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SOME CHARACTERISTICS OF CURVATURE TENSORS OF NONSYMMETRIC AFFINE CONNEXION *

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Abstract

In the work [4] are defined 12 curvature tensors of nonsymmetric affine connexion. In the present work we study their characteristics of the type (2.1) (for an arbitrary nonsymmetric affine connexion) and of the type (4.2) (for a nonsymmetric Riemannian connexion)

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1 Introduction

Let $\mathcal{M}_\mathcal{N}$ be an N-dimensional differentiable manifold, on which a nonsymmetric affine connexion ${}^1\nabla$ is introduced. Then by the mapping (see [1]) ${}^2\nabla : \mathcal{X}(\mathcal{M}_\mathcal{N}) \times \mathcal{X}(\mathcal{M}_\mathcal{N}) \to \mathcal{X}(\mathcal{M}_\mathcal{N})$, given by

$$(1.1) \qquad {}^2\nabla_X Y = {}^1\nabla_Y X + [X, Y]$$

another non-symmetric affine connexion is defined ($X, Y \in \mathcal{X}(\mathcal{M}_\mathcal{N})$, where $\mathcal{X}(\mathcal{M}_\mathcal{N})$ is the set of smooth vector fields on $\mathcal{M}_\mathcal{N}$). This means that following conditions are in force:

$$(1.2) \qquad a)\, {}^\theta\nabla_{X+V} Y = {}^\theta\nabla_X Y + {}^\theta\nabla_V Y, \qquad b)\, {}^\theta\nabla_{fX} Y = f\, {}^\theta\nabla_X Y,$$

$$c)\, {}^\theta\nabla_X (Y + V) = {}^\theta\nabla_X Y + {}^\theta\nabla_X V, \quad d)\, {}^\theta\nabla_X (fY) = Xf \cdot Y + f\, {}^\theta\nabla_X Y,$$

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for $\theta = 1,2$; $X, Y, V \in \mathcal{X}(\mathcal{M}_N), \{ \in \mathcal{F}(\mathcal{M}_N)$, where $\mathcal{F}(\mathcal{M}_N)$ is the set of smooth real functions on $\mathcal{M}_N$. As it is proved in [5], by equations

$$\overset{\theta}{R}(X,Y)Z = {}^\theta\nabla_X {}^\theta\nabla_Y Z - {}^\theta\nabla_Y {}^\theta\nabla_X Z - {}^\theta\nabla_{[X,Y]}Z, \quad \theta = 1,2,$$

$$\overset{3}{R}(X,Y)Z = {}^2\nabla_X {}^1\nabla_Y Z - {}^1\nabla_Y {}^2\nabla_X Z + {}^2\nabla_{{}^1\nabla_Y X} Z - {}^1\nabla_{{}^2\nabla_X Y} Z, \tag{1.3}$$

$$\overset{4}{R}(X,Y)Z = {}^2\nabla_X {}^1\nabla_Y Z - {}^1\nabla_Y {}^2\nabla_X Z + {}^2\nabla_{{}^2\nabla_Y X} Z - {}^1\nabla_{{}^1\nabla_X Y} Z$$

four curvature tensor fields on $\mathcal{M}_N$ are defined.

In the work [4] we obtained also the following curvature tensors of non-symmetric affine connexion:

$$\overset{5}{R}(X,Y)Z = {}^1\nabla_X {}^1\nabla_Y Z + {}^2\nabla_Y {}^2\nabla_X Z - {}^1\nabla_Y {}^2\nabla_X Z - {}^2\nabla_Y {}^1\nabla_X Z - {}^1\nabla_{[X,Y]}Z,$$

$$\overset{6}{R}(X,Y)Z = {}^2\nabla_X {}^2\nabla_Y Z + {}^1\nabla_Y {}^1\nabla_X Z - {}^1\nabla_Y {}^2\nabla_X Z - {}^2\nabla_Y {}^1\nabla_X Z - {}^2\nabla_{[X,Y]}Z,$$

$$\overset{7}{R}(X,Y)Z = \frac{1}{2}\{{}^1\nabla_X {}^1\nabla_Y Z + {}^2\nabla_X {}^2\nabla_Y Z - {}^1\nabla_Y {}^2\nabla_X Z - {}^2\nabla_Y {}^1\nabla_X Z - $$
$$- {}^1\nabla_{[X,Y]}Z - {}^2\nabla_{[X,Y]}Z\},$$

$$\overset{8}{R}(X,Y)Z = \frac{1}{2}\{{}^1\nabla_X {}^2\nabla_Y Z + {}^2\nabla_X {}^1\nabla_Y Z - {}^1\nabla_Y {}^2\nabla_X Z - {}^2\nabla_Y {}^1\nabla_X Z - $$
$$- {}^1\nabla_{[X,Y]}Z - {}^2\nabla_{[X,Y]}Z\},$$

$$\overset{9}{R}(X,Y)Z = \frac{1}{2}\{{}^1\nabla_X {}^2\nabla_Y Z + {}^2\nabla_X {}^1\nabla_Y Z - {}^1\nabla_Y {}^1\nabla_X Z - {}^2\nabla_Y {}^2\nabla_X Z - \tag{1.4}$$
$$- {}^1\nabla_{[X,Y]}Z - {}^2\nabla_{[X,Y]}Z\},$$

$$\overset{10}{R}(X,Y)Z = \frac{1}{6}\{2({}^1\nabla_X {}^2\nabla_Y Z + {}^2\nabla_X {}^1\nabla_Y Z + {}^2\nabla_X {}^2\nabla_Y Z - {}^1\nabla_Y {}^2\nabla_X Z -$$

$$-{}^2\nabla_Y{}^1\nabla_X Z - {}^2\nabla_Y{}^2\nabla_X Z - {}^1\nabla_{[X,Y]}Z - 2^2\nabla_{[X,Y]}Z)+$$
$$+{}^1\nabla_{{}^1\nabla_X Y}Z + {}^2\nabla_{{}^1\nabla_Y X}Z - {}^1\nabla_{{}^2\nabla_X Y}Z - {}^2\nabla_{{}^2\nabla_Y X}Z\},$$

$$\overset{11}{R}(X,Y)Z = {}^1\nabla_X{}^2\nabla_Y Z + {}^2\nabla_X{}^1\nabla_Y Z - {}^1\nabla_X{}^1\nabla_Y Z - {}^2\nabla_Y{}^2\nabla_X Z -$$
$$-{}^2\nabla_{[X,Y]}Z,$$

$$\overset{12}{R}(X,Y)Z = {}^1\nabla_X{}^2\nabla_Y Z + {}^2\nabla_X{}^1\nabla_Y Z - {}^2\nabla_X{}^2\nabla_Y Z - {}^1\nabla_Y{}^1\nabla_X Z - {}^1\nabla_{[X,Y]}Z,$$

2 Curvature tensors of nonsymmetric connexion

2.0. In the case of symmetric connexion ${}^0\nabla = {}^1\nabla = {}^2\nabla$ all the tensors $\overset{\theta}{R}\,(\theta = 1,...,12)$ reduce to the curvature tensor $\overset{0}{R}$ of this symmetric connexion. As it is known, for $\overset{0}{R}$ the next relations are valid:

$$\text{(2.1}a\text{)} \qquad \overset{0}{R}(X,Y)Z = -\overset{0}{R}(Y,X)Z,$$

$$\text{(2.1}b\text{)} \qquad C_{XYZ}\,\overset{0}{R}(X,Y)Z \equiv \overset{0}{R}(X,Y)Z + \overset{0}{R}(Y,Z)X + \overset{0}{R}(Z,X)Y = 0,$$

and we have to examine these properties for $\overset{\theta}{R}\,(\theta = 1,...,12)$.

