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ON THE CONTINUOUS ELEMENTS OF THE LATTICE

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Abstract

Meet-continuous ($\wedge$-continuous) element of a lattice L is an element a which satisfies

$$a \wedge \bigvee_{i \in I} x_i = \bigvee_{i \in I} (a \wedge x_i),$$

for every chain $\{x_i | i \in I\}$ of a lattice; an element with the dual property is called join-continuous ($\vee$-continuous), and element with both properties - continuous. We give some properties of these elements, and prove some statements on lattice identities containing infinitary operations.

In the second part we apply these lattice theoretic results on the lattice of weak congruences of an algebra. Namely, the diagonal relation of this lattice is always a meet-continuous element in the lattice of weak congruences. We consider some well known algebraic properties, such as CEP, CIP and infinite CIP (*CIP) and their connection with the continuity of the diagonal relation. Particularly, we prove some results on transfering these properties from an algebra to it's subalgebras or factor algebras.

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1. Preliminaries

1.1. Special elements

Here we will give definitions of some special elements of lattices, i.e. elements which satisfy some identities, and among them definitions of continuous elements of lattices. Continuity of elements is a link between finite and analogous infinite properties of them.

An element a of a lattice L is **distributive** if for every $x, y \in L$,

$$a \vee (x \wedge y) = (a \vee x) \wedge (a \vee y).$$

An element a which satisfies the dual law is called **codistributive**. Element a is codistributive if and only if the mapping $m_a : L \longrightarrow a \downarrow$ defined with $m_a(x) = a \wedge x$ is a lattice homomorphism. This homomorphism induces a congruence on L, and if the congruence class of an $x \in L$ has the top element, we shall denote it with $\overline{x}$. The dual homomorphism connected with a distributive element a will be denoted with n_a, and the bottom element of the class to which an element x belongs with $\underline{b}$

An element a of a lattice L is **infinitely distributive** ([4]) if for every family $\{x_i | i \in I\} \subseteq L$:

$$a \vee (\bigwedge_{i \in I} x_i) = \bigwedge_{i \in I}(a \vee x_i).$$

An element a of a lattice L is **neutral** if for every $x, y \in L$, $(a \vee x) \wedge (a \vee y) \wedge (x \vee y) = (a \wedge x) \vee (a \wedge y) \vee (x \wedge y)$.

An element a is neutral iff it is distributive, codistributive and **cancellable** (from $(x \wedge a = y \wedge a$ and $x \vee a = y \vee a)$ it follows that $x = y$).

Let L be a complete lattice. An element $a \in L$ is said to be **meet -continuous** ($\wedge$-continuous) if

$$a \wedge \bigvee_{x \in I} x_i = \bigvee_{i \in I}(a \wedge x_i)$$

for every chain $\{x_i | i \in I\} \subseteq L$.([6]).

An element a satisfying the dual property is called **join-continuous** ($\vee$-continuous).([6]).

An element a is **continuous** iff it is both meet -continuous and join -continuous.([6])

1.2. Weak congruence lattice

Let $\mathcal{A} = (A, F)$ be an algebra. A **lattice of weak congruences** of $\mathcal{A}$ ($Cw\mathcal{A}$) is a lattice of all weakly reflexive, symmetric, transitive, and compatible relations on $\mathcal{A}$. Weak congruence lattice is thus a lattice of all congruences on all subalgebras on $\mathcal{A}$. This lattice is algebraic. Diagonal relation Δ is always a codistributive element in $Cw\mathcal{A}$.

Algebra $\mathcal{A}$ has the **congruence extension property (CEP)** if every congruence on a subalgebra of $\mathcal{A}$ is a restriction of a congruence on $\mathcal{A}$. Recall that an algebra has the CEP iff Δ is a costandard element in $Cw\mathcal{A}$ iff Δ is a comodular element in $Cw\mathcal{A}$ iff Δ is a cancellable element in $Cw\mathcal{A}$.

