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AN ATTEMPT FOR CONSTRUCTION OF A TRIPLE OF PAIRWISE MUTUALLY ORTHOGONAL LATIN SQUARES ON 10 ELEMENTS

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Abstract

Abstract. A well-motivated attempt is made to construct a triple of mutually dual latin squares of order 10 by using resolvable Mendelsohn triple systems. Two different representations of resolvable Mendelsohn triple systems are considered. The first one is analogous to the representation used for constructing pairs of mutually dual latin squares of order 10 in [3] and [4].

Our conclusion is that the approach used here does not give the required triple.

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1. Introduction

The problem ([2]) of existence of a triple of pairwise mutually orthogonal latin squares of order 10 is still open, although numerous attempts have been made during the past decades. For example, Parker ([6]) has constructed a latin square of order 10 with 5504 transversals and with about one million orthogonal mates (sets of 10 disjoint transversals). However, he was also able to prove that no two of these mates are mutually orthogonal.

On the other hand, Keedwell ([5]) has constructed a triple of (96,94,90)-orthogonal[1] latin squares of order 10, which additionally have the same column sets.

Given a resolvable MTS ([3], [4]) M, – it can be easily shown that there exist two mutually orthogonal latin squares of order 10, which are naturally associated to M and M^2.

This paper presents a well-motivated (in particular see Theorem 2., Section 3 and Lemma 2., Section 4) attempt for construction of a triple of pairwise mutually orthogonal latin squares of order 10. The construction uses two different representations of resolvable MTS M of order 10, which map these MTS to column latin squares and row latin squares respectively.

2. Preliminaries

Let S denote the set $\{0,1,2,...,9\}$.

Permutations of the set S, that is, permutations of order 10, are represented by means of their cycles.

A *Mendelsohn triple system* (shortly: MTS) of order 10 is a pair (S,T), where T is a collection of 30 cyclically ordered 3-subsets of S, such that each ordered pair of S appears in exactly one cyclic triple of T.

Let $C-$*sequence* denote a sequence $(p_0, p_1, ..., p_9)$ consisting of ten permutations $p_0, p_1, \dots, p_9$ of S, which satisfies:

- $p_i(i) = i \quad (0 \le i \le 9)$,
- $p_i(j) \neq j \quad (\text{for } 0 \le i,j \le 9,\ i \neq j)$.

Each $C-$sequence C can be viewed as a *row latin square* L; each permutation of C is placed into a distinct row of L.

An MTS of order 10 is called *resolvable* if its 30 cyclic triples can be partitioned into ten sets consisting of three pairwise disjoint cyclic triples

[1] This denotation is taken from [1] .

each. A resolvable MTS can be represented by a $C-$sequence $C = (p_0,\ p_1, ..., p_9)$, such that each permutation p_d $(0 \leq d \leq 9)$ of C has the form

$$(a_1\ a_2\ a_3)\ (b_1\ b_2\ b_3)\ (c_1\ c_2\ c_3)(d),$$

where the cyclic triples $(a_1\ a_2\ a_3)$, $(b_1\ b_2\ b_3)$ and $(c_1\ c_2\ c_3)$ belong to one of the ten sets, while d is the fixed element.

The *$n-$th degree* M^n and the *cubic root* $\sqrt[3]{M}$ of a resolvable MTS M are represented by the $C-$sequences, which are obtained by applying the corresponding operations to distinct permutations of the $C-$sequence associated to M.

In the next two sections two representations of $C-$sequences, R_1 and R_2, will be introduced.

If $(p_0\ , ..., \ p_9)$ denotes a $C-$sequence associated to a resolvable MTS M, then the denotations $R_1(p_0, ..., p_9)$ and $R_2(p_0, ..., p_9)$ may be replaced by $R_1(M)$ and $R_2(M)$ respectively.

Similarly, let $(q_0\ , ..., \ q_9)$ and $(q_0^2, ..., q_9^2)$ denote the $C-$sequences associated to $\sqrt[3]{M}$ and $\sqrt[3]{M^2}$ respectively. Then the denotations $R_1(q_0, ..., q_9)$ and $R_1(q_0^2, ..., q_9^2)$ (also $R_2(q_0, ..., q_9)$ and $R_2(q_0^2, ..., q_9^2)$) may be replaced by $R_1(\sqrt[3]{M})$ and $R_1(\sqrt[3]{M^2})$ ($R_2(\sqrt[3]{M})$ and $R_2(\sqrt[3]{M^2})$) respectively.

