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COMPOSITION OF ABEL-GRASSMANN'S 3-BANDS[1]

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Abstract. Abel-Grassmann's groupoids, or shortly AG-groupoids, have been considered in quite a number of papers, although under the different names. In some papers they are named left-almost semigroups, LA-semigroups [3], in other left invertive groupoids [2]. In this paper we introduce the notions of a 3-potent element of an AG-groupoid and of AG-3-band. We describe AG-3-band as an AG-band of Abelian groups of a certain type. Furthermore we define mappings which take part in the construction that illuminates the structure of AG-3-bands. This construction defines the multiplication between the elements from different components, thus it gives the way how to make an AG-3-band.

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1. Introduction

Groupoid S on which the following is true

$$(\forall a, b, c \in S)\ (ab)c = (cb)a, \tag{1}$$

is an Abel-Grassmann's groupoid (AG-groupoid), [1]. It is easy to verify that on every AG-groupoid holds *medial* law

$$(ab)(cd) = (ac)(bd). \tag{2}$$

The Abel-Grassmann's groupoid satisfying $(\forall a, b, c \in S)\ (ab)c = b(ca)$ is an AG^*-groupoid. It is easy to prove that any AG^*-groupoid satisfies permutation identity of a following type

$$(a_1 a_2)(a_3 a_4) = (a_{\pi(1)} a_{\pi(2)})(a_{\pi(3)} a_{\pi(4)}),$$

where π is any permutation of a set $\{1, 2, 3, 4\}$, [7].

Since AG-groupoids satisfy medial law, they belong to the class of entropic groupoids. Entropic groupoids are first introduced by D.C. Murdoch (1941)

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under the name Abelian quasigroups and appeared to be the most investigated class of groupoids. Entropic groupids also appear under the name bisymmetric, J. Aczel and M. Hoszu as well as medial, S. Stein.

Abell-Grassmann's groupoids are not associative in general, however, there is a close relation with semigroups as well as with commutative structures. Introducing new operation on AG-groupoid gives commutative semigroup.

Let $(S, \cdot)$ be AG-groupoid, $a \in S$ fixed element, we can define the "sandwich" operation on S as follows:

$$x \circ y = (xa)y, \;\; x, y \in S.$$

It was verified in [6] that if G is an AG^*-groupoid then $(x \circ y) \circ z = x \circ (y \circ z)$ i.e. $(S, \circ)$ is a commutative semigroup.

Connections mentioned above make AG-groupoids to be among the most interesting nonassociative structures.

As in Semigroup Theory, bands and band decompositions appear as one of the most useful method for research on AG-groupoids.

If on AG-groupoid G every element is an idempotent, then G is an AG-band.

An AG-groupoid G is an *AG-band Y of AG-groupoids* G_α if $G = \bigcup_{\alpha \in Y} G_\alpha$, Y is an AG-band, $G_\alpha \cap G_\beta = \emptyset$ for $\alpha, \beta \in Y$, $\alpha \neq \beta$ and $G_\alpha G_\beta \subseteq G_{\alpha\beta}$.

A congruence ρ on G is called *band congruence* if G/ρ is a band.

2. Band decompositions of AG-3-bands

Let S be a semigroup such that $a^2 = a$ holds for each $a \in S$, i.e. let S be an associative band. If $ab = ba$ holds for all $a, b \in S$, then S is a *semilattice.* A band S, satisfying $a = aba$ for all $a, b \in S$ is the rectangular band. It is a well-known result from Semigroup Theory that the band S is a semilattice of rectangular bands. It is not hard to prove that a commutative AG-band is a semilattice.

Let us now introduce the following notion.

Definition 2.1 *Let G be an AG-groupoid, $a \in G$ arbitrary element if $(aa)a = a(aa) = a$ we say that a is a* 3*-potent. AG-groupoid G is a* 3*-band (or AG-3-band) if all of its elements are* 3*-potents.* □

Some interesting properties of 3-potents in an arbitrary AG-groupoid will be given below as well as their connection with idempotent elements.