An algebra $\mathcal{A}$ has the **congruence intersection property (CIP)** if $(\rho \wedge \theta) \vee \Delta = (\rho \vee \Delta) \wedge (\theta \vee \Delta)$ for all $\rho, \theta \in Cw\mathcal{A}$, i.e. if Δ is a distributive element in the lattice $Cw\mathcal{A}$([3]). An algebra $\mathcal{A}$ has the **infinite congruence intersection property (*CIP)** ([4]) if for an arbitrary family of weak congruences $\{\rho_i | i \in I\}$,

$$\Delta \vee (\bigwedge_{i \in I} \rho_i) = \bigwedge_{i \in I} (\Delta \vee \rho_i).$$

2. Lattice theoretic results

Lemma 1. *In a complete, cocompactly generated lattice every element is join -continuous.*

Proof. Let L be a complete, cocompactly generated lattice, $a \in L$, and $\{x_i | i \in I\}$ an arbitrary chain in L. Since the inequality $a \vee \bigwedge_{i \in I} x_i \leq \bigwedge_{i \in I}(a \vee x_i)$ is always satisfied, it suffices to prove $a \vee \bigwedge_{i \in I} x_i \geq \bigwedge_{i \in I}(a \vee x_i)$. If c is a cocompact element, such that $c \geq a \vee \bigwedge_{i \in I} x_i$, then $c \geq a$ and $c \geq \bigwedge_{i \in I} x_i$. From $c \geq \bigwedge_{i \in I} x_i$ and c cocompact it follows that $c \geq \bigwedge_{i \in J} x_i$ for $J \subseteq I$ and J finite. Since $\{x_i | i \in J\}$ is a chain, $\bigwedge_{i \in J} x_i = x_d$, for $x_d \in \{x_i | i \in J\} \subseteq \{x_i | i \in I\}$ and it follows that $c \geq a \vee x_d \geq \bigwedge_{i \in I}(a \vee x_i)$. Since every cocompact element which is greater than $a \vee \bigwedge_{i \in I} x_i$ is greater than $\bigwedge_{i \in I}(a \vee x_i)$, too, $a \vee \bigwedge_{i \in I} x_i$, as an infima of cocompact elements and therefore, of all cocompact elements greater than itself, is greater than $\bigwedge_{i\in I}(a \vee x_i)$, i.e. $\bigwedge_{i\in I}(a \vee x_i) \leq a \vee \bigwedge_{i\in I} x_i$. □

Proposition 1. *Let L be a complete lattice. Then*

a) *The set of all meet-continuous elements of a lattice L is closed under infima.*

b) *The set of all join-continuous elements of a lattice L is closed under suprema.*

c) *The set of all continuous elements of a lattice L is a continuous sublattice of L.*

Proof. **a)** Let a and b be meet-continuous elements of L and $\{x_i | i \in I\}$ an arbitrary chain in L. Then:

$$a \wedge b \wedge \bigvee_{i\in I} x_i = a \wedge \bigvee_{i\in I}(b \wedge x_i) = \bigvee_{i\in I}(a \wedge b \wedge x_i),$$

since a and b are meet-continuous elements, and since from the fact that $\{x_i | i \in I\}$ is a chain, it follows that $\{b \wedge x_i | i \in I\}$ is a chain as well.

The statement **b)** is dual to the statement **a)**, and **c)** follows directly from **a)** and **b)**. □

The next example shows that the infimum of join-continuous elements need not be join-continuous, i.e. that the set of all join-continuous elements is not closed in general under infima. The similar can be said also for meet-continuous elements.

Example 1.

In the lattice L (Fig. 1) b and c are join-continuous elements, since

$$b \vee \bigwedge x_i = b \vee d = f = \bigwedge(b \vee x_i).$$

For every finite chain, and every subchain of the chain $\{x_i\}$, the required equality is also satisfied. Similarly, c is also join-continuous. However,

$$a \vee \bigwedge x_i = a \vee d = h \neq g = \bigwedge(a \vee x_i),$$

for $a = b \wedge c$.