2.1. The property (2.1a) is evident for $\overset{1}{R},\overset{2}{R},\overset{8}{R}$ on the base of the corresponding equations in (1.3), (1.4). For $\overset{10}{R}$ it is also valid what is obvious for the part in the round bracket, and for the rest members we have

$$\begin{aligned}\mathcal{A}(\mathcal{X},\mathcal{Y})\mathcal{Z} &\equiv {}^\infty\nabla_{{}^\infty\nabla_\mathcal{X}\mathcal{Y}}\mathcal{Z} + {}^\in\nabla_{{}^\infty\nabla_\mathcal{Y}\mathcal{X}}\mathcal{Z} - {}^\infty\nabla_{{}^\in\nabla_\mathcal{X}\mathcal{Y}}\mathcal{Z} - {}^\in\nabla_{{}^\in\nabla_\mathcal{Y}\mathcal{X}}\mathcal{Z} = \\ &= {}^1\nabla_{{}^1\nabla_X Y - {}^2\nabla_X Y}Z + {}^2\nabla_{{}^1\nabla_Y X - {}^2\nabla_Y X}Z = {}^1\nabla_{T(X,Y)}Z - {}^2\nabla_{T(Y,X)}Z = \\ &= -\mathcal{A}(\mathcal{Y},\mathcal{X})\mathcal{Z},\end{aligned}$$

because from (1.1) is

$$\text{(2.2)} \quad {}^1\nabla_X Y - {}^2\nabla_X Y = {}^1\nabla_X Y - {}^2\nabla_Y X - [X,Y] = T(X,Y) = -T(Y,X),$$

where $T(X,Y)$ is the torsion tensor. Therefore,

$$\text{(2.3)} \qquad \overset{\theta}{R}(X,Y)Z = -\overset{\theta}{R}(Y,X)Z, \ \theta \in \{1,2,8,10\}.$$

2.2. For an arbitrary expression $\mathcal{B}(\mathcal{X},\mathcal{Y},\mathcal{Z})$ we introduce the notation

$$C_{XYZ}\mathcal{B}(\mathcal{X},\mathcal{Y},\mathcal{Z}) = \mathcal{B}(\mathcal{X},\mathcal{Y},\mathcal{Z}) + \mathcal{B}(\mathcal{Y},\mathcal{Z},\mathcal{X}) + \mathcal{B}(\mathcal{Z},\mathcal{X},\mathcal{Y}), \tag{2.4}$$

wherefrom

$$C_{XYZ}\mathcal{B}(\mathcal{X},\mathcal{Y},\mathcal{Z}) = \mathcal{C}_{\mathcal{X}\mathcal{Y}\mathcal{Z}}\mathcal{B}(\mathcal{Y},\mathcal{Z},\mathcal{X}) = \mathcal{C}_{\mathcal{X}\mathcal{Y}\mathcal{Z}}\mathcal{B}(\mathcal{Z},\mathcal{X},\mathcal{Y}). \tag{2.4'}$$

We shall examine the expressions $C_{XYZ}\overset{\theta}{R}(X,Y)Z$ $(\theta = 1,\dots,12)$. By virtue of (1.3) and (2.4) for $\theta \in \{1,2\}$ we obtain

$$C_{XYZ}\overset{\theta}{R}(X,Y)Z = C_{XYZ}\{{}^{\theta}\nabla_X({}^{\theta}\nabla_Y Z - {}^{\theta}\nabla_Z Y) - {}^{\theta}\nabla_{[X,Y]}Z\}.$$

Since

$$ {}^{1}\nabla_Y Z - {}^{1}\nabla_Z Y = [Y,Z] + T(Y,Z), \tag{2.5}$$

where $T(Y,Z)$ is the torsion tensor field, using also (2.4′), we get

$$C_{XYZ}\overset{1}{R}(X,Y)Z = C_{XYZ}\{{}^{1}\nabla_X(T(Y,Z) + [Y,Z]) - {}^{1}\nabla_{[Y,Z]}X\}.$$

Applying (2.5), we obtain

$$C_{XYZ}\overset{1}{R}(X,Y)Z = C_{XYZ}\{{}^{1}\nabla_X(T(Y,Z)) + T(X,[Y,Z]) + [X,[Y,Z]]\},$$

and because of the Jacobi equation $C_{XYZ}[X,[Y,Z]] = 0$, we have finally

$$C_{XYZ}\overset{1}{R}(X,Y)Z = C_{XYZ}\{{}^{1}\nabla_X(T(Y,Z)) + T(X,[Y,Z])\}, \tag{2.6}$$

Using (2.4′,5) and the relation

$$({}^{\theta}\nabla_X T)(Y,Z) = {}^{\theta}\nabla_X(T(Y,Z)) - T({}^{\theta}\nabla_X Y, Z) - T(Y, {}^{\theta}\nabla_X Z), \quad (\theta = 1,2) \tag{2.7}$$

for $\theta = 1$, we can give an another form to the equation (2.6):

$$C_{XYZ}\overset{1}{R}(X,Y)Z = C_{XYZ}\{({}^{1}\nabla_X T)(Y,Z) + T(T(X,Y),Z))\}. \tag{2.6'}$$

Because from (1.1) and (2.5) we have

$$ {}^{2}\nabla_Y Z - {}^{2}\nabla_Z Y = [Y,Z] - T(Y,Z), \tag{2.8}$$

then, analogously to (2.6), one obtains

$$C_{XYZ}\overset{2}{R}(X,Y)Z = -C_{XYZ}\{({}^{2}\nabla_X(T(Y,Z)) + T(X,[Y,Z])\}, \tag{2.9}$$

which, applying (2.7) for $\theta = 2$ and (2.8), can be written in the form

$$C_{XYZ}\overset{2}{R}(X,Y)Z = -C_{XYZ}\{({}^2\nabla_X T)(Y,Z) + T(X, T(Y,Z))\}. \tag{2.9'}$$

Further we have

$$\begin{aligned}C_{XYZ}\overset{3}{R}(X,Y)Z &= {}^2\nabla_X {}^1\nabla_Y Z - {}^1\nabla_Y {}^2\nabla_X Z + {}^2\nabla_{{}^1\nabla_Y X} Z - {}^1\nabla_{{}^2\nabla_X Y} Z + \\ &+ {}^2\nabla_Y {}^1\nabla_Z X - {}^1\nabla_Z {}^2\nabla_Y X + {}^2\nabla_{{}^1\nabla_Z Y} X - {}^1\nabla_{{}^2\nabla_Y Z} X + \\ &+ {}^2\nabla_Z {}^1\nabla_X Y - {}^1\nabla_X {}^2\nabla_Z Y + {}^2\nabla_{{}^1\nabla_X Z} Y - {}^1\nabla_{{}^2\nabla_Z X} Y.\end{aligned}$$

