Univ. u Novom Sadu
Zb. Rad. Prirod.–Mat. Fak.
Ser. Mat. 25, 1 (1995), 99-100

Review of Research
Faculty of Science
Mathematics Series

A THEOREM ON RESIDUAL IDEAL OF A COMMUTATIVE RING WITH APARTNESS

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Abstract

In this short note we present a theorem on residual ideal of a commutative ring with apartness in constructive mathematics.

AMS Mathematics Subject Classification (1991): 13A15.
Key words and phrases: Constructive mathematics, coideal of ring, residual ideal of ring.

Let $(A, =, \neq, +, \cdot)$ be a commutative with an apartness in the sense of books [1] and [3] and of the paper [2], in constructive mathematics ([1], [3]). A subset S of the ring A is called a coideal of A if and only if S is strongly extensional and hold

$$\neg(0 \in S), -b \in S \Rightarrow b \in S$$
$$a + b \in S \Rightarrow a \in S \vee b \in S,$$
$$ab \in S \Rightarrow a \in S \wedge b \in S.$$

If a is an arbitrary element of A we write $a\#S$ if and only if $(\forall s \in S)(s \neq a)$. In the paper [2] we proved that the set $\overline{S} \equiv \{a \in A : a\#S\}$ is an ideal of A. Let J and P be an ideal and a coideal of A. Then the set $((P : J)) \equiv \{b \in A : (\exists x \in J)(bx \in P)\}$ is a coideal of A. This paper gives the proof that the ideals $\overline{((P : J))}$ and $(\overline{P} : J)$ are equal.

Theorem 1. *Let J and S be an ideal and a coideal of a commutative ring $(A, =, \neq, +, \cdot)$ with an apartness. Then* $\overline{((S:J))} = (\overline{S}:J)$.

Proof.

(i) Let a be an arbitrary element of $(\overline{S}:J)$. Then $aJ \subseteq \overline{S}$, i. e. $(\forall x \in J)(ax\#S)$. Let u be an arbitrary element of $((S:J))$, i. e. suppose that there exists an element x_u of $J \cap S$ such that $ux_u \in S$. So, $ux_u \neq ax$ $(x \in J)$, and specially $ux_u \neq ax_u$. Thus $u \neq a$. Therefore $a \in \overline{((S:J))}$. So, $(\overline{S}:J) \subseteq \overline{((S:J))}$.

(ii) Let a be an arbitrary element of $\overline{((S:J))}$, i. e. let be $a\#((S:J))$. Let u be an arbitrary element of S. By strongly extrasionality of S, we have $u \neq ax \vee ax \in S$ where $x \in J$. Suppose that there exists an element $x \in J$ such that $ax \in S$. Then we have $a \in ((S:J))$. It is impossible. So, for every x in J is $u \neq ax$, i. e. $ax\#S$. Thus $aJ \subseteq \overline{S}$. Therefore $\overline{((S:J))} \subseteq (\overline{S}:J)$.

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Received by the editors October 3, 1990.