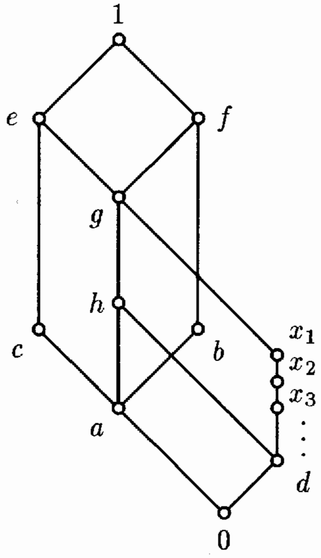

Fig. 1

Proposition 2. [6] *a is a join -continuous and distributive element of a complete lattice L, if and only if it is an infinitely distributive element.* □

Lemma 2. [6] *If a is an infinitely codistributive element of the lattice L and $f(x_1, x_2, ..., x_\alpha, ...)$ is an arbitrary lattice expression*[2] *(possibly with infinitary operations), then for all $x_1, ..., x_\alpha, ... \in L$,*

$$f(x_1, x_2, ..., x_\alpha, ...) \wedge a = f(x_1 \wedge a, x_2 \wedge a, ..., x_\alpha \wedge a, ...).$$

□

The dual proposition is also valid:

Lemma 3. *If a is an infinitely distributive element of the lattice L and $f(x_1, x_2, ..., x_\alpha, ...)$ is an arbitrary lattice expression (possibly with infinitary operations), then for all $x_1, ..., x_\alpha, ... \in L$,*

$$f(x_1, x_2, ..., x_\alpha, ...) \vee a = f(x_1 \vee a, x_2 \vee a, ..., x_\alpha \vee a, ...).$$

□

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[2] Defined as a lattice therm including infinitary operations

Proposition 3. [6] *If a is a neutral and continuous element of a complete lattice L, then a lattice identity (possibly with infinitary operations) is satisfied on L if and only if this identity holds on $a\downarrow$ and on $a\uparrow$.* □

In the sequel we shall prove some statements concerning identities on lattices (possibly with infinitary operations) and in the second part shall give their applications in the weak congruence lattices of algebras. Analogous theorems to Theorem 1 and 2 were proved in [5] for lattice identities with finitary operations.

Lemma 4. *Let L be a complete lattice with the top element 1, $b \in L$ and $f(x_1, x_2, ..., x_\alpha, ...) = g(x_1, x_2, ..., x_\alpha, ...)$ an arbitrary lattice identity (possibly with infinitary operations). If for all $x_2, ..., x_\alpha, ... \in L$,*

$$f(1, x_2, ..., x_\alpha, ...) = g(1, x_2, ..., x_\alpha, ...),$$

then for all $y_2, ..., y_\alpha, ... \in b\downarrow$,

$$f(b, y_2, ..., y_\alpha, ...) = g(b, y_2, ..., y_\alpha, ...).$$

Proof. b is the top element in $b\downarrow$ as 1 is the top element in L. For all $y_2, ..., y_\alpha, ... \in b\downarrow$, we have that

$$f(1, y_2, ..., y_\alpha, ...) = g(1, y_2, ..., y_\alpha, ...).$$

Since $x \wedge 1 = x, x \vee 1 = 1$, for all $x \in b\downarrow$, we prove by induction that the equality above becomes $1 = 1$, or an equality $f_1(y_2, ..., y_\alpha, ...) = g_1(y_2, ..., y_\alpha, ...)$, without 1. Since also, $x \wedge b = x$, and $x \vee b = b$, for all $x \in b\downarrow$, when we replace 1 with b, the equality

$$f(b, y_2, ..., y_\alpha, ...) = g(b, y_2, ..., y_\alpha, ...)$$

becomes $b = b$ or $f_1(y_2, ..., y_\alpha, ...) = g_1(y_2, ..., y_\alpha, ...)$, and this is true by the assumption (the same equality as the one which is obtained for 1). □

