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# POSET VALUED VARIETIES

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**Abstract**

The aim of the paper was to introduce and investigate fuzzy varieties as mappings from a variety to a poset or a lattice.

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## 1. Introduction

Fuzzy algebraic structures can be considered as an application of fuzzy sets in pure mathematics. More precisely, structural analysis of algebras encounters investigation of subalgebras, congruence relations, direct products, and also of partially ordered sets and lattices of these objects. After the introduction of fuzzy substructures, it turned out that these new objects should also be attached to groups, rings, etc. The first concept was the one of fuzzy subgroup, proposed by Rosenfeld [2], who has been followed by many others. Fuzzy classes of algebras, particularly fuzzy varieties, were introduced by Mordeson [1].

The main approach in studying fuzzy structures and corresponding classes is based on fuzzy sets as mappings from an algebra to the unit interval. In the present paper we investigate, more generally, poset valued classes of algebras, particularly poset and lattice valued varieties. The reason for this co-domain is the fact that every fuzzy structure uniquely determines a special partially ordered collection of ordinary substructures. Therefore, to investigate an algebra using its fuzzy substructures, one has to examine all such collections of substructures. In the case of the interval [0,1] as the co-domain of mappings, these collections are chains, which represent only a part of fuzzy substructures.

We give necessary and sufficient conditions under which a function from a variety to a poset determines its fuzzy subvariety. We investigate properties of these poset valued varieties, particularly of the one that we call principal: it maps the variety into its subvariety lattice, associating an algebra with the variety it generates. It turns out that every subvariety of the given variety is the level of the principal (fuzzy) one.

## 2. Preliminaries

If $(P, \leq)$ is a partially ordered and $A$ a nonempty set, then any mapping $\overline{A} : A \to P$ is a **partially ordered fuzzy set ($P$-fuzzy set)** on $A$ (see [4]). A **level, level subset, $p$-cut** of $\overline{A}$, where $p \in P$, is a subset $A_p$ of $A$, such that for $x \in A$, $x \in A_p$ if and only if $\overline{A}(x) \geq p$. The characteristic function of $A_p$ is usually denoted by $\overline{A}_p$, and it is also said to be a $p$-cut (level-function) of $\overline{A}$ (in other words, $\overline{A}_p : A \to \{0,1\}$, and $\overline{A}_p(x) = 1$ if and only if $\overline{A}(x) \geq p$).

If a partially ordered set is a complete lattice $L$ with the bottom element 0 and the top element 1, then the corresponding fuzzy set $\overline{A} : A \to L$ is said to be **lattice valued ($L$-valued).**

Some elementary properties of $P$- and $L$-fuzzy sets are the following (Propositions I to V) (for the proofs, see the papers listed in References).

**I** If $\overline{A} : A \to P$ is a $P$-fuzzy set on $A$, then for $x \in A$

$$\overline{A}(x) = \bigvee(p \in P \mid x \in A_p)$$

(i.e., the supremum on the right exists in $(P, \leq)$ for every $x \in A$ and is equal to $\overline{A}(x)$).

**II** Let $\overline{A} : A \to P$ be a $P$-fuzzy set on $A$. Then,

a) if $p, q \in P$ and $p \leq q$, then $A_q \subseteq A_p$;

In particular, for all $x, y \in A$, $\overline{A}(x) \leq \overline{A}(y)$ if and only if $A_{\overline{A}(y)} \subseteq A_{\overline{A}(x)}$;

b) if for $Q \subseteq P$ there exists a supremum of $Q$ ($\bigvee(p \mid p \in Q)$), then

$$\bigcap(A_p \mid p \in Q) = A_{\bigvee(p \mid p \in Q)};$$

c) $\bigcup(A_p \mid p \in P) = A$;

d) for every $x \in A$, $A_{\overline{A}(x)} = \bigcap(A_p \mid x \in A_p)$.

Note that d) is a special case of b), since by I, in this case the supremum exists in $P$.

