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INFLATIONS OF THE AG-GROUPOIDS [1]

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Abstract. Inflations of semigroups are considered by Clifford [4] and Petrich [7]. The notion of n-inflation was inrtoduced by S.Bogdanović and S.Milić in [3] In this paper we made the construction for the n-inflation of the AG-groupoid, and gave some of its properties.

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1. Introduction

Before we consider the construction for the n-inflation we shall give the definition of an AG-groupoid. After that we shall introduce some notions such as retract extension, inflation, strong inflation, etc.

The groupoid S on which the following is true

$$(\forall a, b, c \in S)\ (ab)c = (cb)a, \tag{1}$$

is an AG-groupoid (Abel-Grassmann's groupoid), [5]. On an AG-groupoid holds *medial* law

$$(ab)(cd) = (ac)(bd) \tag{2}$$

for every $a, b, c, d \in S$. An AG-groupoid B whose all elements are idempotents we shall call an AG-band.

A subset I of S is a left (right, two sided) ideal of S if $SI \subseteq I$ ($IS \subseteq I$, $SIS \subseteq I$).

Definition 1.1 *Let S and T be two disjoint groupoids, and suppose that T has a zero element. AG-groupoid V is said to be an (ideal) extension of S by T if it contains S as an ideal and the Rees factor $V \mid S$ is isomorphic to T. If, in addition, there is partial homomorphism $\varphi : T - 0 \longrightarrow S$ such that for all $A, B \in T - \{0\}$ and $c, d \in S$:*

$$A \circ B = \begin{cases} AB, & \text{for} \quad AB \neq 0 \quad \text{in } T \\ \varphi(A)\varphi(B), & \text{for} \quad AB = 0 \quad \text{in } T, \end{cases}$$

$A \circ c = \varphi(A)c$, $c \circ A = c\varphi(A)$, $c \circ d = cd$, then we say that the extension V is determined by the partial homomorphism.

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Definition 1.2 *Let V be an extension of S. We say that V is a retract extension if there exists the homomorphism φ of V onto S such that $\varphi(x) = x$, for all $x \in S$. In this case we call φ a retraction.*

M. Petrich in [7] proved that an extension V of a semigroup S by semigroup T with zero is determined by a partial homomorphism iff it is a retract. S. Bogdanović and S. Milić [3] gave one more characterization for retract extension of semigroups; we shall carry it out for AG-groupoids. Authors of this paper made such a constructions for the AG^*-groupoids in paper [8].

Clifford in [4] have defined a notion of an inflation of a semigroup, and we shall extend it to an inflation of an arbitrary groupoid. A groupoid G is an inflation of a groupoid T if T is a subgroupoid of G and there exists the mapping $\varphi : G \to T$ for which $\varphi(a) = a, a \in T$ and $xy = \varphi(x)\varphi(y), x, y \in G$.

We can also give a notion of a strong inflation of a groupoid which is based on the definition of M. Petrich [7] for a strong inflation of semigroups. Let T be a groupoid. To any $a \in T$ we associate the sets X_a and Y_a with the following properties:

$$a \in X_a,\ X_a \cap X_b = Y_a \cap Y_b = \emptyset,\ \text{for } a \neq b,\ X_a \cap Y_b = \emptyset \text{ for all } a, b \in T.$$

For $x \in Y_a$, $y \in Y_b$ let the element $\varphi^{(a,b)}(x, y) \in X_{ab}$. Let $Z_a = X_a \cup Y_a$, $G = \bigcup_{a \in T} Z_a$ and define an operation $*$ on G in the following way:

$$x * y = \begin{cases} \varphi^{(a,b)}(x, y), & x \in Y_a, y \in Y_b \\ ab, & \text{in other case} \end{cases}$$

where $x \in Z_a, y \in Z_b$. The groupoid G is a strong inflation of the groupoid T.

Definition 1.3 *Let G be an arbitrrary groupoid, for $n \in Z^+$ we can define the set $G^{(n)}$ in the following way:*

$$G^{(1)} = G;\ G^{(2)} = G^2 = \{xy : x, y \in G\};\ G^{(n+1)} = GG^{(n)} \cup G^{(n)}G \ \textit{for}\ n > 2$$

In other words, $G^{(n)}$ contains all products of a length n with all possible combinations of brackets, and we can call it general power of G. It is obvious that for all $n \in Z^+$, G^n is an ideal in G. For if $x \in G$, $a \in G^{(n)}$, we can suppose without loss of generality that $a = ((x_1x_2)x_3)\ldots)x_n)$, then $xa = x(((x_1x_2)x_3)\ldots)x_n)) \in G^{(n)}$ and $ax = (((x_1x_2)x_3)\ldots)x_n))x \in G^{(n)}$, since $x_1x_2 \in G$. We also have that $G^{(n+1)} \subseteq G^{(n)}$ for all $n \in Z^+$.