The dual statement is also satisfied:

Lemma 5. *Let L be a complete lattice with the bottom element 0, $b \in L$ and $f(x_1, x_2, ..., x_\alpha, ...) = g(x_1, x_2, ..., x_\alpha, ...)$ an arbitrary lattice identity (possibly with infinitary operations). If for all $x_2, ..., x_\alpha, ... \in L$,*

$$f(0, x_2, ..., x_\alpha, ...) = g(0, x_2, ..., x_\alpha, ...),$$

then for all $y_2, ..., y_\alpha, ... \in b \uparrow$,

$$f(b, y_2, ..., y_\alpha, ...) = g(b, y_2, ..., y_\alpha, ...).$$

□

Theorem 1. *Let* a *be a neutral and continuous element of a lattice* L, *such that the classes of the congruence induced by the homomorphism* m_a *have the top elements, and* $f(x_1, x_2, ..., x_\alpha, ...) = g(x_1, x_2, ..., x_\alpha, ...)$ *an arbitrary lattice identity (possibly with infinitary operations), and* $b \in a \downarrow$.

If $f(a, x_2, ..., x_\alpha, ...) = g(a, x_2, ..., x_\alpha, ...)$ *is satisfied on* $a \downarrow$ *and on* $a \uparrow$, *then* $f(b, x_2, ..., x_\alpha, ...) = g(b, x_2, ..., x_\alpha, ...)$ *is satisfied on* $\bar{b} \downarrow$.

Proof. For $x_2, ..., x_\alpha, ... \in \bar{b} \downarrow$,

$$f(b, x_2, ..., x_\alpha, ...) \wedge a = f(b \wedge a, x_2 \wedge a, ..., x_\alpha \wedge a, ...) =$$

$$= g(b \wedge a, x_2 \wedge a, ..., x_\alpha \wedge a, ...) = g(b, x_2, ..., x_\alpha, ...) \wedge a,$$

by Lemma 2 and Lemma 4, since $b \wedge a = b$, and $x_2 \wedge a, ..., x_\alpha \wedge a \in b \downarrow$.

Similarly, by Lemma 3,

$$f(b, x_2, ..., x_\alpha, ...) \vee a = g(b, x_2, ..., x_\alpha, ...) \vee a.$$

Since a is a neutral element of L, it follows that:

$$f(b, x_2, ..., x_\alpha, ...) = g(b, x_2, ..., x_\alpha, ...).$$

□

The dual statement is also satisfied:

Theorem 2. *Let* a *be a neutral and continuous element of a lattice* L, *such that the classes of the congruence induced by the homomorphism* n_a *have the bottom elements, and* $f(x_1, x_2, ..., x_\alpha, ...) = g(x_1, x_2, ..., x_\alpha, ...)$ *an arbitrary lattice identity (possibly with infinitary operations), and* $b \in a \uparrow$.

If $f(a, x_2, ..., x_\alpha, ...) = g(a, x_2, ..., x_\alpha, ...)$ *is satisfied on* $a \downarrow$ *and on* $a \uparrow$, *then* $f(b, x_2, ..., x_\alpha, ...) = g(b, x_2, ..., x_\alpha, ...)$ *is satisfied on* $\underline{b} \uparrow$.

Theorem 3. *Let a be a distributive and continuous element of a lattice L, and $b \in a\uparrow$. If for all $x_1, x_2, ..., x_\alpha, ... \in a\uparrow$,*

$$\bigwedge_{i\in I}(b \vee f_1^i(x_1, x_2, ..., x_\alpha, ...)) = \bigwedge_{j\in J}(b \vee f_2^j(x_1, x_2, ..., x_\alpha, ...)),$$

then the same identity is satisfied for all $x_1, x_2, ..., x_\alpha, ... \in L$.