Proposition 2.1 *Let G be an AG-groupoid, if $a \in G$ is a* 3*-potent then a^2 is an idempotent.*

Proof. Let $a \in G$ be 3-potent i.e. $(aa)a = a(aa) = a$ then

$$a^2a^2 = (aa)(aa) = ((aa)a)a = aa = a^2.$$

Whence a^2 is an idempotent. □

Remark 2.1. *Let G be an AG-groupoid, $a, b \in G$ arbitrary elements. Suppose $a^2 = b^2$ is an idempotent, then, $(ab)^2 = (ab)(ab) = (aa)(bb) = a^2b^2 = a^2a^2 = a^2$, consequently, $(ab)^2 = a^2 = b^2$.*

One of the best approaches to study of one type of algebraic structure is to connect it with the other type which is better explored. This aim is accomplished for *AG*-3-bands by the next theorem, which connects *AG*-3-bands with Abelian groups.

Connections between 3-potents and idempotents mentioned above give us the motivation to introduce relation $\mathcal{K}$ on arbitrary *AG*-groupoid, as follows,

$$a\mathcal{K}b \Leftrightarrow a^2 = b^2. \tag{3}$$

It follows immediately from its definition that $\mathcal{K}$ is reflexive, symmetric and transitive, i.e. it is an equivalence relation.

Theorem 2.1 *Let G be an AG-groupoid. G is an AG-3-band if and only if it can be decomposed to an AG-band Y of Abelian groups $S_\alpha, \alpha \in Y$ satisfying $a^2 = e_\alpha$ for all $a \in S_\alpha$.*

Proof. Let G be an *AG*-3-band, $\mathcal{K}$ congruence relation defined above. Since G is an *AG*-3-band, by Proposition 2.1. a^2 is an idempotent for all $a \in G$ so $\mathcal{K}$ is a band congruence on G. Therefore, G is an *AG*-band $Y = S/_{\mathcal{K}}$ of *AG*-groupoids, by $S_\alpha, \alpha \in Y$ we shall mean equivalence classes of $\mathcal{K}$. Let $b \in S_\alpha$ be an arbitrary element by $e_\alpha = b^2$, we shall mean (the unique) idempotent element of the class S_α. It is now clear that $a^2 = e_\alpha$ for all $a \in S_\alpha$. The fact that e_α is the unique idempotent follows from the fact that $\mathcal{K}$ is the idempotent separating congruence.

Since G is an *AG*-3-band we have $a^2a = aa^2 = a$ for all $a \in G$, which means that a^2 is a neutral for a. Denote $a^2 = e$, $b \in \mathcal{K}_a$ then $be = eb = b$ since $b^2 = a^2 = e$, whence e is the identity element in $\mathcal{K}_a$. Clearly, $a^2 = e$ holds for all $a \in \mathcal{K}_a$. Since *AG*-groupoid with the identity is a semigroup we have that $\mathcal{K}_a$ is a semigroup with identity $a^2 = e$.

Let $a, b \in G$ be elements such that $a\mathcal{K}b$, then $ab = (a^2a)(bb^2)$ since a, b are 3-potent furthermore, $(a^2a)(bb^2) = (a^2b)(ab^2) = (b^2b)(aa^2) = ba$, because $a^2 = b^2$. By the above we have that $ab = ba$ for all $a, b \in \mathcal{K}_a$, whence $\mathcal{K}_a$ is a commutative semigroup.

Conversely, suppose that $G = \cup_{\alpha \in Y} S_\alpha$ where $S_\alpha, \alpha \in Y$ are commutative semigroups satisfying $a^2 = e_\alpha$ for all $a \in S_\alpha$, Y an *AG*-band. We are going to prove that G is an *AG*-3-band. Let $x \in G$ be an arbitrary element, then there exist $\beta \in Y$ such that $x \in S_\beta$. If e_β is the identity element of S_β then $x^2 = e_\beta$, consequently $x(xx) = xe_\beta = x$ and $(xx)x = e_\beta x = x$. By the above it follows that $(xx)x = x(xx) = x$ holds for all $x \in G$, so G is an *AG*-3-band. □