It is easy to show that if a groupoid G is a strong inflation of a groupoid T then G is a retract extension of T and $G^{(3)} \subset T$.

Definition 1.4 *Groupoid G is an n-nilpotent if $G^{(n)} = 0$ for some $n \in Z^+$.*

If $T = 0$ and G is a strong inflation of T then G is an n-nilpotent, and degree of nilpotency is $n \leq 3$.

Although we defined inflations on an arbitrary groupoid, in this paper we shall discuss only inflations of the AG-groupoids.

2. n-inflation of the AG-groupoids

In this paragraph we shall modify the construction of the n-inflation of semigroup made by S. Bogdanović and S. Milić to an n-inflation of the AG-groupoid.Let us remark that for $n = 1$ we obtain Clifford's inflation and for $n = 2$ we obtain Petrich's (strong) inflation.

Lemma 1 *Let T be an AG-groupoid, to any $a \in T$ we associate a family of sets X_i^a, $i = 1, 2, \ldots, n$ such that $a \in x_r^a$ for some $r \in 1, 2, \ldots, n$ and*

$$X_i^a \cap X_j^a = \emptyset \quad for \quad i \neq j; \quad X_i^a \cap X_j^b = \emptyset \quad for \quad a \neq b. \tag{3}$$

For the nonempty sets X_i^a and X_j^b let

$$\begin{array}{l} \phi_{(i,j)}^{(a,b)} : X_i^a \times X_j^b \to \cup_{\nu=1}^n X_\nu^{ab}, \ if \quad i+j \leq n \\ \phi_{(i,j)}^{(a,b)}(x,y) = ab, \ if \quad i+j > n \\ \phi_{(i,j)}^{(a,b)}(a,y) = \phi_{(i,j)}^{(a,b)}(x,b) = ab \end{array} \tag{4}$$

be the mappings for which it holds:

$$(\forall s \geq i+j)(\forall t \geq k+j) \quad \phi_{(s,k)}^{(ab,c)}(\phi_{(i,j)}^{(a,b)}(x,y),z) = \phi_{(t,i)}^{(cb,a)}(\phi_{(k,j)}^{(c,b)}(z,y),x) \tag{5}$$

for all $a, b, c \in T$ where $i + j \leq n$ or $j + k \leq n$ or $s + k \leq n$ or $t + i \leq n$. Let $Y_a = \cup_{i=1}^n X_i^a$, on $S = \cup_{a \in T} Y_a$, define operation $*$ with:

$$x * y = \phi_{(i,j)}^{(a,b)}(x,y), x \in X_i^a, y \in X_j^b, 1 \leq i, j \leq n.$$

Then $(S, *)$ is an AG-groupoid.

Proof. Let $x \in Y_a$, $y \in Y_b$, $z \in Y_c$ i.e. $x \in X_i^a$, $y \in X_j^b$, $z \in X_k^c$, $1 \leq i, j, k \leq n$. Let $i + j \leq n$, $j + k \leq n$. Then we have:

$$\begin{aligned} (x * y) * z = & \ \phi_{(i,j)}^{(a,b)}(x,y) * z = \phi_{(s,k)}^{(ab,c)}(\phi_{(i,j)}^{(a,b)}(x,y),z) \\ = & \ \phi_{(t,i)}^{(cb,a)}(\phi_{(k,j)}^{(c,b)}(z,y),x) = \phi_{(k,j)}^{(c,b)}(z,y) * x \\ = & \ (z * y) * x \end{aligned}$$

In other cases we can prove in a similar way that $(x * y) * z = (z * y) * x$, so $(S, *)$ is an AG-groupoid. □

Definition 2.1 *An AG-groupoid S constructed in Lemma 2.1. is called an n-inflation of the AG-groupoid T.*

Theorem 1. *An AG-groupoid S is an n-inflation of the groupoid T iff $S^{(n+1)} \subset T$ and S is a retract extension of T.*