Proof. Since a is a distributive and continuous element, it is infinitely distributive by Proposition 2. Since $a \vee b = b$, for $x_1, x_2, ...x_\alpha, ... \in L$ we have that:

$$\bigwedge_{i\in I}(b \vee f_1^i(x_1, x_2, ..., x_\alpha, ...)) = \bigwedge_{i\in I}(b \vee a \vee f_1^i(x_1, x_2, ..., x_\alpha, ...)) =$$

$$\bigwedge_{i\in I}(b\vee f_1^i(x_1\vee a, x_2\vee a, ..., x_\alpha\vee a, ...)) = \bigwedge_{j\in J}(b\vee f_2^j(x_1\vee a, x_2\vee a, ..., x_\alpha\vee a, ...)) =$$

$$= \bigwedge_{j\in J}(b \vee a \vee f_2^j(x_1, x_2, ..., x_\alpha, ...)) = \bigwedge_{j\in J}(b \vee f_2^j(x_1, x_2, ..., x_\alpha, ...)).$$

□

The dual statement is also satisfied:

Theorem 4. *Let a be a codistributive and continuous element of a lattice L, and $b \in a\downarrow$. If for all $x_1, x_2, ..., x_\alpha, ... \in a\downarrow$,*

$$\bigvee_{i\in I}(b \wedge f_1^i(x_1, x_2, ..., x_\alpha, ...)) = \bigvee_{j\in J}(b \wedge f_2^j(x_1, x_2, ..., x_\alpha, ...)),$$

then the same identity is satisfied for all x_1, $x_2, ..., x_\alpha \in L$. □

Theorem 5. *Let a be a neutral and continuous element of a lattice L. If for $b \in a\uparrow$, $f_1(b, x_2, ..., x_\alpha, ...) = f_2(b, x_2, ..., x_\alpha, ...)$ is satisfied on $a\uparrow$, and $f_1(a, x_2, ..., x_\alpha, ...) = f_2(a, x_2, ..., x_\alpha, ...)$ is satisfied on $a\downarrow$, then*

$$f_1(b, x_2, ..., x_\alpha, ...) = f_2(b, x_2, ..., x_\alpha, ...)$$

is satisfied on L.

Proof. Since a is a neutral and continuous element of a lattice L, it is infinitely distributive, and infinitely codistributive, as well. For $x_2, ..., x_\alpha, ... \in L$, by Lemmas 2 and 3, we obtain that:

$$f_1(b, x_2, ..., x_\alpha, ...) \vee a = f_1(b, x_2 \vee a, ..., x_\alpha \vee a, ...) =$$

$$= f_2(b, x_2 \vee a, ..., x_\alpha \vee a, ...) = f_2(b, x_2, ..., x_\alpha, ...) \vee a,$$

and, similarly,

$$f_1(b, x_2, ..., x_\alpha, ...) \wedge a = f_2(b, x_2, ..., x_\alpha, ...) \wedge a.$$

Since a is neutral,

$$f_1(b, x_2, ..., x_\alpha, ...) = f_2(b, x_2, ..., x_\alpha, ...).$$

□

Theorem 6 is dual to Theorem 5:

Theorem 6. *Let a be a neutral and continuous element of a lattice L. If for $b \in a\downarrow$, $f_1(b, x_2, ..., x_\alpha, ...) = f_2(b, x_2, ..., x_\alpha, ...)$ is satisfied on $a\downarrow$, and $f_1(a, x_2, ..., x_\alpha, ...) = f_2(a, x_2, ..., x_\alpha, ...)$ is satisfied on $a\uparrow$, then*

$$f_1(b, x_2, ..., x_\alpha, ...) = f_2(b, x_2, ..., x_\alpha, ...)$$

is satisfied on L. □

3. Application to algebra

Proposition 4. *Δ is a meet -continuous element in the lattice of weak congruences.*