Example 2.1 Let G be an *AG*-groupoid given by the following table

$\cdot$	1	2	3	4	5	6	7	8
1	1	2	7	8	3	4	5	6
2	2	1	8	7	4	3	6	5
3	5	6	3	4	7	8	1	2
4	6	5	4	3	8	7	2	1
5	7	8	1	2	5	6	3	4
6	8	7	2	1	6	5	4	3
7	3	4	5	6	1	2	7	8
8	4	3	6	5	2	1	8	7

By using an AG-test [6] we can easily verify that G is an AG-groupoid, but G is not a semigroup since, for example, $(3 \cdot 2) \cdot 8 = 6 \cdot 8 = 3$ and $3 \cdot (2 \cdot 8) = 3 \cdot 5 = 7$. It is easy to verify that G is an AG-3-band as well. Consequently, G is decomposable to an AG-band $T = \{1, 3, 5, 7\}$ of a commutative inverse semigroups $S_\alpha = \{\alpha, \alpha+1\}, \alpha \in T$ with the identity element. Band T is isomorphic with the unique AG-band of order 4 (T_4 [8]), and multiplication in G_α is given with $\alpha\alpha = (\alpha+1)(\alpha+1) = \alpha$ and $\alpha(\alpha+1) = (\alpha+1)\alpha = \alpha+1, \quad \alpha \in T$. Obviosly, the semigroups S_α satisfy $x^2 = \alpha$.

In Theorem 2.1. we have described the class of AG-3-bands as the AG-band of Abelian groups. This description, however, did not illuminate the actual structure of AG-3-bands. Thus, our task now is to provide the structural description.

Similarly as in Semigroup Theory an important role in making compositions plays the concept of inner translations. Left and right inner translations on the class of AG^*-groupoids are discussed by the authors of this paper in [8], here we give a few more general properties which will be useful in the construction that comes.

Definition 2.2. [8] *Let G be an AG-groupoid and $a \in S$ an arbitrary element. Mapping $\lambda_a : G \longrightarrow G$ defined with $\lambda_a(x) = ax$ is the inner left translation on G. Dually we can define the inner right translation $\rho_a : G \longrightarrow G$ with $\rho_a(x) = xa$.*

Lemma 2.1 *Let G be an arbitrary AG-groupoid, $a, b \in G$ arbitrary elements, $\lambda_a, \rho_a, \lambda_b, \rho_b$ and λ_{ab} inner translations, then:*

1. $\rho_a \circ \rho_b = \lambda_{ab}$,
2. $\rho_a \circ \lambda_b = \rho_b \circ \lambda_a$,
3. $\rho_e \circ \rho_e = \lambda_e$, *if* $e \in E(G)$.

Proof. (1) Let $x \in G$ be arbitrary element, then

$$(\rho_a \circ \rho_b)(x) = \rho_a(\rho_b(x)) = (xb)a = (ab)x = \lambda_{ab}(x).$$

(2) Similarly,

$$(\rho_a \circ \lambda_b)(x) = \rho_a(\lambda_b(x)) = (bx)a = (ax)b = \rho_b(\lambda_a(x)) = (\rho_b \circ \lambda_a)(x).$$

(3) Let $e \in G$ be an idempotent then by 1. we have

$$\rho_e \circ \rho_e = \lambda_{ee} = \lambda_e.$$

Lemma 2.2 *Let G be an AG-groupoid, $e \in E(G)$. Inner translations λ_e and ρ_e are automorphisms on G.*

Proof. Let $x, y \in G$ be arbitrary elements, then

$$\lambda_e(xy) = e(xy) = (ee)(xy) = (ex)(ey) = \lambda_e(x)\lambda_e(y).$$

Similarly,

$$\rho_e(xy) = (xy)e = (xy)(ee) = (xe)(ye) = \rho_e(x)\rho_e(y).$$

By the above it follows that λ_e and ρ_e are homomorphisms from G to G i.e. automorphisms. □

From now on we assume that G is an *AG*-3-band or *AG*-band of Abelian groups, which is the same by Theorem 2.1. Let Y be an *AG*-band isomorphic with $E(G)$. Let $\alpha \longrightarrow e_\alpha$ be the isomorphism of Y upon $E(G)$. Thus $e_\alpha e_\beta = e_{\alpha\beta}$. Elements of G_α will be denoted by $a_\alpha, b_\alpha, \ldots$. In the next three lemmas we shall introduce two families of mappings between the components in decomposition of *AG*-3-band. We shall also give the properties of those mappings which are naturally connected with the structure of *AG*-3-band.