Proof Let S be an n-inflation of a semigroup T. By (5) T is an ideal of S. Let $u \in S^{(n+1)}$. Without loss of generality we can suppose that $u = (\ldots(s_1 * s_2) * s_3) * \ldots) * s_{n+1}$ and $s_r \notin T, r = 1,2,\ldots,n+1$. Let $s_r \in X_1^{a_r}, a_r \in T$, then:

$$u = (\ldots(s_1 * s_2) * s_3) * \ldots) * s_{n+1} = (\ldots(\phi_{(1,1)}^{(a_1,a_2)}(s_1,s_2) * s_3)\ldots) * s_{n+1}$$

If $2 > n$ then $\phi_{(1,1)}^{(a_1,a_2)}(s_1,s_2) = u_1 \in T$, so $u \in T$. If $2 \leq n$ then:

$$u = (\ldots(u_1 * s_3) * \ldots) * s_{n+1} = (\ldots(\phi_{(t_1,1)}^{(a_1a_2,a_3)}(u_1,s_3)) * s_4)\ldots) * s_{n+1},$$

where $u_1 \in X_{t_1}^{a_1a_2}, 2 \leq t_1 \leq n$. If $t_1 + 1 > n$ then $\phi_{(t_1,1)}^{(a_1a_2,a_3)}(u_1,s_3) = u_2 \in T$, so $u \in T$. If $t_1 + 1 \leq n$ then:

$$u = (\ldots(u_2 * s_4) * s_5)\ldots) * s_{n+1}, \quad u_2 \in X_{t_2}^{(a_1a_2)a_3}, \quad 3 \leq t_2 \leq n.$$

By continuing this procedure we obtain that if $t_{n-2} + 1 > n$, then:

$$\phi_{(t_{n-2},1)}^{((\ldots(a_1a_2)a_3)\ldots)a_{n-1},a_n)}(u_{n-2},s_n) = u_{n-1} \in T,$$

so $u \in T$ and if $t_{n-2} + 1 \leq n$ then:

$$u = \phi_{(n,1)}^{((\ldots((a_1a_2)a_3)\ldots)a_n,a_{n+1})}(u_{n-1},s_{n+1}) \in T,$$

since $n - 1 \leq t_{n-2} \leq n$ and $t_{n-2} + 1 = n$.

Other cases can be proved simillarly, so $S^{(n+1)} \subset T$. Let us define the mapping $\phi : S = \cup_{a\in T} Y_a \longrightarrow T$ by $\phi(x) = a$ for all $x \in Y_a$. Let $x, y \in S$, then there exist the elements $a, b \in T$ such that $x \in Y_a$, $y \in Y_b$ i.e. $x \in X_i^a$, $y \in X_j^b$, for some $1 \leq i, j \leq n$. Since $\phi_{(i,j)}^{(a,b)}(x,y) \in X_k^{ab} \subset Y_{ab}, \quad i + j \leq k \leq n$ we have:

$$\phi(x * y) = \phi(\phi_{(i,j)}^{(a,b)}(x,y)) = ab = \phi(x)\phi(y),$$

so ϕ is a homomorphism. It is clear that $\phi(a) = a$ for all $a \in T$ so ϕ is a retraction and S is a retract extension of T.

Conversely let n be the smallest positive integer such that $S^{(n+1)} \subset T$ and let $\phi : S \longrightarrow T$ be a retraction from S to T. An arbitrary element $a \in T$ must be in one of the sets $S - S^{(2)}, \quad S^{(2)} - S^{(3)}, \ldots, S^{(n-1)} - S^{(n)}, \quad S^{(n)}$, for example $a \in S^{(n-r)} - S^{(n-r+1)}, \quad 0 \leq r \leq n-1$. Let us define the sets $Y_a = \phi^{-1}(a)$ and

$$\begin{aligned} X_1^a &= Y_a \cap (S - S^{(2)}) \\ X_2^a &= Y_a \cap (S^{(2)} - S^{(3)}) \\ &\ldots \\ &\ldots \\ X_{n-r-1}^a &= Y_a \cap (S^{(n-r-1)} - S^{(n-r)}) \\ X_{n-r}^a &= Y_a \cap S^{(n-r)} \\ X_{n-r+1}^a &= X_{n-r+2}^a = \ldots = X_n^a = \emptyset. \end{aligned}$$

For $a \in T$, we have that $Y_a = \cup_{i=1}^{n} X_i^a$ and $S = \cup_{a\in T} Y_a$. For $x, y \in S$ there exist the elements $a, b \in T$ such that $x \in Y_a$, $y \in Y_b$. By Proposition 1.1 $Y_a Y_b \subset Y_{ab}$.