**III** Let $P$ be a family of subsets of a nonempty set $A$, union of which is also $A$, and such that for every $x \in A$, $\bigcap(p \in P \mid x \in p) \in P$. Let $\overline{A} : A \to P$ be defined with

$$\overline{A}(x) := \bigcap(p \in P \mid x \in p).$$

Then, $\overline{A}$ is a $P$-fuzzy set on $A$, where $(P, \leq)$ is a dual of $(P, \subseteq)$, and for every $p \in P$, $p = A_p$.

**IV** Let $\overline{A}$ be an $L$-fuzzy set on $A$. Then, the collection $\{A_p \mid p \in L\}$ of level subsets of $\overline{A}$ is closed under the set-intersection and contains $A$, hence it is a lattice under the set inclusion.

**V** Let $A$ be a nonempty set and $F$ a family of its subsets closed under arbitrary intersections and containing $A$. Let also $L$ be the lattice dual to $(F, \subseteq)$ and $\overline{A} : A \to L$ an $L$-valued subset of $A$ defined with

$$\overline{A}(x) := \bigcap\{f \in F \mid x \in f\}.$$

Then, the lattice of level subsets of $\overline{A}$ is isomorphic with $(F, \subseteq)$, and for every $f \in F$, $f = A_f$.

## 3. Results

Let $\mathcal{V}$ be an arbitrary variety, and $P$ a partially ordered set (poset). A mapping $\overline{\mathcal{V}} : \mathcal{V} \to P$ is said to be a **poset valued** (**$P$-valued, fuzzy**) subvariety of $\mathcal{V}$ (or a poset valued variety *on* $\mathcal{V}$) if every level of $\overline{\mathcal{V}}$ is an (ordinary) subvariety of $\mathcal{V}$. In particular, if $P$ is a lattice (usually denoted by $L$), then $\overline{\mathcal{V}}$ is a **lattice valued** (**$L$-valued**) subvariety of $\mathcal{V}$ (lattice valued variety *on* $\mathcal{V}$). Some general properties of poset ($P$ and $L$) valued varieties are as follows.

**Proposition 1.** *Let $\overline{\mathcal{V}}$ be a $P$-valued subvariety of $\mathcal{V}$, and $\mathcal{A}, \mathcal{B}$ algebras from $\mathcal{V}$, such that*

*$\mathcal{B}$ is a subalgebra of $\mathcal{A}$, or*

*$\mathcal{B}$ is a homomorphic image of $\mathcal{A}$, or*

*$\mathcal{B} = \mathcal{A} \times \mathcal{A}$.*

*Then, $\overline{\mathcal{V}}(\mathcal{A}) \leq \overline{\mathcal{V}}(\mathcal{B})$.*

*Proof.* Let $\mathcal{A} \in \mathcal{V}$, and let $\mathcal{B}$ be an algebra satisfying any of the foregoing conditions. Then, by the definition of the level set (which is in this case an ordinary subvariety of $\mathcal{V}$), $\mathcal{A} \in \mathcal{V}_{\overline{\mathcal{V}}(\mathcal{A})}$. Since $\mathcal{V}_{\overline{\mathcal{V}}(\mathcal{A})}$ is an ordinary variety, together with $\mathcal{A}$ it contains $\mathcal{B}$. Hence, $\mathcal{B} \in \mathcal{V}_{\overline{\mathcal{V}}(\mathcal{A})}$, and thus $\overline{\mathcal{V}}(\mathcal{B}) \geq \overline{\mathcal{V}}(\mathcal{A})$. □

**Proposition 2.** *Let $\overline{\mathcal{V}}$ be a $P$-valued subvariety of $\mathcal{V}$, and $\mathcal{A}, \mathcal{B}$ two isomorphic algebras from $\mathcal{V}$. Then, $\overline{\mathcal{V}}(\mathcal{A}) = \overline{\mathcal{V}}(\mathcal{B})$.*

*Proof.* By the same argument as in the previous proposition,

$\mathcal{A} \in \mathcal{V}_{\overline{\mathcal{V}}(\mathcal{B})}$, and $\mathcal{B} \in \mathcal{V}_{\overline{\mathcal{V}}(\mathcal{A})}$.