Both proposed representations were tested on the collection of 135 non-equivalent resolvable MTS's of order 10 (21 without repeated blocks and 114 with repeated blocks) [3, 4]. The (mainly negative) results of these testing are descibed in Section 5.

3. Representation 1 (18 possibilities for a row of a third root of M)

Let $C = (p_0, ... , p_9)$ denote a $C-$sequence. Then the representation $R_1(p_0, ..., p_9) = A$ is defined by

$$A[i,j] \stackrel{def}{=} p_j\ (i) \quad \text{for} \quad 0 \leq i,j \leq 9\ ,$$

where $p_j\ (i)$ denotes the image of element i under permutation p_j .

Consequence. $R_1(p_0, ..., p_9)$ is a *column latin square.*

This representation was used in papers [3] and [4].

If $(p_{i_1},...,p_{i_l})$ is a subsequence of a a C-sequence, then $R_1(p_{i_1},...,p_{i_l})$ is defined in an analogous way.

We proceed with some elementary properties of Representation 1:

Property 1. ([3],[4]) *Given a resolvable MTS M, the representations $R_1(M)$ and $R_1(M^2)$ are latin squares.*

The proofs follow from the definitions of MTS and resolvability. □

Property 2. ([3],[4]) *Given a resolvable MTS M, the latin squares $R_1(M)$ and $R_1(M^2)$ are mutually orthogonal.*

Proof. Each permutation p_j associated with M gives ten distinct pairs of the form $(p_j(i), p_j^2(i))$ $(0 \leq i,j \leq 9)$. By definition of MTS, all the hundred pairs obtained in this manner are distinct.

The proof is completed by observing that elements $p_j(i)$ and $p_j^2(i)$ are on the positions (i,j) of the latin squares $R_1(M)$ and $R_1(M^2)$ respectively. □

Let p denote one of the ten permutations of the C-sequence associated to a resolvable MTS and let q be a cubic root of p. There are exactly 18 possibilities for q:

Theorem 1. *There exist 18 distinct cubic roots of a permutation p of the form*

$$p = (a_1\ a_2\ a_3)\ (b_1\ b_2\ b_3)\ (c_1\ c_2\ c_3)(d).$$

Proof. Each third root q of p can be represented as a permutation of the form

$$q = (a_1\ x_1\ y_1\ a_2\ x_2\ y_2\ a_3\ x_3\ y_3)(d),$$

where $x_2 = p(x_1)$, $x_3 = p(x_2)$, $y_2 = p(y_1)$, $y_3 = p(y_2)$. There are six possible choices for x_1, as well as three additional choices for y_1 in each one of the six cases. □

Remark. Note that there are merely six mutually independent third roots among the eighteen; if q is any third root of a permutation p of the form mentioned in Theorem 1, then q^4 and q^7 are another two third roots of p, while q^2, q^5 and q^8 are some three third roots of p^2.

The next theorem (Theorem 2., which is preceded by a lemma) was the main source of motivation for considering Representation 1:

Lemma 1. *Given a $C-$sequence $(q_0, ..., q_9)$, a necessary and sufficient condition that $R_1(q_0, ..., q_9)$ is a latin square is that there do not exist some four integers*
j_1, j_2, k, l in S, so that $q_{j_1}(k) = l$ and $q_{j_2}(k) = l$.

Proof. If the integers j_1, j_2, k, l do exist, then the element l is placed both on the positions (k, j_1) and (k, j_2). □

Theorem 2. *Given a resolvable MTS M of order 10, suppose that $R_1(\sqrt[3]{M})$ and $R_1(\sqrt[3]{M^2})$ are latin squares* [2]. *Then the latin squares $R_1(\sqrt[3]{M})$, $R_1(\sqrt[3]{M^2})$ and $R_1(M)$ constitute a triple of mutually orthogonal latin squares.*

Proof. The proof of the theorem is based on the following three statements:

1. If $R_1(\sqrt[3]{M})$ and $R_1(\sqrt[3]{M^2})$ are not orthogonal, then $R_1(\sqrt[3]{M})$ is not a latin square.
2. If $R_1(\sqrt[3]{M})$ and $R_1(M)$ are not orthogonal, then $R_1(\sqrt[3]{M^2})$ is not a latin square.
3. If $R_1(\sqrt[3]{M^2})$ and $R_1(M)$ are not orthogonal, then $R_1(\sqrt[3]{M})$ is not a latin square.