Let $x \in X_i^a$, $y \in X_j^b$, $a \in S^{(n-r)} - S^{(n-r+1)}$, $b \in S^{(n-p)} - S^{(n-p+1)}$, $0 \leq r, p \leq n-1$, then

$$x \in X_i^a = Y_a \cap (s^{(i)} - S^{(i+1)}), \quad y \in X_j^b = Y_b \cap (s^{(j)} - S^{(j+1)}),$$

where $1 \leq i \leq n-r$, $1 \leq j \leq n-p$. By above we have that $xy \in S^{(i)} S^{(j)} \subset S^{(i+j)}$, if $i+j \leq n$ then $xy \in \cup_{\nu=i+j}^{n} X_\nu^{ab}$ and if $i+j > n$ then $xy = ab \in T$. For $x \in X_i^a$, $b \in T$ $xb = ab$ and $bx = ba$. In this way the mappings $\phi_{(i,j)}^{(a,b)}$ are defined and the condition (6) holds. □

If we put in Lemma 2.1 that the mappings $\phi_{(i,j)}^{(a,b)} : X_i^a \times X_j^b \to X_r^{ab}$, where $i + j \leq r \leq n$, then we obtain a strong inflation.

We can replace the condition (6) in Lemma 2.1. with some other one and obtain an n-inflation of other class of groupoids. If we make construction without the condition (6), then we obtain an n-inflation of groupoid in general. However, in this paper we shall deal only with n-inflations of the AG-grupoids.

Example 2.1 The AG-groupoid $S = \{1, 2, 3, 4, 5, 6, \}$ whose multiplication is given with the following table is a 3-inflation of an AG-groupoid $T = \{1, 2. 3\}$.

·	1	2	3	4	5	6
1	2	3	1	2	2	3
2	1	2	3	1	1	2
3	3	1	2	3	3	1
4	2	3	1	2	6	3
5	2	3	1	6	2	3
6	1	2	3	1	1	2

Table 1.

The sets X_i^j are:

$$X_1^1 = \{1\}, \quad X_2^1 = \{4\}, \quad X_3^1 = \{5\}$$
$$X_1^2 = \emptyset, \quad X_2^2 = \{2\}, \quad X_3^2 = \{6\}$$
$$X_1^3 = \emptyset, \quad X_2^3 = \emptyset, \quad X_3^3 = \{3\}.$$

It is obvious that $S^{(3)} \subset T$ and that there exist retraction from S to T. This inflation is also a strong inflation.

3. n-inflations of the orthogonal sum of AG-groupoids

In [7] M. Petrich proved that inflation (1-inflation) of semigroups is compatible with an orthogonal sum. In Theorem 3.1. we shall prove this for inflation of the AG-groupoids (groupoids in general) and give necessary and sufficient conditions for the n-inflation to be compatible with an orthogonal sum (Theorem 3.2).

Let S be an AG-groupoid, 0 element not belonging to S, define $x0 = 0x = 00 = 0$ for all $x \in S$, then $S \cup \{0\}$ is an AG-groupoid with zero. With S^0 we shall denote S if it has a zero element and $S \cup \{0\}$ if S does not have a zero element. By S^* we shall denote the set $S^0 - 0$.

Definition 3.1 *We say that an AG-groupoid with the zero S^0 is an orthogonal sum of AG-groupoids $\{S_\alpha, \alpha \in Y\}$ and denote $S = \Sigma_{\alpha \in Y} S_\alpha$, if $S = \cup_{\alpha \in Y} S_\alpha$, $S_\alpha \cap S_\beta = 0$ and $S_\alpha S_\beta = 0$ for all $\alpha, \beta \in Y$, $\alpha \neq \beta$.*

Proposition 3.1 *The AG-groupoid S^0 is an inflation of the orthogonal sum of the AG-groupoids $T_\alpha, \alpha \in Y$ iff $S = \Sigma_{\alpha \in Y} S_\alpha$ where S_α are inflations of T_α, $\alpha \in Y$.*