Lemma 2.3 *Mappings $\rho_{\alpha,\beta} : S_\beta \longrightarrow S_{\beta\alpha}$, defined by $\rho_{\alpha,\beta}(a_\beta) = a_\beta e_\alpha$ and $\lambda_{\alpha,\beta} : S_\beta \longrightarrow S_{\alpha\beta}$, defined by $\lambda_{\alpha,\beta}(a_\beta) = e_\alpha a_\beta$, are homomorphisms for all $\alpha, \beta \in Y$. Moreover, $\rho_{\alpha,\alpha}$ and $\lambda_{\alpha,\alpha}$ are identity mappings.*

Proof. Let $a_\beta, b_\beta \in G_\beta$ be arbitrary elements, then we have

$$\rho_{\alpha,\beta}(a_\beta b_\beta) = (a_\beta b_\beta)e_\alpha = (a_\beta b_\beta)(e_\alpha e_\alpha) = (a_\beta e_\alpha)(b_\beta e_\alpha) = \rho_{\alpha,\beta}(a_\beta)\rho_{\alpha,\beta}(b_\beta).$$

Similarly,

$$\lambda_{\alpha,\beta}(a_\beta b_\beta) = e_\alpha(a_\beta b_\beta) = (e_\alpha e_\alpha)(a_\beta b_\beta) = (e_\alpha a_\beta)(e_\alpha b_\beta) = \lambda_{\alpha,\beta}(a_\beta)\lambda_{\alpha,\beta}(b_\beta).$$

Obviously, $\lambda_{\alpha,\alpha}$ and $\rho_{\alpha,\alpha}$ are identity mappings since e_α is the identity in S_α.□

Lemma 2.4 *Let mappings $\rho_{\alpha,\beta}$ and $\lambda_{\alpha,\beta}$ be defined like in Lemma 2.3 then the following identities hold*

(a) $\rho_{\alpha,\beta\gamma} \circ \lambda_{\beta,\gamma} = \rho_{\beta,\alpha\gamma} \circ \lambda_{\alpha,\gamma}$,
(b) $\rho_{\alpha,\gamma\beta} \circ \rho_{\beta,\gamma} = \lambda_{\alpha\beta,\gamma}$.

Proof. Let $a_\gamma \in S_\gamma$ be arbitrary element, then we have

$$\begin{aligned}(\rho_{\alpha,\beta\gamma} \circ \lambda_{\beta,\gamma})(a_\gamma) &= \rho_{\alpha,\beta\gamma}(\lambda_{\beta,\gamma}(a_\gamma)) = \rho_{\alpha,\beta\gamma}(e_\beta a_\gamma) = (e_\beta a_\gamma)e_\alpha = (e_\alpha a_\gamma)e_\beta \\ &= \lambda_{\alpha,\gamma}(a_\gamma)e_\beta = \rho_{\beta,\alpha\gamma}(\lambda_{\alpha,\gamma}(a_\gamma)) = (\rho_{\beta,\alpha\gamma} \circ \lambda_{\alpha,\gamma})(a_\gamma).\end{aligned}$$

Similarly,

$$\begin{aligned}(\rho_{\alpha,\gamma\beta} \circ \rho_{\beta,\gamma})(a_\gamma) = & \ \rho_{\alpha,\gamma\beta}(\rho_{\beta,\gamma}(a_\gamma)) = \rho_{\alpha,\gamma\beta}(a_\gamma e_\beta) = (a_\gamma e_\beta)e_\alpha \\ = & \ (e_\alpha e_\beta)a_\gamma = (e_{\alpha\beta})a_\gamma = \lambda_{\alpha\beta,\gamma}(a_\gamma).\end{aligned}$$

The following corollary gives the connection between the mappings ρ and λ, in other words it gives the possibility to represent the mappings from λ by mappings from ρ.

