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A CAGD METHOD FOR ELLIPTIC CROSS SECTIONS OF CIRCULAR PARABOLIC QUADRICS

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Abstract

A Computer Aided Geometric Design (CAGD) method is developed for obtaining an elliptic cross-section between a conus and a plane, and between a cylinder and a plane. The method is based on procedures of constructive geometry. The process of drawing the bodies and planes is interpreted using analytic geometry. The mathematical relations are implemented in a Turbo Pascal program.

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1. Introduction

The problem of constructing the intersection between a conus and a plane, and also a cylinder and a plane has been well known for a long time. It has been successfully solved by the methods of constructive geometry which make possible to show the three-dimensional body in two-dimensional plane as, well as the analogous problem regarding a cross-section in a plane. Recent investigations show that the problem can be solved much more correctly and faster by computer than by drawing with ruler and callipers. Many methods of Computer Aided Graphic Design (CAGD) for obtaining various parabolic quadrics have been developed and grouped as in [1]. The body can be designed by computer in a lot of manners. The most frequently applied method is based on forming Bezier surfaces [2]. They are generated by one-parameter curves and lines. The advantage of the method is that it gives a computer program in a very simple way, because the parameters are directly applied to plotting. But, the disadvantage of the mentioned procedure is that the circle can not be plotted correctly (see [3]). The method of contour is based on the rules of differential geometry as considered in [4]. Eisenhart [5] shows that this procedure is very complex and it requires many long-time computations for each point and it slows down the plotting. In addition, a serious disadvantage is the fact that the contour line of rotating bodies is not obtained, as it can be seen in Fig. 1. So, the circular form of the body can not be seen in the picture.

A new method for plotting bodies and cross sections graphical design is introduced in this paper. The design process of the projection is mathematically described by analytical geometry, and the corresponding computer program is formed. The affine and polar properties of quadrics are applied. Two examples are presented: the cross-section of a cone and a plane, and a cross-section of a cylinder and a plane.

2. Working Principle

A coordinate system Oxy_kz is introduced with the origin O_x and O_y, according to the coordinate system of screen Ex_cy_c of the computer screen. The rotating body is plotted as suggested in paper [6]. Using the affine relation between the projections the both orthogonal projections of conus and cylinder are plotted. The points of the body are constructed. For the given angle γ and shortening $y_k : y = Y_{sk} : Y_s$ (see Fig. 2) the coordinates of the point A are

$$y_{Ak} = y_A \frac{Y_{sk}}{Y_s}, \tag{1}$$

$$x_{A1} = x_A - y_{Ak} \cos\gamma, \tag{2}$$

$$y_{A1} = y_{Ak} \sin\gamma. \tag{3}$$

The equations (1)-(3) lead to affine transformation of projections. Transforming the three-dimensional system into a two-dimensional system, the point A_K is given by the coordinates x_{A1} (eq.(2)) and z_{A1} where

$$z_{A1} = z_A - y_{A1}. \tag{4}$$

Eqs. (2) and (4) define all the points in the figures. The intersecting plane is parallel to the horizontal plane Oxy_k. The intersection between this plane and frontal plane is along a line which is parallel to $x-$axis, and the intersection with the profile plane is along a line parallel to y or y_k axis. These two lines (one parallel to y_k and other to x) define the plane in projection.

The intersection between the body and the plane is formed basing on the first and second orthogonal projections. Using the affine relations (1)-(3) the intersection is plotted. The correctness of the plotting increases with increasing the number of points on the intersection curve. To satisfy this requirement, a point is considered at each degree on the basis of the body. These points form an intersecting curve. It is necessary to show how to obtain the coordinates of one point of the intersection.

3. Coordinates of the point on the intersection curve

3.1. Intersection of a conus and a plane

Let the orthogonal projections of the parabolic quadrics and the positions of plane in first and second projections be known. The plane is given with the coordinates $\alpha(\alpha_1, \infty, \alpha_3)$. The basis is shown in its real dimensions in the first orthogonal projection and is divided into 360 equal parts. A side line is drawn to the end part on the basis. The intersection between the side line and the plane is obtained in the second projection. Mathematically, it represents an intersection of two lines: one corresponding to the side line and the other corresponding to the projection of the plane. The equation of the side line in the second projection is

$$y = V_Z \frac{x - X_{BK}}{V_Z - X_{BK}}, \tag{5}$$

while the equation of projection of plane is

$$y = -x\frac{\alpha_3}{\alpha_1} + \alpha_3, \tag{6}$$

where $(X_{BK}, Y_{BK}, 0)$ are the coordinates of a point of the side line and (V_X, V_Y, V_Z) are the coordinates of the top of the conus. The coordinates of the intersection point $P_2(X_1, Z_1)$ in the second projection are

$$X_1 = \frac{\alpha_3 + AX_{BK}}{A + \frac{\alpha_3}{\alpha_1}}, \tag{7}$$

$$Z_1 = A(X_1 - X_{BK}), \tag{8}$$

where $A = V_Z/(V_X - X_{BK})$.