Hence, $\overline{\mathcal{V}}(\mathcal{A}) = \overline{\mathcal{V}}(\mathcal{B})$. □

**Corollary 1.** *If $\mathcal{B}$ is embeddable into $\mathcal{A}$, or $\mathcal{B}$ is isomorphic to a power of $\mathcal{A}$, and $\mathcal{A} \in \mathcal{V}$, then for a $P$-valued subvariety $\overline{\mathcal{V}}$, $\overline{\mathcal{V}}(\mathcal{A}) \leq \overline{\mathcal{V}}(\mathcal{B})$.* □

**Corollary 2.** *If the algebra $\mathcal{A}$ generates the variety $\mathcal{V}$, and $\overline{\mathcal{V}}$ is a $P$-valued subvariety of $\mathcal{V}$, then for every $\mathcal{B} \in \mathcal{V}$, $\overline{\mathcal{V}}(\mathcal{A}) \leq \overline{\mathcal{V}}(\mathcal{B})$.* □

**Proposition 3.** *Let $\mathcal{V}$ be a variety and $\mathcal{A} = (A, F)$ an algebra from $\mathcal{V}$ with a one-element subalgebra. If $\overline{\mathcal{V}}$ is a $P$-valued subvariety of $\mathcal{V}$, then $\overline{\mathcal{V}}(\mathcal{A}) = \overline{\mathcal{V}}(\mathcal{A}^I)$, for every power $\mathcal{A}^I$ of $\mathcal{A}$.*

*Proof.* By Proposition 1, $\overline{\mathcal{V}}(\mathcal{A}) \leq \overline{\mathcal{V}}(\mathcal{A}^I)$. Further on, since $\mathcal{A}$ has a one-element subalgebra, say $(\{a\}, F)$, for every $n$-ary operation $f \in F$,

$f(a,...,a) = a$, and $\mathcal{A}$ is isomorphic to the subalgebra $\mathcal{A}_1 = (A_1, F)$ of $\mathcal{A}^I$, where

$$A_1 = \{X \in A^I \mid X(i) = a, \text{ for all } i \neq i_0 \in I\}.$$

Hence, by Corollary 1, $\overline{\mathcal{V}}(\mathcal{A}^I) \leq \overline{\mathcal{V}}(\mathcal{A})$, and the equality is proved. □

The converse of Proposition 1 also holds.

**Proposition 4.** *Let $\mathcal{V}$ be a variety and $P$ a poset. Further on, let $\overline{\mathcal{V}}$ be a $P$-fuzzy set on $\mathcal{V}$, which satisfies the following: for all $\mathcal{A}, \mathcal{B} \in \overline{\mathcal{V}}$, if*

*$\mathcal{B}$ is a subalgebra of $\mathcal{A}$, or*

*$\mathcal{B}$ is a homomorphic image of $\mathcal{A}$, or*

*$\mathcal{B} = \mathcal{A}^2$,*

*then $\overline{\mathcal{V}}(\mathcal{A}) \leq \overline{\mathcal{V}}(\mathcal{B})$.*

*Then, $\overline{\mathcal{V}}$ is a $P$-valued subvariety of $\mathcal{V}$.*

*Proof.* We have to prove that for every $p \in P$, the level $\mathcal{V}_p$ is a subvariety of $\mathcal{V}$. Since for every $p$, $\mathcal{V}_p$ is a subset of $\mathcal{V}$, it suffices to prove that $\mathcal{V}_p$ is closed under formation of subalgebras, homomorphic images and direct products. Hence, if $\mathcal{A} \in \mathcal{V}_p$, then $\mathcal{V}(\mathcal{A}) \geq p$. Further on, if $\mathcal{B}$ is a subalgebra (homomorphic image, square) of $\mathcal{A}$, then, by the assumption, $\overline{\mathcal{V}}(\mathcal{A}) \leq \overline{\mathcal{V}}(\mathcal{B})$, and hence also $\overline{\mathcal{V}}(\mathcal{B}) \geq p$, i.e., $\mathcal{B} \in \mathcal{V}_p$. □.

We can sum up Propositions 1 and 4, as follows.