With respect to (1.1) we obtain

$$\begin{aligned}{}^2\nabla_X {}^1\nabla_Y Z &= {}^1\nabla_{{}^1\nabla_Y Z} X + [X, {}^1\nabla_Y Z], \\ {}^1\nabla_Y {}^2\nabla_X Z &= {}^2\nabla_{{}^2\nabla_X Z} Y + [Y, {}^2\nabla_X Z],\end{aligned} \tag{2.10}$$

and then

$$\begin{aligned}C_{XYZ}\overset{3}{R}(X,Y)Z &= {}^1\nabla_{{}^1\nabla_Y Z} X + [X, {}^1\nabla_Y Z] - {}^2\nabla_{{}^2\nabla_X Z} Y - [Y, {}^2\nabla_X Z] + \\ &+ {}^2\nabla_{{}^1\nabla_Y X} Z - {}^1\nabla_{{}^2\nabla_X Y} Z + + {}^1\nabla_{{}^1\nabla_Z X} Y + [Y, {}^1\nabla_Z X] - {}^2\nabla_{{}^2\nabla_Y X} Z - \\ &- [Z, {}^2\nabla_Y X] + {}^2\nabla_{{}^1\nabla_Z Y} X - {}^1\nabla_{{}^2\nabla_Y Z} X + {}^1\nabla_{{}^1\nabla_X Y} Z + [Z, {}^1\nabla_X Y] - \\ &- {}^2\nabla_{{}^2\nabla_Z Y} X - [X, {}^2\nabla_Z Y] + {}^2\nabla_{{}^1\nabla_X Z} Y - {}^1\nabla_{{}^2\nabla_Z X} Y = \\ &= C_{XYZ}\{{}^1\nabla_{{}^1\nabla_Y Z - {}^2\nabla_Y Z} X + {}^2\nabla_{{}^1\nabla_Z Y - {}^2\nabla_Z Y} X + [X, {}^1\nabla_Y Z - {}^2\nabla_Z Y]\}.\end{aligned}$$

From here, by applying (2.2) and (1.1), it follows that

$$\begin{aligned}C_{XYZ}\overset{3}{R}(X,Y)Z &= C_{XYZ}\{{}^1\nabla_{T(Y,Z)} X + {}^2\nabla_{T(Z,Y)} X + [X, [Y, Z]]\} = \\ &= C_{XYZ} T(T(Y,Z), X) + 0,\end{aligned}$$

i.e.

$$C_{XYZ}\overset{3}{R}(X,Y)Z = C_{XYZ} T(T(X,Y), Z) \tag{2.11}$$

In the same manner, we obtain for the R^4

$$C_{XYZ}\overset{4}{R}(X,Y)Z = C_{XYZ}[X, {}^1\nabla_Y Z - {}^2\nabla_Z Y] = C_{XYZ}[X, [Y, Z]] = 0$$

i.e.

$$C_{XYZ}\overset{4}{R}(X,Y)Z = 0 \tag{2.12}$$

Further, one obtains

$$
\begin{aligned}
&C_{XYZ}\overset{5}{R}(X,Y)Z = \\
&= C_{XYZ}\{{}^1\nabla_X({}^1\nabla_Y Z - {}^2\nabla_Z Y) + {}^2\nabla_X({}^2\nabla_Z Y - {}^1\nabla_Z Y) - {}^1\nabla_{[X,Y]}Z\} = \\
&= C_{XYZ}\{{}^1\nabla_X[Y,Z] + {}^2\nabla_X(T(Y,Z)) - {}^1\nabla_{[X,Y]}Z\}.
\end{aligned}
$$

Since because of (2.4′) and (2.5)

$$
\begin{aligned}
&C_{XYZ}\{{}^1\nabla_X[Y,Z] - {}^1\nabla_{[X,Y]}Z\} = C_{XYZ}\{{}^1\nabla_X[Y,Z] - {}^1\nabla_{[Y,Z]}X\} = \\
&= C_{XYZ}\{[X,[Y,Z]] + T(X,[Y,Z])\} = C_{XYZ}T(X,[Y,Z]),
\end{aligned}
$$

we obtain

$$
C_{XYZ}\overset{5}{R}(X,Y)Z = C_{XYZ}\{{}^2\nabla_X(T(Y,Z)) + T(X,[Y,Z])\}, \tag{2.13}
$$

what one can write in the form

$$
C_{XYZ}\overset{5}{R}(X,Y)Z = C_{XYZ}\{({}^2\nabla_X T)(Y,Z) + T(X,T(Y,Z))\}. \tag{2.13′}
$$

By the similar procedure one obtains

$$
C_{XYZ}\overset{6}{R}(X,Y)Z = -C_{XYZ}\{({}^1\nabla_X T)(Y,Z) + T(T(X,Y),Z)\}, \tag{2.14}
$$

$$
C_{XYZ}\overset{5}{R}(X,Y)Z = -C_{XYZ}\{({}^1\nabla_X T)(Y,Z) + T(T(X,Y),Z)\}. \tag{2.14′}
$$

In relation to (1.4,1), (2.4′,5,8) we get

$$
\begin{aligned}
&C_{XYZ}\overset{7}{R}(X,Y)Z = \frac{1}{2}C_{XYZ}\{{}^1\nabla_X({}^1\nabla_Y Z - {}^2\nabla_Z Y) + \\
&+{}^2\nabla_X({}^2\nabla_Y Z - {}^1\nabla_Z Y) - ({}^1\nabla_{[X,Y]}Z + {}^2\nabla_{[X,Y]}Z)\} = \\
&= \frac{1}{2}C_{XYZ}\{{}^1\nabla_X[Y,Z] + {}^2\nabla_X[Y,Z] - ({}^1\nabla_{[Y,Z]}X + {}^2\nabla_{[Y,Z]}X)\},
\end{aligned}
$$

$$
C_{XYZ}\overset{7}{R}(X,Y)Z = 0 \tag{2.15}
$$

and similarly

$$
C_{XYZ}\overset{8}{R}(X,Y)Z = \frac{1}{2}C_{XYZ}T(T(X,Y),Z), \tag{2.16}
$$

(2.17) $$C_{XYZ}\overset{9}{R}(X,Y)Z = 0,$$

(2.18) $$C_{XYZ}\overset{10}{R}(X,Y)Z =$$

$$= -\frac{1}{6}C_{XYZ}\{2\,{}^1\nabla_X(T(Y,Z)) + T(X, T(Y,Z) + 2[Y,Z])\},$$

(2.19) $$C_{XYZ}\overset{11}{R}(X,Y)Z = -C_{XYZ}\{{}^1\nabla_X(T(Y,Z)) + T(X,[Y,Z])\},$$

that is

(2.19′) $$C_{XYZ}\overset{11}{R}(X,Y)Z = -C_{XYZ}\{({}^1\nabla_X T)(Y,Z) + T(T(X,Y),Z)\},$$

(2.20) $$C_{XYZ}\overset{12}{R}(X,Y)Z = C_{XYZ}\{{}^2\nabla_X(T(Y,Z)) + T(X,[Y,Z])\},$$

or

(2.20′) $$C_{XYZ}\overset{12}{R}(X,Y)Z = C_{XYZ}\{({}^2\nabla_X T)(Y,Z) + T(X,T(Y,Z))\}.$$