Proof. Let S^0 be an inflation of $T = \Sigma_{\alpha \in Y} T_\alpha$, and $\varphi : S^0 \longrightarrow T$ a retraction associated with it. We shall fix $\beta \in Y$, for $\alpha \in Y$, $\alpha \neq \beta$ let

$$S_\alpha = \{x \in S : \varphi(x) \in T_\alpha^*\} \cup 0, \quad \varphi_\alpha = \varphi \mid S_\alpha$$

and

$$S_\beta = \{x \in S : \varphi(x) \in T_\beta\}, \quad \varphi_\beta = \varphi \mid S_\beta.$$

If $x, y \in S_\beta$, then $\varphi(xy) = \varphi(x)\varphi(y) \in T_\beta$ so $xy \in S_\beta$. If $x, y \in S_\alpha^*$, $\alpha \neq \beta$, then $\varphi(x), \varphi(y) \in T_\alpha^*$. If $\varphi(xy) = 0$ then $xy = 0$ (since all $u \in S$ such that $u \neq 0$ and $\varphi(u) = 0$ belong to S_β) so $xy \in S_\alpha$. If $\varphi(xy) \neq 0$ then $\varphi(xy) = \varphi(x)\varphi(y) \in T_\alpha^*$ so $xy \in S_\alpha$.

Let $x \in S_\alpha$, $y \in S_\gamma$ and $xy \neq 0$, we have $xy = \varphi(xy) = \varphi(x)\varphi(y) \neq 0$, since $S^2 \subseteq T$ and $\varphi(x) \in T_\alpha$, $\varphi(y) \in T_\gamma$ it will be $\alpha = \gamma$. Consequently, $S = \Sigma_{\alpha \in Y} S_\alpha$, and obviously S_α is an inflation of T_α with the retractions φ_α, $\alpha \in Y$.

Conversely, let S_α be inflations of T_α and $\varphi_\alpha : S_\alpha \longrightarrow T_\alpha$ associated retractions, we can define the mapping $\varphi : S = \Sigma_{\alpha \in Y} S_\alpha \longrightarrow T = \Sigma_{\alpha \in Y} T_\alpha$ with

$$\varphi(x) = \begin{cases} \varphi_\alpha(x), & \text{for } \ x \in S_\alpha^* \\ 0, & \text{for } \ x = 0, \end{cases}$$

It is obvious that S is an inflation of T. □

However for an n-inflation of AG-groupoid with zero we have to introduce some restrictions to make it compatible with the orthogonal sum.

Theorem 2 *The AG-groupoid S is an orthogonal sum of the AG-groupoids S_α which are n-inflations of the AG-groupoids T_α, $\alpha \in Y$ iff S is an n-inflation of $T = \Sigma_{\alpha \in Y} T_\alpha$ and $X_0^i = \emptyset$, $1 \leq i \leq n$.*

In other words, we should not blow zero element from S.

Proof. Suppose that S is an n-inflation of T, $\varphi : S \longrightarrow T$ associate retraction. Similarly as in proof of Proposition 3.1 we make the sets

$$S_\alpha = \{x \in S : \varphi(x) \in T_\alpha^*\} \cup 0$$

and the mappings

$$\varphi_\alpha = \varphi\,|_{S_\alpha},\ \alpha \in Y.$$

If $x, y \in S_\alpha^*$ then $\varphi(xy) = \varphi(x)\varphi(y) \in T_\alpha^*$ so $xy \in S_\alpha^*$. If $x \in S_\alpha$, $y \in S_\beta$ and $xy \neq 0$ then $\varphi(x)\varphi(y) = \varphi(xy) \neq 0$, since $\varphi(x) \in T_\alpha$, $\varphi(y) \in T_\beta$ it follows that $\alpha = \beta$. Therefore, S is an orthogonal sum of $\{S_\alpha,\ \alpha \in Y\}$.

Let $y \in S_\alpha^{(n+1)} \subseteq S^{(n+1)}$, since S is an n-inflation of T it holds $S^{(n+1)} \subseteq T$, so $u \in T$ whence $\varphi(u) = u$. Furthermore, since $u \in S_\alpha$ we have $\varphi(u) \in T_\alpha^*$, whence $u = \varphi(u) \in T_\alpha^*$. From the above it follows $S_\alpha^{(n+1)} \subseteq T_\alpha$ and since φ_α are retractions from S_α to T_α by Theorem 2.1 S_α is an n-inflation of T_α.