Proposition 5. [6] *An algebra $\mathcal{A}$ has *CIP iff it has CIP and Δ is a $\vee$-continuous element in $Cw\mathcal{A}$.* □

Proposition 6. [6] *If an algebra $\mathcal{A}$ has CEP and *CIP then an arbitrary lattice identity (possibly with infinitary operations) is satisfied on $Cw\mathcal{A}$ if and only if this identity holds on $Sub\mathcal{A}$ and on $Con\mathcal{A}$.* □

Proposition 7. *Let $\mathcal{A}$ be an algebra which has CEP and *CIP, let*

$$f(x_1, x_2, ..., x_\alpha, ...) = g(x_1, x_2, ..., x_\alpha, ...)$$

be an arbitrary lattice identity (possibly with infinitary operations), and $\mathcal{B}$ a subalgebra of $\mathcal{A}$.

If $f(\Delta, x_2, ..., x_\alpha, ...) = g(\Delta, x_2, ..., x_\alpha, ...)$ is satisfied on $Sub\mathcal{A}$ and on $Con\mathcal{A}$, then

$$f(\Delta_B, x_2, ..., x_\alpha, ...) = g(\Delta_B, x_2, ..., x_\alpha, ...)$$

is satisfied on $Cw\mathcal{B}$.

Proof. The proposition is a consequence of Theorem 1. Namely, since $\mathcal{A}$ has CEP and *CIP, Δ is a neutral and continuous element of $Cw\mathcal{A}$. In a weak congruence lattice classes of the congruence induced by mapping m_a always have the top elements (squares of subalgebras). Conditions of the theorem are thus fulfilled. For every weak congruence ρ, $\overline{\rho}$ is the square of the underlining set of a subalgebra $\mathcal{B}$ on which ρ is a congruence, and $\overline{\rho}\downarrow$ is in fact $Cw\mathcal{B}$. □.

Corollary 1. *If algebra $\mathcal{A}$ has CEP and *CIP, then every subalgebra $\mathcal{B}$ of $\mathcal{A}$ has CEP and *CIP as well.*

Proof. CEP and *CIP are identities of the form

$$f(\Delta, x_2, ..., x_\alpha, ...) = g(\Delta, x_2, ..., x_\alpha, ...),$$

which are satisfied on the whole $Cw\mathcal{A}$ (and thus on $Con\mathcal{A}$ and on $Sub\mathcal{A}$). CEP and *CIP on a subalgebra $\mathcal{B}$ are the same identities, where Δ is replaced with Δ_B. □.

Proposition 8. *Let $\mathcal{A}$ be an algebra which has CEP and *CIP, let*

$$f(x_1, x_2, ..., x_\alpha, ...) = g(x_1, x_2, ..., x_\alpha, ...)$$

be an arbitrary lattice identity (possibly with infinitary operations), and $\theta \in Con\mathcal{A}$.

If $f(\Delta, x_2, ..., x_\alpha, ...) = g(\Delta, x_2, ..., x_\alpha, ...)$ is satisfied on $Sub\mathcal{A}$ and on $Con\mathcal{A}$, then

$$f(\theta, x_2, ..., x_\alpha, ...) = g(\theta, x_2, ..., x_\alpha, ...)$$

is satisfied on $\underline{\theta}\uparrow$.

Proof. Since the algebra has *CIP, classes of the congruence induced by n_a have bottom elements. The rest of the proof is a direct consequence of Theorem 2. □

Corollary 2. *If algebra $\mathcal{A}$ has CEP and *CIP, and $\theta \in Con\mathcal{A}$ is a congruence which satisfies $Cw\mathcal{A}/\theta \cong \underline{\theta}\uparrow$, then the factor algebra $\mathcal{A}/\theta$ has CEP and *CIP as well.*

Proof. If $Cw\mathcal{A}/\theta \cong \underline{\theta}\uparrow$, under the isomorphism f, then $f(\Delta) = \theta$. Instead of CEP and *CIP in $Cw\mathcal{A}/\theta$ we consider identities of the type

$$f(\theta, x_2, ..., x_\alpha, ...) = g(\theta, x_2, ..., x_\alpha, ...)$$

in the lattice $\underline{\theta}\uparrow$, which are satisfied by Proposition 8. □

Remark. Conditions under which $Cw\mathcal{A}/\theta \cong \underline{\theta}\uparrow$, is satisfied have been given in [7].