We shall prove in detail merely the first one of these statements, the proofs of the other two being quite analogous:

Denote $R_1(\sqrt[3]{M})$ and $R_1(\sqrt[3]{M^2})$ by X and Y respectively. Suppose that the latin squares X and Y are not orthogonal. Then there exist two pairs of indices (i_1, j_1) and (i_2, j_2), $0 \leq i_1, i_2, j_1, j_2 \leq 9$, such that $X(i_1, j_1) = X(i_2, j_2) = k$ and $Y(i_1, j_1) = Y(i_2, j_2) = l$ for some k and l.

Consider the permutations q_{j_1} and q_{j_2} of the $C-$sequence $(q_0, ..., q_9)$, associated to $\sqrt[3]{M}$. Since $q_{j_1}(i_1) = k$ and $q^2_{j_1}(i_1) = l$, it follows that

[2] Note that the use of denotations $R_1(\sqrt[3]{M})$ and $R_1(\sqrt[3]{M^2})$ takes into account the existence of the underlying $C-$sequences $(q_0, ..., q_9)$ and $(q_0^2, ..., q_9^2)$.

$q_{j_1}(k) = l$. It holds similarly that $q_{j_2}(k) = l$. It follows that $R_1(\sqrt[3]{M})$ is not a latin square; the condition of Lemma 1 is violated. □

The following statement shows that the second assumption of Theorem 2. is independent of the first one.

Statement 1. *If* $R_1(q_0, ..., q_9)$ *is a latin square, then* $R_1(q_0^2, ..., q_9^2)$ *need not be a latin square.*

Proof. Consider the latin square $R_1(p_0, ..., p_9)$, which represents the resolvable MTS 2.2.1. [3]. It has $p_0 = (123)(485)(697)(0)$ and $p_6 = (132)(409)(587)(6)$.

Let q_0 and q_6 denote the following two third roots of p_0 and p_6 respectively:
$q_0 = (146289357)(0)$ and $q_6 = (184370259)(6)$.

The permutations q_0 and q_6 determine the latin rectangle $R_1(q_0, q_6)$. They do not contradict the possible assumption that $R_1(q_0, ..., q_9)$ is a latin square.

However, $R_1(q_0^2, q_6^2)$ is not a latin rectangle. The representation $A = R_1(q_0^2, ..., q_9^2)$ satisfies that $A[2,0] = A[2,6] = 9$ (also $A[5,0] = A[5,6] = 1$ and $A[8,0] = A[8,6] = 3$) and it consequently is not a latin square. □

The main part of our construction was an attempt to associate latin squares to the configurations $R_1(q_0, ..., q_9)$ and $R_1(q_0^2, ..., q_9^2)$, where the C-sequence $(q_0, ..., q_9)$ corresponds to one of the possible third roots of M. In the case of success of such an attempt, Theorem 2 would give the existence of the required triple of pairwise mutually orthogonal latin squares.

However, it turns out that the method does not give the required triple. Namely, the computer search has shown:

Statement 2. *There does not exist a third root of a resolvable MTS, represented by a C-sequence* $(q_0, ..., q_9)$, *so that* $R_1(q_0, ..., q_9)$ *and* $R_1(q_0^2, ..., q_9^2)$ *are latin squares.*

Moreover, the computer search has additionally shown that the following stronger statement is also true:

Statement 3. *Let* $(q_0, ..., q_9)$ *be the C-sequence, which corresponds to any one of the* 18^{10} *third roots of* M, *where* M *is an arbitrary one of the 135 non-equivalent resolvable MTS. Then* $R_1(q_0, ..., q_9)$ *is not a latin square.*

4. Representation 2 (162 possibilities for a row of a third root of M)

Let $(p_0, \dots , p_9)$ denote a C-sequence. Then the representation $R_2(p_0, ..., p_9) = A$ is defined by

$$A[i,j] \stackrel{def}{=} p_i\,[j] \quad \text{for} \quad 0 \leq i,j \leq 9\,,$$

where $p_i\,[j]$ denotes the $j-$th component of the vector obtained by removing brackets from the cycle denotation of the permutation p_i .