Conversely, if S_α are n-inflations of T_α, $\varphi_\alpha : S_\alpha \longrightarrow T_\alpha$ associate retractions and $T = \Sigma_{\alpha\in Y} T_\alpha$, it is easy to prove that $\varphi : S \longrightarrow T$ defined as in the proof of Proposition 3.1 is a retraction. Obviously, $S^{(n+1)} \subseteq T$, so $S = \Sigma_{\alpha\in Y} S_\alpha$ is an n-inflation of T. □

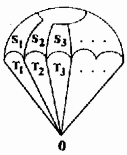

Figure 1.

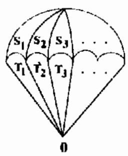

Figure 2.

4. Inflations of the *AG*-bands

Now we shall characterize inflations of the *AG*-bands and semilattices.

Theorem 3 *On an AG-groupoid S the following conditions are equivalent:*
(i) S is an inflation of an AG-band,
(ii) $S^{(2)}$ *is an AG-band,*
(iii) S is an AG-band Y of zero semigroups S_α, $\alpha \in Y$, *and* $Y \cong E(S) = S^{(2)}$
(iv) $(\forall x, y \in S)\ xy = x^2y^2 = (xy)^2$.

Proof. *(i)* ⇒ *(ii)* Let S be an inflation of the *AG*-band T. Then $S^{(2)} \subseteq T$, T is an ideal in S and there exists a retraction φ from S onto T. Clearly $S^{(2)} = T$.

(ii) ⇒ *(i)* Suppose that $S^{(2)}$ is an *AG*-band. The mapping φ defined by $\varphi(x) = x^2$ is a homomorphism from S to $S^{(2)}$ because $\varphi(xy) = (xy)^2 = (xy)(xy) = (xx)(yy) = x^2y^2 = \varphi(x)\varphi(y)$. Since $S^{(2)}$ is an *AG*-band it follows that $\varphi(x) = x^2 = x$ so φ is a retraction and by Theorem 2.1 S is an inflation of $S^{(2)}$.

(ii) ⇒ *(iii)* Since φ is a homomorphism from S to $S^{(2)}$, $ker(\varphi)$ is a congruence on S. From $\varphi(x) = x^2 = x^2x^2 = (x^2)^2 = \varphi(x^2)$ it follows that $x\,ker(\varphi)\,x^2$ for all $x \in S$, so $ker(\varphi)$ is a band congruence and $S\,|_{(ker(\varphi))}$ is an *AG*-band Y.

For $x, y \in S_\alpha$, $\alpha \in Y$ it holds $xy = \varphi(xy) = \varphi(x)\varphi(y) = e_\alpha e_\alpha = e_\alpha$ and $xe_\alpha = \varphi(xe_\alpha) = \varphi(x)\varphi(e_\alpha) = e_\alpha$. Similarly, $e_\alpha x = e_\alpha$, so S_α is a zero semigroup with zero e_α.

$(iii) \Rightarrow (ii)$ Follows immediately.

$(iv) \Rightarrow (ii)$ Suppose that for all $x, y \in S$ it holds $xy = x^2y^2$ then

$$xy = x^2y^2 = (xx)(yy) = (xy)(xy) = (xy)^2,$$

so xy is an idempotent and S^2 is an AG-band.

$(ii) \Rightarrow (iv)$ Let S^2 be an AG-band, then for all $x, y \in S$ it holds

$$xy = (xy)^2 = (xy)(xy) = (xx)(yy) = x^2y^2. \qquad \square$$

Example. Let the AG-groupoid S be given by the following table.

$\cdot$	1	2	3	4	5	6
1	1	4	2	3	1	4
2	3	2	4	1	3	2
3	4	1	3	2	4	1
4	2	3	1	4	2	3
5	1	4	2	3	1	4
6	3	2	4	1	3	2

Table 2.

This groupoid is an inflation of the AG-band $T = \{1, 2, 3, 4\}$ by sets $X^1 = \{1, 5\}$; $X^2 = \{2, 6\}$; $X^3 = \{3\}$; $X^4 = \{4\}$. We also have that S is an AG-band T of the zero semigroups S_α, $\alpha \in T$, where $S_\alpha = X^\alpha$. Condition (iv) from above Theorem holds too.

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