According to explained, we have

(2.21) $$C_{XYZ}\overset{\theta}{R}(X,Y)Z = 0 \quad \text{for} \quad \theta \in \{4,7,9\}$$

and the next theorem is valid:

Theorem 1. *For the curvature tensors* $\overset{\theta}{R}(X,Y)Z$ $(\theta = 1, ..., 12)$, *defined by equations* (1.3, 4), *where* ${}^1\nabla$ *is a nonsymmetric affine connexion, and* ${}^2\nabla$ *the corresponding nonsymmetric connexion, defined by* (1.1), *the characteristic* (2.3) *is valid for* $\overset{1}{R}$, $\overset{2}{R}$, $\overset{8}{R}$, $\overset{10}{R}$, *and the expressions* $C_{XYZ}\overset{\theta}{R}(X,Y)Z$, *defined with respect to* (2.4), *are given by the equations* (2.6, 9, 11 − 20) *or* (2.6′, 9′, 13′, 14′, 19′, 20′), *that is the characteristic* (2.21) *is valid for* $\overset{4}{R}$, $\overset{7}{R}$, $\overset{9}{R}$.

3 Generalized Riemannian space

Def.3.1. If on a differentiable manifold $\mathcal{M}_N$ is given a nonsymmetric tensor field $g(X,Y)$ of the type (0,2) and if the conditions

$$((\forall p \in \mathcal{M}_N)(\forall \mathcal{X} \in \mathcal{X}(\mathcal{M}_N))\}(\mathcal{X},\mathcal{Y}) = \prime) \Leftrightarrow \mathcal{Y} = \prime$$

and

$$((\forall p \in \mathcal{M}_N)(\forall \mathcal{X} \in \mathcal{X}(\mathcal{M}_N))\}(\mathcal{Y},\mathcal{X}) = \prime) \Leftrightarrow \mathcal{Y} = \prime$$

are valid, then we call the pair $(\mathcal{M}_N, \})$ a *generalized Riemannian space* and designate it GR_N. The field $g(X,Y)$ is the *basic tensor field.*
Def.3.2. An affine connexion ∇ defined on a GR_N, is a *Riemannian connexion,* if it satisfies the condition: for $X, Y, Z \in \mathcal{X}(\mathcal{M}_N)$ is

$$Z \circ h(X,Y) = h(\nabla_Z X, Y) + h(X, \nabla_Z Y), \tag{3.1}$$

where

$$h(X,Y) = \frac{1}{2}[g(X,Y) + g(Y,X)] = h(Y,X) \tag{3.2}$$

is a symmetric field.

Let us prove first of all the next lemma.

Lemma. *If* ${}^1\nabla$ *is a Riemannian connexion, then* ${}^2\nabla$*, defined by* (1.1)*, is a Riemannian connexion too, and conversely.*

Proof. Suppose that ${}^1\nabla$ is a Riemannian connexion, i.e. for $\nabla = {}^1\nabla$ (3.1) is valid. With respect to the equation (14) in [1,§3.5]: if on differentiable manifold a nonsymmetric connexion ∇ with a torsion tensor $T(X,Y)$ is given, $h(X,Y) = h(Y,X)$ is a symmetric basic tensor field on the manifold, ${}^0\nabla$ Levi-Civita connexion corresponding to the field h, then

$$\begin{aligned} &h(X, \nabla_Y Z) = \\ &= h(X, {}^0\nabla_Y Z) + \frac{1}{2}[h(X, T(Y,Z)) - h(Y, T(Z,X)) + h(Z, T(X,Y))]. \end{aligned} \tag{3.3}$$

If $\tilde{\nabla}$ is another such a connexion with the torsion tensor $\tilde{T}$, then the corresponding equation (3.3) is valid, wherefrom

$$h(X, \tilde{\nabla}_Y Z) = h(X, \nabla_Y Z) + \frac{1}{2}[h(X, \tilde{T}(Y,Z) - T(Y,Z)) -$$

$$-h(Y,\tilde{T}(Z,X)-T(Z,X))+h(Z,\tilde{T}(X,Y)-T(X,Y))]. \tag{3.4}$$

Taking $\nabla = {}^1\nabla$, $\tilde{\nabla} = {}^2\nabla$, by virtue of (3.4) one obtains

$$\begin{aligned}
&h({}^2\nabla_Z X, Y) + h(X, {}^2\nabla_Z Y) = \\
&= h({}^1\nabla_Z X, Y) + \frac{1}{2}[h(Y, \overset{2}{T}(Z,X) - \overset{1}{T}(Z,X)) - \\
&-h(Z, \overset{2}{T}(X,Y) - \overset{1}{T}(X,Y)) + h(X, \overset{2}{T}(Y,Z) - \overset{1}{T}(Y,Z))] + \\
&+h(X, {}^1\nabla_Z Y) + \frac{1}{2}[h(X, \overset{2}{T}(Z,Y) - \overset{1}{T}(Z,Y)) - \\
&-h(Z, \overset{2}{T}(Y,X) - \overset{1}{T}(Y,X)) + h(Y, \overset{2}{T}(X,Z) - \overset{1}{T}(X,Z))] = \\
&= h({}^1\nabla_Z X, Y) + h(X, {}^1\nabla_Z Y) = Z \circ h(X,Y),
\end{aligned}$$

since with respect to (1.1)

$$\overset{2}{T}(X,Y) = -\overset{1}{T}(X,Y) = T(X,Y). \tag{3.5}$$

So, if (3.1) is valid for $\nabla = {}^1\nabla$, it is valid for $\nabla = {}^2\nabla$ too. The converse assertion can be proved in the same way.

In other to find the components of Riemannian connexion ∇, we put into (3.1) $X = \partial_i$, $Y = \partial_j$, $Z = \partial_k$ and obtain

$$\partial_k h_{ij} = h(\nabla_{\partial_k}\partial_i, \partial_j) + h(\partial_i, \nabla_{\partial_k}\partial_j) = h(\Gamma^p_{ik}\partial_p, \partial_j) + h(\partial_i, \Gamma^p_{jk}\partial_p),$$

that is

$$h_{ij,k} = \Gamma^p_{ik} h_{pj} + \Gamma^p_{jk} g_{ip}, \tag{3.6}$$

where

$$\nabla_{\partial_k}\partial_i = \Gamma^p_{ik}\partial_p, \quad h_{ij,k} = \frac{\partial}{\partial x^k} h_{ij}$$

and analogously in other cases. Denoting

$$\Gamma_{i.jk} = h_{ip}\Gamma^p_{jk}, \tag{3.6'}$$

from (3.6) one gets

$$\Gamma_{i.jk} + \Gamma_{j.ik} = h_{ij,k}. \tag{3.7}$$

Since ∇ is nonsymmetric by supposition, in this case we have generally $\Gamma^i_{jk} \neq \Gamma^i_{kj}$ and $\Gamma_i.jk \neq \Gamma i.kj$. The equation (3.7) is satisfied for

$$\Gamma_{i.jk} = \frac{1}{2}(g_{ji,k} - g_{jk,i} + g_{ik,j}). \tag{3.8}$$

Supposing that $det(h_{ij}) \neq 0$, we can determine $(h^{ij}) = (h_{ij})^{-1}$ and then from (3.6′) is

$$(3.6'') \qquad \Gamma^i_{jk} = h^{ip}\Gamma_{p.jk}$$

The magnitudes $\Gamma_{i.jk}$, Γ^i_{jk} are called *generalized Christoffel symbols*, because by virtue of (3.8,6″) they are transformed as corresponding Christoffel symbols in Riemannian spaces. By the tensors h_{ij} and h^{ij} one performs respectively the lowering and raising of indices in GR_N (see for example [2],[3],[6],[7]).