Consequence. $R_2(p_0, ..., p_9)$ is a *row latin square.*

Similarly as R_1, the denotation R_2 may also be applied to the subsequences of a C-sequence.

Specially, the cycle denotation of a permutation p^2 is assumed to have the cycles with the same *first* elements as the cycle denotation of the permutation p.

Remark. Given a resolvable MTS M, there are 162 different possibilities for each row of $R_2(\sqrt[3]{M})$. Namely, there are 9 different possibilities for a row associated to each one of the 18 third roots of a permutation in the C-sequence associated to M. In addition, it is required that the only fixed element of each one of the ten permutations is placed in the same column (say, in the tenth one).

Example. Let $(a_1\ a_2\ a_3)\ (b_1\ b_2\ b_3)\ (c_1\ c_2\ c_3)(d)$ be the cycle denotation of the permutation p_d of the C-sequence associated to a resolvable MTS M. Further, let $q = (a_1\ b_1\ c_1\ a_2\ b_2\ c_2\ a_3\ b_3\ c_3)(d)$ be the cycle denotation of some third root of the permutation p_d. Then the $d-th$ rows of the representations $R_2(q_0, ..., q_9)$ and $R_2(q_0^2, ..., q_9^2)$ are equal to $a_1\ b_1\ c_1\ a_2\ b_2\ c_2\ a_3\ b_3\ c_3\ d$ and $a_1\ c_1\ b_2\ a_3\ c_3\ b_1\ a_2\ c_2\ b_3\ d$ respectively.

It is quite obvious with this representation that:

Lemma 2. *If* $R_2\ (q_0, ..., q_9)$ *is a latin square, then both* $R_2\ (q_0^2, ..., q_9^2)$ *and* $R_2\ (q_0^3, ..., q_9^3)$ *are latin squares.*

Proof. The last two representations are obtained by permuting the columns of the first one. □

Moreover, the neighbouring columns of the first latin square are on distance 2 with the second and on distance 3 with the third one.

The following lemma implies that the statement analogous to Theorem 2 is not possible with Representation 2.

Lemma 3. *If* $R_2\ (q_0, ..., q_9)$ *and* $R_2\ (q_0^2, ..., q_9^2)$ *are latin squares, then they are not orthogonal.*

Proof. The latin squares $R_2\ (q_0, ..., q_9)$ and $R_2\ (q_0^2, ..., q_9^2)$ have by construction the same both the first and the tenth columns. This implies that the pair (i, i) , for $0 \leq i \leq 9$, can be found twice in this pair of latin squares. □

Remark. The statement of Lemma 3 would be true even if the condition that the corresponding cycles of the cycle denotations of the permutations p and p^2 have the same first elements – would be abandoned. Namely, it can be checked that the vector $(a_1,\ ...\ , a_9)$ has one common component with each one of the nine vectors: $(a_1,\ a_3,\ a_5,\ a_7,\ a_9,\ a_2,\ a_4,\ a_6,\ a_8\ )$, $(a_2,\ a_4,\ a_6,\ a_8,\ a_1,\ a_3,\ a_5,\ a_7,\ a_9\ )$, $(a_3,\ a_5,\ a_7,\ a_9,\ a_2,\ a_4,\ a_6,\ a_8,\ a_1\ )$, ... , $(a_9,\ a_2,\ a_4,\ a_6,\ a_8,\ a_1,\ a_3,\ a_5,\ a_7\ )$.