Theorem 7. *Let algebra $\mathcal{A}$ has *CIP and $\theta \in Con\mathcal{A}$. If*

$$\bigwedge_{i\in I}(\theta \vee f_1^i(x_1, x_2, ..., x_\alpha, ...)) = \bigwedge_{j\in J}(\theta \vee f_2^j(x_1, x_2, ..., x_\alpha, ...)),$$

is satisfied on $Con\mathcal{A}$, then it is also satisfied in the lattice $Cw\mathcal{A}$.

Proof. This is a direct consequence of Theorem 3. □

Corollary 3. *If an algebra $\mathcal{A}$ has *CIP, θ is an infinitely distributive element of $Con\mathcal{A}$ and $Cw\mathcal{A}/\theta$ is a complete sublattice of $Cw\mathcal{A}$, then the factor algebra $\mathcal{A}/\theta$ has *CIP as well.*

Proof. Since θ corresponds to the diagonal element in $Cw\mathcal{A}/\theta$ and $Cw\mathcal{A}/\theta$ is a complete sublattice of $Cw\mathcal{A}$, and since θ is infinitely distributive element in $Cw\mathcal{A}$, by Theorem 7, $\mathcal{A}/\theta$ has *CIP. □

Theorem 8. *Let $\mathcal{B} \in Sub\mathcal{A}$ for an algebra $\mathcal{A}$. If the identity*

$$\bigvee_{i\in I}(\mathcal{B} \wedge f_1^i(x_1, x_2, ..., x_\alpha, ...)) = \bigvee_{j\in J}(\mathcal{B} \wedge f_2^j(x_1, x_2, ..., x_\alpha, ...)),$$

is satisfied on $Sub\mathcal{A}$ then it is also satisfied on the lattice $Cw\mathcal{A}$.

Proof. This is a direct consequence of Theorem 4. □

Theorem 9. *Let $\mathcal{A}$ be an algebra satisfying CEP and *CIP. If for $\theta \in Con\mathcal{A}$ $f_1(\theta, x_2, ..., x_\alpha, ...) = f_2(\theta, x_2, ..., x_\alpha, ...)$ is satisfied on $Con\mathcal{A}$, and $f_1(\Delta, x_2, ..., x_\alpha, ...) = f_2(\Delta, x_2, ..., x_\alpha, ...)$ is satisfied on $Sub\mathcal{A}$, then*

$$f_1(\theta, x_2, ..., x_\alpha, ...) = f_2(\theta, x_2, ..., x_\alpha, ...)$$

is satisfied on $Cw\mathcal{A}$.

Proof. This is a direct consequence of Theorem 5. □

Theorem 10. *Let $\mathcal{A}$ be an algebra satisfying CEP and *CIP. If for $\mathcal{B} \in Sub\mathcal{A}$, $f_1(\mathcal{B}, x_2, ..., x_\alpha, ...) = f_2(\mathcal{B}, x_2, ..., x_\alpha, ...)$ is satisfied on $Sub\mathcal{A}$, and $f_1(\Delta, x_2, ..., x_\alpha, ...) = f_2(\Delta, x_2, ..., x_\alpha, ...)$ is satisfied on $Con\mathcal{A}$, then*

$$f_1(\mathcal{B}, x_2, ..., x_\alpha, ...) = f_2(\mathcal{B}, x_2, ..., x_\alpha, ...)$$

is satisfied on $Cw\mathcal{A}$.

Proof. Direct consequence of Theorem 6. □

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