We shall examine the determination of components of a Riemannian connexion by means of (3.1). If besides (3.7) we have also

$$(3.7') \qquad \tilde{\Gamma}_{i.jk} + \tilde{\Gamma}_{j.ik} = h_{ij,k}$$

then from (3.7,7′) one concludes

$$(3.9) \qquad \tilde{\Gamma}_{i.jk} - \Gamma_{i.jk} = -(\tilde{\Gamma}_{j.ik} - \Gamma_{j.ik}),$$

and we can put

$$(3.10) \qquad \tilde{\Gamma}_{i.jk} = \Gamma_{i.jk} + P_{i.jk},$$

where with respect to (3.9) $P_{i.jk} = -P_{j.ik}$ is some tensor.

Using in (3.1) ${}^2\nabla$ (defined by (1.1) instead of $\nabla \equiv {}^1\nabla$ (which is possible by virtue of the previous Lemma), then, instead of (3.5), we obtain

$$h_{ij,k} = \Gamma^p_{ki}h_{pj} + \Gamma^p_{kj}h_{ip},$$

because

$${}^2\nabla_{\partial_i}\partial_j = {}^1\nabla_{\partial_j}\partial_i = \Gamma^p_{ij}\partial_p .$$

Therefore,

$$\Gamma_{j.ki} + \Gamma_{i.kj} = h_{i.jk}$$

or

$$\Gamma_{i.jk} + \Gamma_{k.ji} = h_{ik,j}$$

and also

$$\tilde{\Gamma}_{i.jk} + \tilde{\Gamma}_{k.ji} = h_{ik,j}.$$

As in the previous case, now is

$$(3.9') \qquad \tilde{\Gamma}_{i.jk} - \Gamma_{i.jk} = -(\tilde{\Gamma}_{k.ji} - \Gamma_{k.ji}).$$

So, in relation to (3.10), we have $P_{i.jk} = -P_{k.ji}$.

From the other side, if $\overset{o}{\Gamma}{}^i_{jk}$ are Christoffel's symbols formed by h_{ij} and

$$\tau^i_{jk} = \frac{1}{2}(\Gamma^i_{jk} - \Gamma^i_{kj}), \quad \tilde{\tau}^i_{jk} = \frac{1}{2}(\tilde{\Gamma}^i_{jk} - \tilde{\Gamma}^i_{kj}) \tag{3.11}$$

we can write

$$\Gamma^i_{jk} = \overset{o}{\Gamma}{}^i_{jk} + \tau^i_{jk}, \quad \tilde{\Gamma}^i_{jk} = \overset{o}{\Gamma}{}^i_{jk} + \tilde{\tau}^i_{jk}.$$

Herefrom and from (3.10,11) it follows that

$$P_{i.jk} = \tilde{\Gamma}_{i.jk} - \Gamma_{i.jk} = \tilde{\tau}_{i.jk} - \tau_{i.jk} = -P_{i.kj}.$$

So, with respect to exposed previously, one concludes

$$P_{i.jk} = -P_{j.ik} = -P_{k.ji} = -P_{i.kj}. \tag{3.12}$$

In this manner we proved

Theorem 2. *Let on differentiable manifold GR_N be given nonsymmetric basic tensor field $g(X,Y)$, whose components in local coordinates are $g_{ij} = g(\partial_i, \partial_j) \neq g_{ji}$. Nonsymmetric affine connexion ∇, defined by virtue of* (3.1, 2), *in local coordinates is defined by the functions $\tilde{\Gamma}^i_{jk} = h^{ip}\tilde{\Gamma}_{p.jk}$, where $\tilde{\Gamma}_{p.jk}$ is defined with respect to the equations* (3.10, 8), *and P is some tensor satisfying* (3.12). *For $P = 0$ $\tilde{\Gamma}$ reduces to the generalized Christoffel symbols, given by* (3.6″, 8).

Remark. In the works [2], [3], [6], [7] the connexion coefficients are introduced by definition with respect to (3.8).

4 Covariant curvature tensors

4.0. By means of curvature tensors $\overset{\theta}{R}(X,Y)Z$ of the type (1,3) given in (1.3,4) we can define in GR_N curvature tensors of the type (0,4):

$$\overset{\theta}{R}(X,Y,Z,V) = h(\overset{\theta}{R}(X,Y)Z,V). \tag{4.1}$$

Substituting $X = \partial_l$, $Y = \partial_k$, $Z = \partial_j$, $V = \partial_i$, for components is obtained

$$\overset{\theta}{R}(\partial_l, \partial_k, \partial_j, \partial_i) = h_{pi}\overset{\theta}{R}{}^p{}_{jkl} = \overset{\theta}{R}ijkl. \tag{4.1'}$$

In the case of the usual Riemannian space R_N for the curvature tensor of the type (0,4) the next characteristics are valid:

$$\overset{0}{R}(X,Y,Z,V) = -\overset{0}{R}(Y,X,Z,V) = -\overset{0}{R}(X,Y,V,Z), \tag{4.2a}$$

$$\overset{0}{R}(X,Y,Z,V) = -\overset{0}{R}(Z,V,X,Y) \tag{4.2b}$$

$$C_{LMN}\overset{0}{R}(X,Y,Z,V) = 0, \quad \{L,M,N\} \subset \{X,Y,Z,V\}, \tag{4.2c}$$

and we have to examine these characteristics for GR_N.

4.1. According to (4.1) and (2.3), the property

$$\overset{\theta}{R}(X,Y,Z,V) = -\overset{\theta}{R}(Y,X,Z,V)$$

is in effect for $\theta \in \{1,2,8,10\}$. The second equality of the type (4.2a) for $\overset{1}{R}$, $\overset{2}{R}$ one proves by standard way. Such equality is valid for $\overset{3}{R}$, $\overset{4}{R}$ $\overset{8}{R}$, $\overset{10}{R}$ too. We shall prove it for $\overset{3}{R}$.