These nine vectors denote different possibilities for some row of of the latin square $R_2\ (q_0^2, ..., q_9^2)$. It follows that some pair (i, i) appears both in the first nine and in the tenth columns of the latin squares $R_2\ (q_0, ..., q_9)$ and $R_2\ (q_0^2, ..., q_9^2)$. □

The computer search has shown that Representation 2 would not help finding the required triple of mutually orthogonal latin squares even if third roots of M and M^2 would be chosen independently of each other:

Statement 4. *Let* $(r_0, ..., r_9)$ *be any of the* 162^{10} *possible* 10×10 *matrices, which are obtained in the following manner:*

The row r_i, $0 \le i \le 9$, *is one of the nine possible cycle denotations of the permutation* q_i, $0 \le i \le 9$, *where* $(q_0, ..., q_9)$ *is the C-sequence, which corresponds to any of the* 18^{10} *third roots of an arbitrary one of the 135 non-equivalent resolvable MTS.*

Then $(r_0, ..., r_9) = R_2(q_0, ..., q_9)$ *is not a latin square.*

4.1. Representation 2a (324 possibilities for a row of a third root of M)

A variation of Representation 2. would be to use *movable* cycles.

If we keep the previous representation (by means of cycles) and do NOT require that all the fixed elements belong to the same column, then the number of possibilities increases.

There are two possible positions (the first and the last one) for the fixed element beside a cycle of length 9. It follows that total number of possibilities for a row (of the matrix representation of $\sqrt[3]{M}$ or $\sqrt[3]{M^2}$) – is equal to $162 \cdot 2 = 324$.

Given a permutation of the form $(a_1\ a_2\ a_3)\ (b_1\ b_2\ b_3)\ (c_1\ c_2\ c_3)(d)$ (of the matrix representation of M), there are even $4! \cdot 3^3 = 648$ possibilities to write down their cycles.

However, it is even not clear to us that there necessarily exists a possibility (out of 648^{10}) which gives a latin square associated to M.

5. Algorithms and computational results

In this section the algorithms for testing representations R_1 and R_2 are sketched and the related computational results are cited. In particular, the derivations of Statements 2. and 3. (related to Representation 1.) are described in the subsection 5.1., while the derivation of Statement 4. (related to Representation 2.) is described in the subsection 5.2.

The algorithms for checking the three statements are similar: all of them use backtracking on rows of the input MTS. The total running time of the three algorithms on a PC-AT was about two hours. This running time is negligible in comparision with the exponential number of possibilities to be checked; this is the consequence of strong cutting during backtracking.

The input MTS's were chosen out of the collection of 135 non-equivalent resolvable MTS's of order 10. Each resolvable MTS M is represented as a C-sequence $(p_0, ..., p_9)$.

5.1. Testing R_1

When the representation R_1 is considered, third roots of M are also represented as C-sequences $(q_0, ..., q_9)$, where q_i is one of the 18 third roots of p_i, $0 \leq i \leq 9$.

A 3-fold sequence $seq[i][num][j]$ is used for handling these third roots, where:
i = index of the row (permutation) p_i within MTS $0 \leq i \leq 9$,
num = index of the third root, $1 \leq num \leq 18$,
j = index of the field of the permutation associated to the num–th third root of p_i $1 \leq j \leq 9$.

In accordance with Theorem 2, an attempt was made to construct two latin squares, which are equal to $R_1(\sqrt[3]{M})$ and $R_1(\sqrt[3]{M^2})$ respectively, for some of 18^{10} possible third roots of a given MTS M. This is performed by expanding the latin rectangles R_1 $(q_0, ..., q_{k-1})$ and R_1 $(q_0^2, ..., q_{k-1}^2)$ of size $10 \times k$, which are associated to the first k permutations $p_0, ..., p_k$ of M.

Two auxiliary sequences $s_0^d, ..., s_9^d$ $(1 \leq d \leq 2)$ of sets are used during backtracking. The element x belongs to s_i^d $(0 \leq i \leq 9,\ 1 \leq d \leq 2)$ iff there exists a third root q_j $(0 \leq j \leq k-1)$, so that $q_j^d\ (i) = x$. It is easy to observe that the set s_i^d is the set of currently present elements in the i–th row of R_1 $(q_0^d, ..., q_{k-1}^d)$.

A third root q_k of the permutation p_k is being searched among all the 18 candidates, so that $q_k\ (i) \notin s_i^1 \cup s_i^2$, for $0 \leq i \leq 9$. Namely, the $i-th$ row of some of the rectangles R_1 $(q_0, ..., q_k)$ and R_1 $(q_0^2, ..., q_k^2)$ would in the opposite case contain two same elements and the condition for the latin rectangle would be violated.