**Theorem 1.** *Let $\mathcal{V}$ be a variety and $P$ a poset. A $P$-valued subset $\overline{\mathcal{V}}$ of $\mathcal{V}$ is a $P$-fuzzy subvariety of $\mathcal{V}$ if and only if*

$$\overline{\mathcal{V}}(\mathcal{A}) \leq \overline{\mathcal{V}}(\mathcal{B}), \tag{1}$$

*for all $\mathcal{A}, \mathcal{B} \in \mathcal{V}$, whenever $\mathcal{B}$ is (up to the isomorphism) a subalgebra, homomorphic image, or a square of $\mathcal{A}$.* □

A more precise description of fuzzy varieties by the formula (1) can be given in terms of identities.

**Proposition 5.** *Let $\overline{\mathcal{V}}$ be a $P$-valued subvariety of a variety $\mathcal{V}$. Further on, let $\mathcal{A}, \mathcal{B} \in \mathcal{V}$ and $\overline{\mathcal{V}}(\mathcal{A}) \leq \overline{\mathcal{V}}(\mathcal{B})$. Then, every identity satisfied by $\mathcal{A}$ is also satisfied by $\mathcal{B}$.*

*Proof.* Let $\mathcal{A}, \mathcal{B} \in \mathcal{V}$ and $\overline{\mathcal{V}}(\mathcal{A}) \leq \overline{\mathcal{V}}(\mathcal{B})$. Then, by the general properties of level sets, $\mathcal{B}$ belongs to every level subvariety of $\mathcal{V}$ to which $\mathcal{A}$ belongs. Hence, every identity satisfied by $\mathcal{A}$ is also satisfied by the algebra $\mathcal{B}$. □

Let $\overline{\mathcal{V}} : \mathcal{V} \to P$ be a poset-valued subvariety of a variety $\mathcal{V}$ and

$$[\mathcal{V}] := \{\mathcal{V}_p \mid p \in P\}.$$

$[\mathcal{V}]$ is a collection of levels of $\overline{\mathcal{V}}$ and it can be ordered by the set-inclusion. For $\mathcal{A} \in \mathcal{V}$, let

(2) $$\langle \mathcal{A} \rangle^{[v]} := \bigcap (\mathcal{V}_p \in [\mathcal{V}] \mid \mathcal{A} \in \mathcal{V}_p).$$

By the property II d) (Introduction) the poset $[\mathcal{V}]$ is closed under the set intersection, and hence $\langle \mathcal{A} \rangle^{[v]} \in [\mathcal{V}]$. In other words, the variety $\langle \mathcal{A} \rangle^{[v]}$ is also a level of $\overline{\mathcal{V}}$. Hence, $\langle \mathcal{A} \rangle^{[v]}$ is a subvariety of $\mathcal{V}$, **generated in** $[\mathcal{V}]$ **by** $\mathcal{A}$. As shown in the sequel, varieties generated in the collection of levels by algebras from $\mathcal{V}$, are precisely the levels indexed by the corresponding algebras.

**Proposition 6.** *Let $\overline{\mathcal{V}} : \mathcal{V} \to P$ be a poset-valued subvariety of a variety $\mathcal{V}$. Then, for every $\mathcal{A} \in \mathcal{V}$, $\langle \mathcal{A} \rangle^{[v]} = \mathcal{V}_{\overline{\mathcal{V}}(\mathcal{A})}$.* [1]

*Proof.* Straightforward: by (3), (2), by II d), and by I (Introduction), for an algebra $\mathcal{A} \in \mathcal{V}$

$$\langle \mathcal{A} \rangle^{[v]} = \bigcap (\mathcal{V}_p \in [\mathcal{V}] \mid \mathcal{A} \in \mathcal{V}_p) = \mathcal{V}_{\bigvee (p \mid \mathcal{A} \in \mathcal{V}_p)} = \mathcal{V}_{\overline{\mathcal{V}}(\mathcal{A})}. \quad \square$$

In the following theorem we show that varieties generated by single algebras are essential for the construction of fuzzy varieties.