In order to prove that $\overset{\theta}{R}(X,Y,Z,V) = -\overset{\theta}{R}(X,Y,V,Z)$, it is sufficiently to prove that $\overset{\theta}{R}(X,Y,Z,Z) = 0$. From (4.1) and (1.3) we get

$$\overset{\theta}{R}(X,Y,Z,Z) = h(\overset{\theta}{R}(X,Y)Z,Z) =$$

$$= h({}^2\nabla_X{}^1\nabla_Y Z, Z) - h({}^1\nabla_Y{}^2\nabla_X Z, Z) + \tag{4.3}$$
$$+ h({}^2\nabla_{{}^1\nabla_Y X} Z - {}^1\nabla_{{}^2\nabla_X Y} Z, Z).$$

Applying (1.1), we have

$${}^2\nabla_{{}^1\nabla_Y X} Z - {}^1\nabla_{{}^2\nabla_X Y} Z = {}^1\nabla_Z{}^1\nabla_Y X + [{}^1\nabla_Y X, Z] - {}^2\nabla_Z{}^2\nabla_X Y - [{}^2\nabla_X Y, Z] =$$

$$= {}^1\nabla_Z{}^1\nabla_Y X - {}^2\nabla_Z({}^1\nabla_Y X + [X,Y]) + [{}^1\nabla_Y X - {}^2\nabla_X Y, Z] = \tag{4.4}$$
$$= {}^1\nabla_Z{}^1\nabla_Y X - {}^2\nabla_Z{}^1\nabla_Y X - {}^1\nabla_{[X,Y]} Z.$$

Since ${}^1\nabla$ and ${}^2\nabla$ are Riemannian connexion, we have from (3.1)

$$X \circ h({}^1\nabla_Y Z, Z) = {}^2\nabla_X(h({}^1\nabla_Y Z, Z)) =$$

$$= h({}^2\nabla_X {}^1\nabla_Y Z, Z) + h({}^1\nabla_Y Z, {}^2\nabla_X Z),$$

$$Y \circ h({}^2\nabla_X Z, Z) = {}^1\nabla_X (h({}^2\nabla_X Z, Z)) = \tag{4.5}$$

$$= h({}^1\nabla_Y {}^2\nabla_X Z, Z) + h({}^2\nabla_X Z, {}^1\nabla_Y Z),$$

$$Y \circ h(Z, Z) = 2h({}^1\nabla_Y Z, Z) = 2h({}^2\nabla_Y Z, Z),$$

and (4.3) with respect to (4.4,5) gives

$$\overset{3}{R}(X,Y,Z,Z) = X \circ h({}^1\nabla_Y Z, Z) - h({}^1\nabla_Y Z, {}^2\nabla_X Z) -$$
$$Y \circ h({}^2\nabla_X Z, Z) + h({}^2\nabla_X Z, {}^1\nabla_Y Z) +$$
$$+h({}^1\nabla_Z {}^1\nabla_Y Z, Z) - h({}^2\nabla_Z {}^1\nabla_Y X, Z) - h({}^1\nabla_{[X,Y]} Z, Z) =$$
$$X \circ \frac{1}{2} Y \circ h(Z,Z) - Y \circ \frac{1}{2} X \circ h(Z,Z) +$$
$$+h({}^2\nabla_{{}^1\nabla_Y X} Z, Z) + h([Z, {}^1\nabla_Y X], Z) -$$
$$-h({}^1\nabla_{{}^1\nabla_Y X} Z, Z) - h([Z, {}^1\nabla_Y X], Z) - h({}^1\nabla_{[X,Y]} Z, Z) =$$
$$\frac{1}{2} {}^1\nabla_Y X \circ h(Z,Z) - \frac{1}{2} {}^1\nabla_Y X \circ h(Z,Z) = 0.$$

Consequently

$$\overset{3}{R}(X,Y,Z,V) = -\overset{3}{R}(X,Y,V,Z).$$

We prove this characteristic in the same way for $\overset{4}{R}$, $\overset{8}{R}$, $\overset{10}{R}$.

4.2. Further let us examine the characteristic of the type (4.2b). We shall prove that it is valid for $\overset{7}{R}$, $\overset{8}{R}$, $\overset{9}{R}$. In relation to (4.1), (1.4) one obtains

$$\overset{7}{R}(X,Y,Z,V) = h(\overset{7}{R}(X,Y)Z, V) =$$

$$= \frac{1}{2}\{h({}^1\nabla_X {}^1\nabla_Y Z, V) + h({}^2\nabla_X {}^2\nabla_Y Z, V) - h({}^1\nabla_Y {}^2\nabla_X Z, V) - \tag{4.6}$$

$$-h({}^2\nabla_Y {}^1\nabla_X Z, V) - h({}^1\nabla_{[X,Y]} Z + {}^2\nabla_{[X,Y]} Z, V)\}.$$

Since ${}^1\nabla$, ${}^2\nabla$ are Riemannian connexions, we have

$$X \circ h({}^\theta\nabla_Y Z, V) = {}^\omega\nabla_X (h({}^\theta\nabla_Y Z, V)) = \tag{4.7}$$

$$= h({}^\omega\nabla_X {}^\theta\nabla_Y Z, V)) + h({}^\theta\nabla_Y Z, {}^\omega\nabla_X V),\ \omega, \theta \in \{1,2\}.$$

Expressing from here the first addend on the right side and substituting into (4.6), we obtain

$$\overset{7}{R}(X,Y,Z,V)=\frac{1}{2}\{X\circ h({}^1\nabla_Y Z,V)-h({}^1\nabla_Y Z,{}^1\nabla_X V)-$$
$$-X\circ h({}^2\nabla_Y Z,V)-h({}^2\nabla_Y Z,{}^2\nabla_X V)-Y\circ h({}^2\nabla_X Z,V)+$$
$$+h({}^2\nabla_X Z,{}^1\nabla_Y V)-Y\circ h({}^1\nabla_X Z,V)+h({}^1\nabla_X Z,{}^2\nabla_Y V)-$$
$$-h({}^1\nabla_{[X,Y]}Z+{}^2\nabla_{[X,Y]}Z,V)\}$$

Applying (1.1), it follows that

$$\overset{7}{R}(X,Y,Z,V)=\frac{1}{2}\{X\circ h({}^1\nabla_Y Z+{}^2\nabla_Y Z,V)-h({}^1\nabla_Y Z,{}^1\nabla_X V)-$$

$$-h({}^1\nabla_Z Y+[Y,Z],{}^1\nabla_V X+[X,V])-Y\circ h({}^1\nabla_X Z+{}^2\nabla XZ,V)+ \tag{4.8}$$
$$+h({}^1\nabla_Z X+[X,Z],{}^1\nabla_Y V)+h({}^1\nabla_X Z,{}^1\nabla_V Y+[Y,V])-$$
$$-h({}^1\nabla_{[X,Y]}Z+{}^2\nabla_{[X,Y]}Z,V)\}.$$

Herefrom

$$Q(X,Y,Z,V)\equiv\overset{7}{R}(X,Y,Z,V)-\overset{7}{R}(Z,V,X,Y)=$$
$$=\frac{1}{2}\{X\circ h({}^1\nabla_Y Z+{}^2\nabla_Y Z,V)-h({}^1\nabla_Y Z,{}^1\nabla_X V)-h({}^1\nabla_Z Y,{}^1\nabla_V X)-$$
$$-h({}^1\nabla_Z Y,[X,V])-h([Y,Z],{}^1\nabla_V X)-h([Y,Z],[X,V])-$$
$$-Y\circ h({}^1\nabla_X Z+{}^2\nabla_X Z,V)+h({}^1\nabla_Z X,{}^1\nabla_Y V)+h([X,Z],{}^1\nabla_Y V)+$$
$$+h({}^1\nabla_X Z,{}^1\nabla_V Y)+h({}^1\nabla_X Z,[Y,V])-h({}^1\nabla_{[X,Y]}Z+{}^2\nabla_{[X,Y]}Z,V)-$$
$$-Z\circ h({}^1\nabla_V X+{}^2\nabla_V X,Y)+h({}^1\nabla_Y X,{}^1\nabla_Z Y)+h({}^1\nabla_X V,{}^1\nabla_Y Z)+$$
$$+h({}^1\nabla_X V,[Z,Y])+h([V,X],{}^1\nabla_Y Z)+h([V,X],[Z,Y])+$$
$$+V\circ h({}^1\nabla_Z X+{}^2\nabla_Z X,Y)-h({}^1\nabla_X Z,{}^1\nabla_V Y)-h([Z,X],{}^1\nabla_V Y)-$$
$$-h({}^1\nabla_Z X,{}^1\nabla_Y V)-h({}^1\nabla_Z X,[V,Y])+h({}^1\nabla_{[Z,V]}X+{}^2\nabla_{[Z,V]}X,Y)\}.$$