To obtain the coordinates of the intersection point in the first projection a line parallel to $y-$axis in X_1 is settled, and its section with the corresponding side line in the first projection is found. The coordinates of the intersection point $P_1(X_1, Y_1)$ in the first projection are X_1 (eq.(6)) and Y_1

$$Y_1 = K(X_1 - X_{BK}) + Y_{BK}, \tag{9}$$

where $K = (V_Y - Y_{BK})/(V_X - X_{BK})$.

Hence, the coordinates of intersection point are $P(X_1, Y_1, Z_1)$.

To show the point in inclined projection, the affine connections (1)-(3) are used. The coordinates of the point $P(X_{PK}, Y_{PK})$ in the coordinate system of the screen are

$$X_{PK} = X_1 - Y_{PK} \cos\gamma, \tag{10}$$

$$Y_{PK} = Y_1 - Y_{PK2}, \tag{11}$$

where

$$Y_{PK} = \frac{Y_1 Y_{SK}}{Y_S}, \tag{12}$$

$$Y_{PK2} = Y_{PK} \sin\gamma. \tag{13}$$

3.2. Intersection of a cylinder and a plane

Repeating the previous procedure, the point on the intersection of a plane and a cylinder in the first inclined projection is obtained.

Let us assume the position of the centre of the low basis to be $C(X_C, Y_C, Z_C)$ and of the top basis $S(X_S, Y_S, Z_S)$. Then, the coordinates of the intersection point $P(X_1, Y_1, Z_1)$ are

$$X_1 = \frac{\alpha_3 + BX_{BK}}{B + \frac{\alpha_3}{\alpha_1}}, \tag{14}$$

$$Y_1 + M(X_1 - X_{BK}) + Y_{BK}, \tag{15}$$

$$Z_1 = B(X_1 - X_{BK}), \tag{16}$$

where $B = Z_S/(X_S - X_c)$ and $M = (Y_S - Y_C)/(X_S - X_C)$.

Using the affine connection, the coordinates of $P(X_{PK1}, Y_{PK1})$ in the screen coordinate system are obtained.

4. Visibility of intersected conus and cylinder

It is necessary to mark the visible and invisible edges of the body that will make the object look like more natural and three-dimensional. The procedure developed in this paper for obtaining visibility of edges is based on the previously obtained data of contour edges. Two points on the basis ellipse from which boundary lines start are applied. The visibility of the basis and of side lines as the beginning points of the contour edges have to be shown.

The visible part of the basis is marked between the point on the zero degree to the first contour point by a thick line, and, after that, with the same type of line from the second contour point to the starting point. The first contour point corresponds to lower values of the angle on the basis ellipse. The invisible part of the basis is marked with a thin line between the first and the second contour points.

Let us denote the visibility of side lines. As the angle position of contour lines are known it is easy to calculate the coordinates of the points on the ellipses which are linked by a line. The visible part of an edge lies between the first i.e. second, contour point to the opposite point on the intersection ellipse. The other part of the contour is invisible. The invisible parts are marked with a thin line.

The intersections of a conus and a plane, and a cylinder and a plane in the inclined projection are plotted in Fig. 3 and Fig. 4, respectively.

5. Conclusion

The proposed method can be used to obtain a very fast and high-quality construction of the elliptic intersection of parabolic quadrics (conus and cylinder) and a plane. The advantage of the method is that it uses algebraic equations for defining the positions of the points instead of complex differential equations as is usually done. The figure plotted by this method is better for higher numbers of points. The procedure for obtaining the points is short and very simple. The method makes possible to separate the visible and invisible lines of the picture that gives a good vision of a three-dimensional body. On comparing Fig. 1 and Fig. 3 (i.e. Fig. 4), the advantage of the suggested method is obvious.

References

[1] Flaquer, J., Garate, G., and Pargada, M., Envelopes of moving quadric surfaces, Computer Aided Geometric Design, Vol.9, No.4 (1992), 299-312.

[2] Lang, J., Roschel, O., Developable (1,n) - Bezier surfaces, Computer Aided Geometric Design, Vol.9, No.4 (1992), 291-298.

[3] Wang, G.J., Wang, G.Z., The rational cubic Bezier representation of conic, Computer Aided Geometric Design, Vol.9, No.6 (1992), 447-455.

[4] Asteasu, C., Intersections of arbitrary surfaces, Computer Aided Design, Vol.20, No.9 (1988), 533-538.