If the search for a "feasible" third root q_k is successful, then the elements $q_k^d(i)$ are added to the sets s_i^d, for each i, $0 \leq i \leq 9$ and for $d = 1, 2$, while k is replaced by $k+1$ (step forwards).

In the case of an unsuccessful search for a "feasible" q_k , the elements $q^d_{k-1}(i)$ are subtracted from the sets s^d_i, for each i, $0 \leq i \leq 9$ and for $d = 1, 2$, while k is replaced by $k - 1$ (step backwards). A *new* candidate for q_{k-1} should be further searched for (which is lexicographically greater than the current third root of p_{k-1}).

This attempt of construction has finished in failure (Statement 2.); the final value of the parameter k was 0.

The maximum value of k which was reached with $R_1(\sqrt[3]{M})$ and $R_1(\sqrt[3]{M^2})$ was equal to 5, while the corresponding maximum number of backtracking iterations was equal to 217. These values were reached with two resolvable MTS's: 3.10.b.2 and 3.10.b.6 ([4]).

One might observe that the above described attempt of construction is not complete, since the third roots of the permutations p_k and p^2_k have not been chosen independently. In fact, all the considered pairs of third roots were of the special form (q_k , q^2_k), which reduced the total number of possibilities from $(18^2)^{10}$ to 18^{10} . Such an approach also enabled us to avoid using another 3-fold sequence, which would be used for handling third roots of M^2.

It turns out, however, that the complete search, which would use mutually independent third roots of p_k and p^2_k , would also finish in failure (Statement 3). Namely, there does not exist some resolvable MTS M (among the 135 non-equivalent possibilities), such that $R_1(\sqrt[3]{M})$ is a latin square. Such a conclusion is derived by applying backtracking similar to the above one, merely the sets s^2_i , $0 \leq i \leq 9$ should be omitted.

The maximum value of k which was reached with a separate consideration of $R_1(\sqrt[3]{M})$ was equal to 8, while the corresponding maximum number of backtracking iterations was equal to 5707. This value was reached with the resolvable MTS 2.26.3 ([4]).

5.2. Testing R_2

The attempt of the construction with $R_2(\sqrt[3]{M})$ was also unsuccessful. The 3-fold sequence $seq[i][num][j]$ was replaced by the 4-fold sequence $seq[i][num][rot][j]$, where the additional index rot took values between 1 and 9 inclusively. This index was used to denote nine different possibilities (cyclic permutations) for the row associated with a third root.

Let p_k $(0 \leq k \leq 9)$ again denote a permutation belonging to the C-sequence representing M. Each third root q_k of p_k is now chosen out of 162 possibilities (18 possibilities for *num* times 9 possibilities for *rot*). The latin rectangle R_2 $(q_0, ..., q_{k-1})$ is now of size $k \times 10$.

If the pairs of the form $(q_k,\ q_k^2)$, $0 \leq k \leq 9$ are used, then Lemma 2. gives that there is no need for two auxiliary sequences related to $R_2(\sqrt[3]{M})$ and $R_2(\sqrt[3]{M^2})$ reaspectively (the first one is sufficient). However, Lemma 3 implies that the pairs of the form $(q_k,\ q_k^2)$ cannot constitute two orthogonal latin squares obtained by Representation 2.

The possibility of using independent third roots of p_k and p_k^2 should also be taken into account. This possibility was eliminated by using backtracking with auxiliary sets s_i, $0 \leq i \leq 9$. The element x belongs to s_i $(0 \leq i \leq 9)$ iff there exists a third root q_j $(0 \leq j \leq k-1)$, so that i–th field of the permutation q_j is equal to x (the set s_i is now the set of currently present elements in the i–th *column* of latin rectangle of size $k \times 10$ R_2 $(q_0, ..., q_{k-1})$).

The step backwards in this backtracking returns to a new third root q_{k-1}, which is determined by the lexicographically next pair (num, rot).

Such a complete search has showed that there does not exist a resolvable MTS M , such that $R_2(\sqrt[3]{M})$ is a latin square (Statement 4).

The maximum value of k which was reached with $R_2(\sqrt[3]{M})$ was equal to 8, while the corresponding maximum number of backtracking iterations was equal to 1135. This value was reached with the resolvable MTS 2.27.c.2 ([4]).

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