By using of symmetric connexion ${}^0\nabla=\frac{1}{2}({}^1\nabla+{}^2\nabla)$ and arranging, we obtain

$$Q(X,Y,Z,V)=X\circ h({}^0\nabla_Y Z,V)-Y\circ h({}^0\nabla_X Z,V)-h({}^0\nabla_{[X,Y]}Z,V)-$$
$$-Z\circ h({}^0\nabla_V X,Y)-V\circ h({}^0\nabla_Z X,Y)+h({}^0\nabla_{[Z,V]}X,Y)+$$
$$\frac{1}{2}\{h([V,X],{}^1\nabla_Y Z+{}^1\nabla_Z Y)+h([Z,Y],{}^1\nabla_X V+{}^1\nabla_V X)+$$
$$h([Y,V],{}^1\nabla_X Z+{}^1\nabla_Z X)+h([X,Z],{}^1\nabla_Y V+{}^1\nabla_V Y)\},$$

and since by virtue of (1.1)

$$^1\nabla_Y Z + {}^1\nabla_Z Y = {}^1\nabla_Y Z + {}^2\nabla_Y Z + [Z,Y] = 2{}^0\nabla_Y Z + [Z,Y]$$

it follows that

$$\begin{aligned}
&Q(X,Y,Z,V) = X \circ h({}^0\nabla_Y Z, V) - Y \circ h({}^0\nabla_X Z, V) - Z \circ h({}^0\nabla_V X, Y) + \\
&+V \circ h({}^0\nabla_Z X, Y) + h([V,X], {}^0\nabla_Y Z) + \frac{1}{2}h([V,X],[Z,Y]) + \\
&+h([Z,Y], {}^0\nabla_X V) + \frac{1}{2}h([Z,Y],[V,X]) + h([Y,V], {}^0\nabla_X Z) + \\
&+\frac{1}{2}h([Y,V],[Z,X]) + h([X,Z], {}^0\nabla_Y V) + \frac{1}{2}h([X,Z],[V,Y]) + \\
&+h({}^0\nabla_{[Z,V]} X, Y) - h({}^0\nabla_{[X,Y]} Z, V).
\end{aligned}$$

For ${}^0\nabla$ equation of the type (4.7) are valid and one obtains

$$\begin{aligned}
&Q(X,Y,Z,V) = h({}^0\nabla_X {}^0\nabla_Z, V) + h({}^0\nabla_Y Z, {}^0\nabla_X V) - h({}^0\nabla_Y {}^0\nabla_X Z, V) - \\
&-h({}^0\nabla_X Z, {}^0\nabla_Y V) - h({}^0\nabla_Z {}^0\nabla_V X, Y) - h({}^0\nabla_V X, {}^0\nabla_Z Y) + \\
&+h({}^0\nabla_V {}^0\nabla_Z X, Y) + h({}^0\nabla_Z X, {}^0\nabla_V Y) + h([V,X], {}^0\nabla_Y Z) + \\
&+h([Z,Y], {}^0\nabla_X V) + h([Y,V], {}^0\nabla_X Z) + h([X,Z], {}^0\nabla_Y V) + \\
&+h([V,X],[Z,Y]) + h([Y,V],Z,X]) + h({}^0\nabla_{[Z,V]} X, Y) - h({}^0\nabla_{[X,Y]} Z, V) = \\
&= h({}^0\nabla_X {}^0\nabla_Y Z - {}^0\nabla_Y {}^0\nabla_X Z - {}^0\nabla_{[X,Y]} Z, V) - \\
&-h({}^0\nabla_Z {}^0\nabla_V X - {}^0\nabla_V {}^0\nabla_Z X - {}^0\nabla_{[Z,V]} X, Y) + \\
&+h({}^0\nabla_Y Z, {}^0\nabla_X V) - h({}^0\nabla_X V + [V,X], {}^0\nabla_Y Z + [Z,Y]) + \\
&+h({}^0\nabla_Z X, {}^0\nabla_V Y) - h({}^0\nabla_Z X + [X,Z], {}^0\nabla_V Y + [Y,V]) + \\
&+h([V,X], {}^0\nabla_Y Z) + h([Z,Y], {}^0\nabla_X V) + h([Y,V], {}^0\nabla_X Z) + \\
&+h([X,Z], {}^0\nabla_Y V) + h([V,X],[Z,Y]) + h([Y,V],[Z,X]) = \\
&= \overset{0}{R}(X,Y,Z,V) - \overset{0}{R}(Z,V,X,Y) + h({}^0\nabla_Y Z, {}^0\nabla_X V) - \\
&-h({}^0\nabla_X V, {}^0\nabla_Y Z) - h({}^0\nabla_X V, [Z,Y]) - h([V,X], {}^0\nabla_Y Z) - \\
&-h([V,X],[Z,Y]) + h({}^0\nabla_Z X, {}^0\nabla_V X) - h({}^0\nabla_Z X, {}^0\nabla_V Y) - \\
&-h({}^0\nabla_Z X, [Y,V]) - h([X,Z], {}^0\nabla_V Y) - h([X,Z],[Y,V]) + \\
&+h([V,X], {}^0\nabla_Y Z) + h([Z,Y], {}^0\nabla_X V + h([Y,V], {}^0\nabla_X Z) + \\
&+h([X,Z], {}^0\nabla_Y V) + h([V,X],[Z'Y]) + h([Y,V],[Z,X]) = \\
&= h([Y,V], {}^0\nabla_X Z - {}^0\nabla_Z X) + h([X,Z], {}^0\nabla_Y V + [V,Y] + \\
&+[V,Y] - {}^0\nabla_V Y) = 0
\end{aligned}$$

Consequently,

$$\overset{7}{R}(X,Y,Z,V)=\overset{7}{R}(Z,V,X,Y). \tag{4.9}$$

In the same way this characteristic one proves for $\overset{8}{R}$, $\overset{9}{R}$.

4.3. We have to examine the characteristic (4.2c). For $\overset{1}{R}$, $\overset{2}{R}$, $\overset{3}{R}$ this characteristic is not valid. In relation to (2.12) and (4.1) it follows that

$$C_{XYZ}\overset{4}{R}(X,Y,Z,V)=0 \tag{4.10}$$

Further, we have $C_{XYZ}\overset{4}{R}(X,Y,Z,V)=-C_{XYZ}\overset{4}{R}(X,Y,V,Z)$, we have

$$C_{XYV}\overset{4}{R}(X,Y,Z,V)=0. \tag{4.11}$$

Other relations of the type (4.2c) are not valid for $\overset{4}{R}$.