**Theorem 2.** *Let $\overline{\mathcal{V}} : \mathcal{V} \to P$ be a poset-valued subvariety of a variety $\mathcal{V}$, and $([\mathcal{V}], \leq)$ its collection of levels ordered dually to the set inclusion. Further on, let $\overline{\mathcal{V}}'$ be the mapping from $\mathcal{V}$ to $[\mathcal{V}]$, which assigns to every algebra in $\mathcal{V}$ the subvariety generated in $[\mathcal{V}]$ by that algebra:*

(3) $$\overline{\mathcal{V}}'(\mathcal{A}) := \langle \mathcal{A} \rangle^{[v]}.$$

*Then:*

*a) the poset of levels of $\overline{\mathcal{V}}$ coincides with the poset of levels of $\overline{\mathcal{V}}'$;*

[1] $\mathcal{V}_{\overline{\mathcal{V}}(\mathcal{A})}$ is the $\overline{\mathcal{V}}(\mathcal{A})$-cut, level of $\overline{\mathcal{V}}$.

*b) the posets of functional values (co-domains) of $\overline{\mathcal{V}}$ and $\overline{\mathcal{V}}'$ are isomorphic.*

*Proof.* a) $\overline{\mathcal{V}}' : \mathcal{V} \to [\mathcal{V}]$ is a poset-valued subset of $\mathcal{V}$, where the poset is $[\mathcal{V}]$, which consists of the levels of $\overline{\mathcal{V}}$. We prove that for every $\mathcal{V}_p \in [\mathcal{V}]$, the corresponding level of $\overline{\mathcal{V}}'$ is $\mathcal{V}_p$ i.e., that $\mathcal{V}'_{\mathcal{V}_p} = \mathcal{V}_p$. [2] Indeed, for $\mathcal{A} \in \mathcal{V}$,

$$\begin{aligned} \mathcal{A} \in \mathcal{V}'_{\mathcal{V}_p} & \text{ if and only if } \overline{\mathcal{V}}'(\mathcal{A}) \geq \mathcal{V}_p \\ & \text{if and only if } \overline{\mathcal{V}}'(\mathcal{A}) \subseteq \mathcal{V}_p \\ & \text{if and only if } \langle \mathcal{A} \rangle^{[v]} \subseteq \mathcal{V}_p \\ & \text{if and only if } \mathcal{A} \in \mathcal{V}_p, \end{aligned}$$

since the order in the poset of levels of $\overline{\mathcal{V}}'$ is dual to the set-inclusion.

Since the levels of $\overline{\mathcal{V}}'$ coincide with the levels of $\overline{\mathcal{V}}$, which are subvarieties of $\mathcal{V}$, we have proved that $\overline{\mathcal{V}}'$ is also a poset-valued subvariety of $\mathcal{V}$.

b) If $\mathcal{A}, \mathcal{B}$ are algebras from $\mathcal{V}$ and $\overline{\mathcal{V}}(\mathcal{A}) \neq \overline{\mathcal{V}}(\mathcal{B})$, then by I and II d), $\mathcal{V}_{\overline{\mathcal{V}}(\mathcal{A})} \neq \mathcal{V}_{\overline{\mathcal{V}}(\mathcal{B})}$. Hence, by Proposition 6, the correspondence $\overline{\mathcal{V}}(\mathcal{A}) \mapsto \overline{\mathcal{V}}'(\mathcal{A})$ from the poset of values of $\overline{\mathcal{V}}$ to the poset of values of $\overline{\mathcal{V}}'$ is one-to-one. Moreover, by II a) both, this mapping and its inverse preserve the order. Hence, it is an isomorphism. □

Theorem 2 shows that for every poset-valued subvariety of a variety $\mathcal{V}$, there is another poset-valued subvariety of $\mathcal{V}$, whose co-domain is the collection of levels of the former, and the values are subvarieties generated by the algebras from $\mathcal{V}$. In addition, these two fuzzy varieties have equal levels and isomorphic posets of functional values.

A kind of the converse of Theorem 2 is the following proposition.