Corollary 2.1 *Let $\alpha, \beta \in Y$ be arbitrary elements then we have*

$$\lambda_{\alpha,\beta} = \rho_{\alpha,\beta\alpha} \circ \rho_{\alpha,\beta}.$$

Proof. If we put α instead of β and β instead of γ in (b) of Lemma 2.4. we obtain the above identity. □

Lemma 2.5 *Let mappings $\rho_{\alpha,\beta}$ and $\lambda_{\alpha,\beta}$ be defined like in Lemma 2.3., $a_\beta, b_\beta \in G_\beta$ be arbitrary elements, then*

$$a_\alpha \cdot b_\beta = \rho_{\beta,\alpha}(a_\alpha) \cdot \lambda_{\alpha,\beta}(b_\beta).$$

Proof. Let $a_\alpha \in S_\alpha$ and $b_\beta \in S_\beta$ be arbitrary elements, then we have

$$a_\alpha \cdot b_\beta = (a_\alpha \cdot e_\alpha)(e_\beta \cdot b_\beta) = \rho_{\beta,\alpha}(a_\alpha) \cdot \lambda_{\alpha,\beta}(b_\beta). \qquad \square$$

The next theorem completely describes the structure of *AG*-3-bands and gives the way for the construction of such groupoids.

Theorem 2.2 *Let Y be an AG-band, for each $\alpha \in Y$ let us assign an Abelian group G_α such that $a_\alpha^2 = e_\alpha$ for all $a_\alpha \in S_\alpha$ and $G_\alpha \cap G_\beta = \emptyset$ if $\alpha \neq \beta$ in Y. For each $\alpha, \beta \in Y$ let us introduce the family of mappings $\rho_{\alpha,\beta} : S_\beta \longrightarrow S_{\beta\alpha}$, and $\lambda_{\alpha,\beta} : S_\beta \longrightarrow S_{\alpha\beta}$ such that $\lambda_{\alpha,\alpha} = \rho_{\alpha,\alpha} = i_{S_\alpha}$ and*

$$(a)\rho_{\alpha,\beta\gamma} \circ \lambda_{\beta,\gamma} = \rho_{\beta,\alpha\gamma} \circ \lambda_{\alpha,\gamma}$$

$$(b)\rho_{\alpha,\gamma\beta} \circ \rho_{\beta,\gamma} = \lambda_{\alpha\beta,\gamma}.$$

Then $G = \cup_{\alpha \in Y} S_\alpha$ is an AG-3-band under the operation $\star$ defined by

$$a_\alpha \star b_\beta = \rho_{\beta,\alpha}(a_\alpha) \cdot \lambda_{\alpha,\beta}(b_\beta).$$

Conversely, each AG-3-band can be constructed in the above manner.

Proof. The converse part of the theorem was already proved by Lemmas 2.3.-2.5.

Suppose $G = \cup_{\alpha \in Y} S_\alpha$ is the groupoid constructed like in the direct part of the theorem. First we prove that $(G, \cdot)$ is an *AG*-groupoid. Let $a_\alpha \in S_\alpha$,

$b_\beta \in S_\beta$ and $c_\gamma \in S_\beta$ be arbitrary elements. By using (a), (b), the facts that $S_\alpha, S_\beta, S_\gamma$ are Abelian groups and Y is an AG-band we can obtain the following