[5] Eisenhart, L.P., A treatise on the differential geometry of curves and surfaces, Dover Publications, New York, NY, USA, 1960, 1-15.

[6] Obradović, R., Štulić, R., Circular parabolic quadratics inCAGD: The polar and affine properties and their application to the contour solution (in Serbian), 18th Yugoslav Meeting of Constructive Geometry, Belgrade 1995., Book of papers, 61-63.

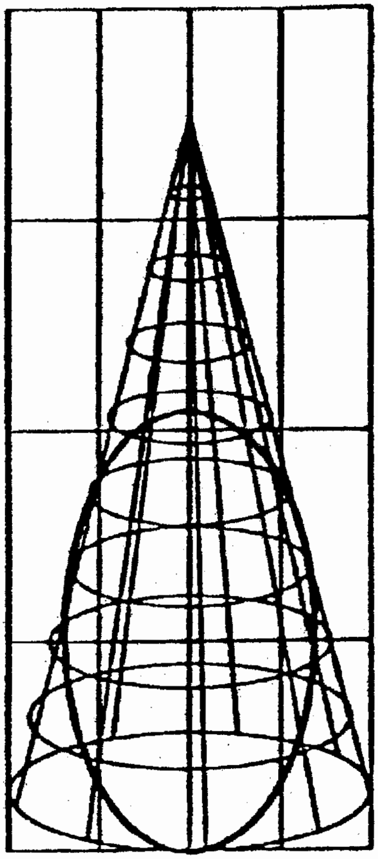

Fig. 1

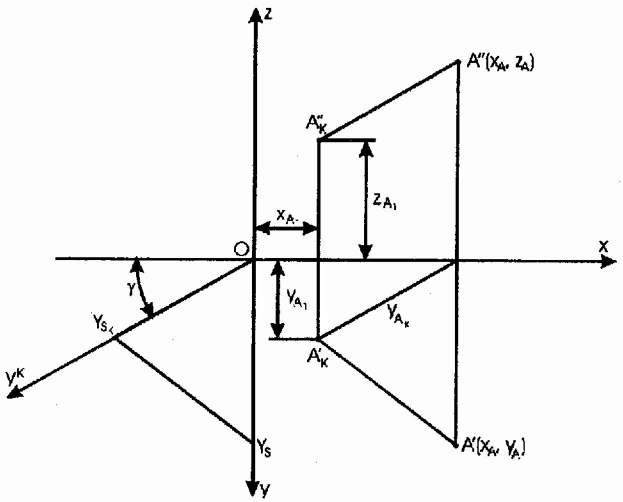

Fig. 2

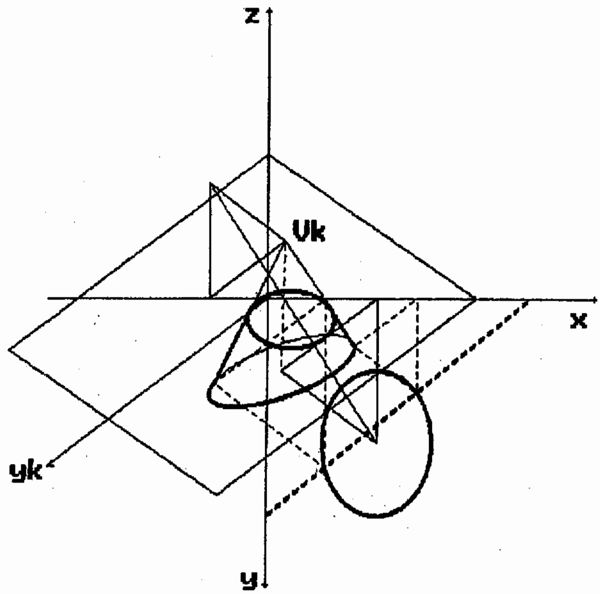

Fig. 3

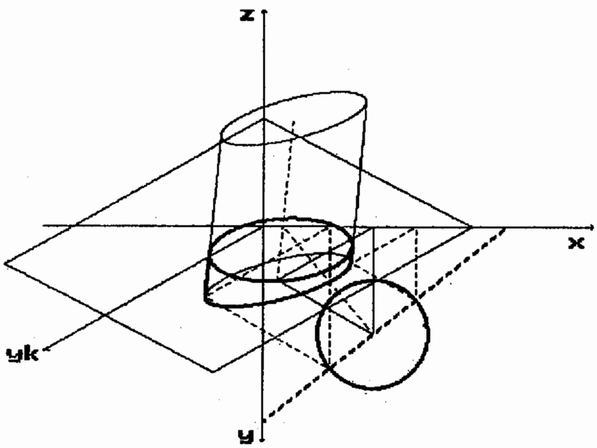

Fig. 4

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