**Theorem 3.** *Let $\mathcal{V}$ be a variety and $\mathcal{F}$ a collection of its subvarieties, such that every algebra $\mathcal{A}$ from $\mathcal{V}$, generates the subvariety $\langle \mathcal{A} \rangle^{\mathcal{F}}$ in $\mathcal{F}$. Then, the mapping $\overline{\mathcal{V}} : \mathcal{V} \to \mathcal{F}$, such that $\overline{\mathcal{V}}(\mathcal{A}) = \langle \mathcal{A} \rangle^{\mathcal{F}}$, is a poset-valued subvariety of $\mathcal{V}$, where the poset is $\mathcal{F}$, ordered dually to the set-inclusion. In addition, for every $f \in \mathcal{F}$, the level-variety $\mathcal{V}_f$ is equal to $f$.*

*Moreover, if $\mathcal{F}$ is closed under arbitrary intersections and contains $\mathcal{V}$, then $\overline{\mathcal{V}}$ is lattice-valued subvariety of $\mathcal{V}$.*

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[2] $\mathcal{V}'_{\mathcal{V}_p}$ is the $\mathcal{V}_p$-cut, level of $\overline{\mathcal{V}}'$.

*Proof.* $\overline{\mathcal{V}}$ is a poset valued subset of $\mathcal{V}$, which satisfies the requried conditions. This follows immediately by III, since for every $\mathcal{A} \in \mathcal{V}$, the collection $\mathcal{F}$ contains by the assumption the intersection of all subvarieties to which $\mathcal{A}$ belongs.

The second part follows by V.

Finally, $\mathcal{V}$ is not only a poset-valued subset, but also a poset-valued subvariety of $\mathcal{V}$, since, as we have just proved, every level is a subvariety of $\mathcal{V}$. □

As a conclusion of the foregoing theorems, we have that every poset-valued subvariety of a variety $\mathcal{V}$ can be represented by a particular collection of its ordinary subvarieties. This collection can also be the lattice $Sub\,\mathcal{V}$ of all subvarieties of $\mathcal{V}$, since it satisfies conditions of Theorem 3. For an algebra $\mathcal{A} \in \mathcal{V}$, let $\langle\mathcal{A}\rangle$ be the subvariety of $\mathcal{V}$, generated by $\mathcal{A}$. [3] As proven in Theorem 3, the mapping $\overline{\mathcal{V}}^* : \mathcal{V} \to Sub\,\mathcal{V}$, such that for every $\mathcal{A} \in \mathcal{V}$,

$$\overline{\mathcal{V}}^*(\mathcal{A}) = \langle\mathcal{A}\rangle,$$

is a poset-valued subvariety of $\mathcal{V}$. We say that $\overline{\mathcal{V}}^*$ is the **principal** fuzzy subvariety of $\mathcal{V}$. As a justification for this name, in the following proposition we show that *every* subvariety of $\mathcal{V}$ is a level of $\overline{\mathcal{V}}^*$.

**Proposition 7.** *The poset of levels of the principal fuzzy subvariety $\overline{\mathcal{V}}^*$ of a variety $\mathcal{V}$ is a lattice dual to the lattice $Sub\,\mathcal{V}$ of all subvarieties of $\mathcal{V}$.*

*Proof.* It is obvious that every level of $\overline{\mathcal{V}}^*$ is a subvariety of $\mathcal{V}$. On the other hand, if $\mathcal{W}$ is an arbitrary subvariety of $\mathcal{V}$ and $\mathcal{V}^*_{\mathcal{W}}$ the corresponding level, then

$$\mathcal{V}^*_{\mathcal{W}} = \{\mathcal{A} \in \mathcal{V} \mid \overline{\mathcal{V}}^*(\mathcal{A}) \geq \mathcal{W}\} =$$

$$\{\mathcal{A} \in \mathcal{V} \mid \langle\mathcal{A}\rangle \subseteq \mathcal{W}\} = \mathcal{W}.$$

Hence, the collection of levels of $\overline{\mathcal{V}}^*$ consists precisely of all subvarieties of $\mathcal{V}$. Since the order in this collection is defined to be dual to the set-inclusion, the proof is complete. □

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[3]This is a well known notion, and it also satisfies the conditions of (2), where $[\mathcal{V}]$ (not explicitely denoted) is $Sub\,\mathcal{V}$.

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