We shall study this trait for the tensor $\overset{7}{R}$. First of all, by virtue (2.15) and (4.1) we have

$$C_{XYZ}\overset{7}{R}(X,Y,Z,V)=0. \tag{4.12}$$

Further, by using of (4.8):

$$\begin{aligned}
&C_{XYZ}\overset{7}{R}(X,Y,Z,V)=C_{XYZ}\{X\circ h({}^0\nabla_Y Z,V)-Y\circ h({}^0\nabla_X Z,V)-\\
&-h({}^0\nabla_{[X,Y]}Z,V)\}+\frac{1}{2}\{-h({}^1\nabla_Y Z,{}^1\nabla_X V)-h({}^1\nabla_Z Y,{}^1\nabla_V X)-\\
&-h({}^1\nabla_Z Y,[X,V])-h([Y,Z],{}^1\nabla_V X)-h([Y,Z],[X,V])+\\
&+h({}^1\nabla_Z X,{}^1\nabla_Y V)+h([X,Z],{}^1\nabla_Y V)+h({}^1\nabla_X Z,{}^1\nabla_V Y)+\\
&+h({}^1\nabla_X Z,[Y,V])-h({}^1\nabla_V Z,{}^1\nabla_Y X)-h({}^1\nabla_Z V,{}^1\nabla_X Y)-\\
&-h({}^1\nabla_Z V,[Y,X])-h([V,Z],{}^1\nabla_X Y)-h([V,Z],[Y,X])+\\
&+h({}^1\nabla_Z Y,{}^1\nabla_V X)+h([Y,Z],{}^1\nabla_V X)+h({}^1\nabla_Y Z,{}^1\nabla_X V)+\\
&+h({}^1\nabla_Y Z,[V,X])-h({}^1\nabla_X Z,{}^1\nabla_V Y)-h({}^1\nabla_Z X,{}^1\nabla_Y V)-\\
&-h({}^1\nabla_Z X,[V,Y])-h([X,Z],{}^1\nabla_Y V)-h([X,Z],[V,Y])+\\
&+h({}^1\nabla_Z V,{}^1\nabla_X Y)+h([V,Z],{}^1\nabla_X Y)+h({}^1\nabla_V Z,{}^1\nabla_Y X)+\\
&+h({}^1\nabla_V Z,[X,Y])\}.
\end{aligned}$$

By arranging and applying of (4.7) to ${}^0\nabla$, the previous equation becomes

$$C_{XYV}\overset{7}{R}(X,Y,Z,V)=C_{XYZ}\{h({}^0\nabla_X{}^0\nabla_Y Z,V)+h({}^0\nabla_Y Z,{}^0\nabla_X V)-$$

$$-h({}^0\nabla_Y {}^0\nabla_X Z, V) - h({}^0\nabla_X Z, {}^0\nabla_Y V) - h({}^0\nabla_{[X,Y]} Z, V) +$$
$$\frac{1}{2}h([X,Z],Y,V]) + \frac{1}{2}h({}^1\nabla_X Z + {}^1\nabla_Z X, [Y,V]\} =$$
$$= C_{XYZ}\{\overset{0}{R}(X,Y,Z,V) + h({}^0\nabla_Y Z, {}^0\nabla_X V) - h({}^0\nabla_X Z, {}^0\nabla_Y V) +$$
$$+\frac{1}{2}h([X,Z] + {}^1\nabla_X Z + {}^1\nabla_Z X, [Y,V])\}.$$

Since

$$[X,Z] + {}^1\nabla_X Z + {}^1\nabla_Z X = {}^1\nabla_X Z + {}^2\nabla_X Z = 2{}^0\nabla_X Z,$$

applying (4.2c), we obtain

(4.13)

$$C_{XYV}\overset{7}{R}(X,Y,Z,V) = C_{XYV}\{h({}^0\nabla_Y Z, {}^0\nabla_X V) - h({}^0\nabla_X Z, {}^0\nabla_Y V) +$$
$$+h({}^0\nabla_X Z, [Y,V])\} = C_{XYV}\{h({}^0\nabla_Y Z, {}^0\nabla_X V) - h({}^0\nabla_X Z, {}^0\nabla_V Y)\} = 0.$$

From this equation and (4.12), applying (4.9), we get

$$C_{XYV}\overset{7}{R}(X,Y,Z,V) = C_{YZV}\overset{7}{R}(X,Y,Z,V) = 0.$$

Further, herefrom and from (4.12,13) is

$$C_{LMN}\overset{7}{R}(X,Y,Z,V) = 0, \ \{L,M,N\} \subset \{X,Y,Z,V\}.$$

In the same way, one can prove that

$$C_{LMN}\overset{9}{R}(X,Y,Z,V) = 0, \ \{L,M,N\} \subset \{X,Y,Z,V\}.$$

On the base of exposed, the next theorem is valid:

Theorem 3. *The curvature tensors of the type* (0,4) *of a nonsymmetric affine connexion, defined by* (4.1), (1.3,4) *have the following characteristics of the type* (4.2)*:*

$$\overset{\theta}{R}(X,Y,Z,V) = -\overset{\theta}{R}(Y,X,Z,V), \quad \theta \in \{1,2,8,10\},$$
$$\overset{\theta}{R}(X,Y,Z,V) = -\overset{\theta}{R}(Y,X,V,Z), \quad \theta \in \{1,2,3,4,8,10\},$$
$$\overset{\theta}{R}(X,Y,Z,V) = \overset{\theta}{R}(Z,V,X,Y), \quad \theta \in \{7,8,9\},$$
$$C_{LMN}\overset{\theta}{R}(X,Y,Z,V) = 0, \quad \{L,M,N\} \subset \{X,Y,Z,V\}, \quad \theta \in \{7,9\},$$
$$C_{XYZ}\overset{4}{R}(X,Y,Z,V) = C_{XYV}\overset{4}{R}(X,Y,Z,V) = 0.$$

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In the paper "**SOME CHARACTERISTICS OF CURVATURE TENSORS OF NONSYMMETRIC AFFINE CONNEXION**" by **Svetislav M. Minčić**, published in the Novi Sad Journal of Mathematics, Vol. 29, No. 3, 1999, 169-186. a number of errors appeared as a consequence of using different version of LATEX. The following table gives a list of the errors and corrections.

page	line	error	correction
169	-8	[1]	[5]
170	1	{	f
171	-7,-5	$\mathcal{X}\mathcal{Y}\mathcal{Z}\infty\in$	$XYZ12$
172	1,2,4	$\mathcal{X}\mathcal{Y}\mathcal{Z}$	XYZ
172	4	$\mathcal{C}$	C
176	4,6,7	$\}\mathcal{X}\mathcal{Y}\prime$	$gXY0$
177	13	other	order
178	-12	(3.5)	(3.6)
179	-1	$Rijkl$	R_{ijkl}
180	4	$= -$	$= +$
183	9	equation	equations
184	-13	C_{XYZ}	C_{XYV}
184	-13	$= C_{XYZ}$	$= C_{XYV}$

We apologize to the authors and readers for these mistakes.

Editors