that K has a normal structure. Let T be a mapping of K into itself which satisfies: for each closed convex subset F of K invariant under T there exists some $\alpha(F)$, $0 \leq \alpha(F) < 1$, such that*

$$||Tx - Ty|| \leq \max\{\delta(x, F), \alpha\delta(F)\}$$

for $x, y \in F$ (where $\delta(x, F) = sup\{||x - y|| : y \in F\}$ and $\delta(F)$ is the diameter of F). Then T has a fixed point.

It is obvious that Theorem 1 follows from Theorem 2.

Remark. The following Example 1 shows that Theorem 2 is more general than Theorem 1.

Example 1. Let $K = [0, 2]$. Define $T : K \to K$ by

$$Tx = \begin{cases} 0, & \text{for } x \neq 2 \\ 3/2 & \text{for } x = 2. \end{cases}$$

Each closed convex subset F of K invariant under T and with $\delta(F) > 0$ will be of the form $[0, \gamma]$, $\gamma < 2$.

For $\gamma < 2$,

$$T([0, \gamma]) = \{0\}$$

and in this case

$$|Tx - Ty| \leq \max\{\delta(x, F), \alpha\delta(F)\},$$

for $x, y \in F$, is satisfied for any $\alpha > 0$.

For $\gamma = 2$,

$$T([0, \gamma]) = \{0, \frac{3}{2}\}$$

hence for $\alpha = 3/4$

$$|Tx - Ty| \leq \max\{\delta(x,K), \alpha\delta(K)\},$$

for $x, y \in K$, is satisfied and $T(0) = 0$. Since $|T(2) - T(1)| = \frac{3}{2}$, $|T(2) - T(1)| \leq |2 - 1|$, for $1, 2 \in K$, is not satisfied.

In fact,

$$\sup_{y \in F} |Tx - Ty| \leq \sup_{y \in F} |x - y|, \; x \in F$$

is also not satisfied for $F = K$.

References

[1] Chih - Sen Chang, Uniform convexity and the fixed point property, Univ. u Novom Sadu Zb. Rad. Prirod. - Mat. Fak. Ser. Mat., 21,1(1991), 105 - 115.

[2] Pai, D.V., Veeramani, P., On some fixed point theorems in Banach spaces, Internat. J. Math. and Math. Sci. 5, 1 (1982), 113 - 122.

REZIME

O TEOREMI O NEPOKRETNOJ TAČKI CHANGA

U radu se daju neke napomene o teoremi o nepokretnoj tački iz [1].

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