$$\begin{aligned}
(a_\alpha \cdot b_\beta) \cdot c_\gamma = & \quad (\rho_{\beta,\alpha}(a_\alpha) \cdot \lambda_{\alpha,\beta}(b_\beta)) \cdot c_\gamma \\
= & \quad \rho_{\gamma,\alpha\beta}(\rho_{\beta,\alpha}(a_\alpha) \cdot \lambda_{\alpha,\beta}(b_\beta)) \cdot \lambda_{\alpha\beta,\gamma}(c_\gamma) \\
= & \quad (\rho_{\gamma,\alpha\beta}(\rho_{\beta,\alpha}(a_\alpha)) \cdot \rho_{\gamma,\alpha\beta}(\lambda_{\alpha,\beta}(b_\beta))) \cdot \lambda_{\alpha\beta,\gamma}(c_\gamma) \\
= & \quad ((\rho_{\gamma,\alpha\beta} \circ \rho_{\beta,\alpha})(a_\alpha) \cdot (\rho_{\gamma,\alpha\beta} \circ \lambda_{\alpha,\beta})(b_\beta)) \cdot \lambda_{\alpha\beta,\gamma}(c_\gamma) \\
= & \quad \lambda_{\gamma\beta,\alpha}(a_\alpha) \cdot (\rho_{\gamma,\alpha\beta} \circ \lambda_{\alpha,\beta})(b_\beta) \cdot (\rho_{\alpha,\gamma\beta} \circ \rho_{\beta,\gamma})(c_\gamma) \\
= & \quad (\rho_{\alpha,\gamma\beta} \circ \rho_{\beta,\gamma})(c_\gamma) \cdot (\rho_{\gamma,\alpha\beta} \circ \lambda_{\alpha,\beta})(b_\beta) \cdot \lambda_{\gamma\beta,\alpha}(a_\alpha) \\
= & \quad (\rho_{\alpha,\gamma\beta} \circ \rho_{\beta,\gamma})(c_\gamma) \cdot (\rho_{\alpha,\gamma\beta} \circ \lambda_{\gamma,\beta})(b_\beta) \cdot \lambda_{\gamma\beta,\alpha}(a_\alpha) \\
= & \quad \rho_{\alpha,\gamma\beta}(\rho_{\beta,\gamma}(c_\gamma) \cdot \lambda_{\gamma,\beta}(b_\beta)) \cdot \lambda_{\gamma\beta,\alpha}(a_\alpha) \\
= & \quad (\rho_{\beta,\gamma}(c_\gamma) \cdot \lambda_{\gamma,\beta}(b_\beta)) \cdot a_\alpha \\
= & \quad (c_\gamma \cdot b_\beta) \cdot a_\alpha
\end{aligned}$$

For elements $a_\alpha, b_\alpha, c_\alpha$ Abel-Grassmann's law follows directly since S_α is an Abelian group.

Let $a_\alpha \in S_\alpha$ be an arbitrary element. Since $\rho_{\alpha,\alpha}$ and $\lambda_{\alpha,\alpha}$ are identity mappings on S_α and $a_\alpha^2 = e_\alpha$, it follows that

$$(a_\alpha \star a_\alpha) \star a_\alpha = (\rho_{\alpha,\alpha}(a_\alpha) \cdot \lambda_{\alpha,\alpha}(a_\alpha)) \star a_\alpha = (a_\alpha \cdot a_\alpha) \star a_\alpha = e_\alpha \star a_\alpha = e_\alpha \cdot a_\alpha = a_\alpha.$$

Consequently, G is an AG-3-band. □

Since by Corollary 2.1. follows that each homomorphism $\lambda_{\alpha,\beta} : S_\beta \longrightarrow S_{\alpha\beta}$ can be replaced by the composition of appropriate "right" homomorphisms we can state the next theorem which makes the construction of AG-3-bands much easier. Now we need only one family of mappings.

Theorem 2.3 *Let Y be an AG-band, for each $\alpha, \beta \in Y$ let us assign Abelian groups G_α, G_β such that $a_\alpha^2 = e_\alpha$ for all $a_\alpha \in S_\alpha$ and $G_\alpha \cap G_\beta = \emptyset$ if $\alpha \neq \beta$. Let us introduce the family of mappings $\rho_{\alpha,\beta} : S_\beta \longrightarrow S_{\beta\alpha}$, such that $\rho_{\alpha,\alpha} = i_{S_\alpha}$ and*

$$\rho_{\alpha,\beta\gamma} \circ (\rho_{\beta,\gamma\beta} \circ \rho_{\beta,\gamma}) = \rho_{\beta,\alpha\gamma} \circ (\rho_{\alpha,\gamma\alpha} \circ \rho_{\alpha,\gamma})$$

Then $G = \cup_{\alpha \in Y} S_\alpha$ is an AG-3-band under the operation defined by

$$a_\alpha \cdot b_\beta = \rho_{\beta,\alpha}(a_\alpha) \cdot \rho_{\alpha,\beta\alpha} \circ \rho_{\alpha,\beta}(b_\beta).$$

Conversely, each AG-3-band can be constructed in above manner.

Proof. Follows from Corollary 2.1. and Theorem 2.1. □

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