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FIXED POINT THEOREMS IN D-METRIC SPACE THROUGH SEMI-COMPATIBILITY

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Abstract. The objective of this paper is to introduce the notion of semi-compatible maps in D-metric spaces and deduce fixed point theorems through semi-compatibility using orbital concept, which improve extend and generalize the results of Ume and Kim [8], Rhoades [7] and Dhage et. al [6]. All the results of this paper are new.

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1. Introduction

Generalizing the notion of metric space, Dhage [3] introduced D-metric space and proved the existence of a unique fixed point of a self-map satisfying a contractive condition. Rhoades [7] generalized Dhage's contractive condition by increasing the number of factors and proved the existence of a unique fixed point of a self-map in a D-metric space. Recently, Ume and Kim [8] have introduced the notion of D-compatible maps in a D-metric space and proved the existence of a unique common fixed point of a pair of D-compatible self maps satisfying the contraction of [7].

In [2] Cho, Sharma and Sahu introduced the concept of semi-compatible maps in d-topological spaces. They define a pair of self-maps (S, T) to be semi-compatible if two conditions $(i) Sy = Ty$ implies $STy = TSy$ (ii) $\{Sx_n\} \to x, \{Tx_n\} \to x$ implies $STx_n \to Tx$, as $n \to \infty$, hold. However, (ii) implies (i), taking $x_n = y$ and $x = Ty = Sy$. So, in D-metric space, we define the semi-compatibility of the pair (S, T) by condition (ii) only.

The second section of this paper formulates the definition of a semi-compatible pair of self-maps in a D-metric space and discusses its relationship with a D-compatible pair of self-maps with an example. While doing so, we observe that, if T is continuous, then (S, T) is D-compatible implies (S, T) is semi-compatible. However, the semi-compatibility of the pair of (S, T) does not imply its D-compatibility, even if T is continuous (example 2.1). Hence it is necessary to discuss the existence of common fixed points of semi-compatible pair of self-maps in fixed point theory.

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In the light of above observations we establish two fixed point theorems in the third section, which generalize, extend and improve the results of [6], [7] and [8]. Moreover, these theorems restrict the domain of x, y and also that of boundedness and completeness considerably. Further, corollary 3.4 of our main result improves and corrects the result of Dhage et al. [6].

2. Preliminaries

Throughout this paper we use the symbols and basic definitions of Dhage [3]. In what follows, (X, D) will denote a D-metric space and N stands for the set of all natural numbers.

Definition 2.1. Let X be a non-empty set and $D : X \times X \times X \to R^+$(the set of non-negative real numbers). The pair (X, D) is said to be a D-metric space if,

(D-1) $D(x, y, z) = 0$ if and only if $x = y = z$;
(D-2) $D(x, y, z) = D(y, x, z) = D(z, y, x) = \cdots$;
(D-3) $D(x, y, z) \leq D(x, y, a) + D(x, a, z) + D(a, y, z), \forall x, y, z, a \in X$.

Definition 2.2. Let (X, D) be a D-metric space and S be a non-empty subset of X. We define the diameter of S as
$\delta_d(S) = Sup\{D(x, y, z) : x, y, z \in S\}$.

Definition 2.3. ([9]) Let T be a multi-valued map on D-metric space (X, D). Let $x_0 \in X$. A sequence $\{x_n\}$ in X is said to be an orbit of T at x_0 denoted by $O(T, x_0)$ if $x_{n-1} \in T^{n-1}(x_0)$, i. e. $x_n \in Tx_{n-1}, \forall n \in N$. An orbit $O(T, x_0)$ is said to be bounded if its diameter is finite. It is said to be complete if every Cauchy sequence in it converges to some point of X.

Definition 2.4. ([3]) A sequence $\{x_n\}$ in a D-metric space is said to converge to a point $x \in X$ if for $\epsilon > 0$, there exists a positive integer n_0 such that $D(x_n, x_m, x) < \epsilon, \forall n, m > n_0$.

Definition 2.5. ([3]) A sequence $\{x_n\}$ is said to be a D-Cauchy sequence in X if for each $\epsilon > 0$, there exists a positive integer n_0 such that $D(x_n, x_{n+p}, x_{n+p+t})$ $< \epsilon, \forall n > n_0, \forall p, t \in N$.

Definition 2.6. ([8]) A pair (S, T) of self-maps on a D-metric space (X, D) is said to be D-compatible if for all x, y and $z \in X$ and for some $\alpha \in (0, \infty)$

$$D(STx, STy, TSz) \leq \alpha D(Tx, Ty, Sz) \tag{2.1}$$

Definition 2.7. A pair (S, T) of self-mappings of a D-metric space is said to be semi-compatible if $lim_{n\to\infty} STx_n = Tx$, whenever $\{x_n\}$ is a sequence in X such that $lim_{n\to\infty} Tx_n = lim_{n\to\infty} Sx_n = x$. In other words, a pair of self-maps (S, T) is said to be semi-compatible if $lim_{n\to\infty} D(Sx_n, Sx_{n+p}, x) = 0$ and $lim_{n\to\infty} D(Tx_n, Tx_{n+p}, x) = 0$ imply $lim_{n\to\infty} D(STx_n, STx_{n+p}, Tx) = 0$.

Proposition 2.1. *Let (S, T) be a D-compatible pair of self maps on a D-metric space (X, D) and T be continuous. Then the pair (S, T) is semi-compatible.*

Proof. Let $\{Sx_n\} \to u, \{Tx_n\} \to u$. To show $STx_n \to Tu$. As T is continuous, $TSx_n \to Tu$. As (S, T) is D-compatible, for some $\alpha \in (0, \infty)$

$$D(STx, STy, TSz) \leq \alpha D(Tx, Ty, Sz), \forall x, y, z \in X.$$

Putting $x = x_n, y = x_{n+p}$ and $z = x_n$ in above condition, we get

$$D(STx_n, STx_{n+p}, TSx_n) \leq \alpha D(Tx_n, Tx_{n+p}, Sx_n),$$

which implies $lim_{n\to\infty} D(STx_n, STx_{n+p}, Tu) = 0$. Therefore $lim_{n\to\infty} STx_n = Tu$. Hence (S, T) is semi-compatible. □

Remark 2.1. *In the following example we observe that,*

(i) The pair of self-maps (S, T) is semi-compatible yet it is not D-compatible even though T is continuous.

(ii) The pair (S, T) is semi-compatible but (T, S) is not semi-compatible.

(iii) $ST = TS$, still (T, S) is not semi-compatible.

Example 2.1. *Let (X, D) be a D-metric space with $X = R^+$, and let $D : X \times X \times X \to R^+$ be defined as*

$$D(x, y, z) = Max\{|x - y|, |y - z|, |z - x|\}, \forall x, y, z \in X.$$

Define self-maps S and T on X as follows: $Sx = 0$, if $x > 0$, and $S(0) = 1$, $Tx = x$, $\forall x \in R^+$, and $x_n = \frac{1}{n}$. Then $Sx_n, Tx_n \to 0$ as $n \to \infty$.

(i) Now,

$STx_n = Sx_n \to 0 = T(0)$ i.e. $STx_n \to T(0)$.

Also as $T = I$, for any sequence $\{x_n\}$ such that $\{Sx_n\} \to u$ and $\{Tx_n\} \to u$, as $n \to \infty, \{STx_n\} = \{Sx_n\} \to u(= Tu)$ i. e. $STx_n \to Tu$. Therefore (S, T) is semi-compatible.

Further as $T = I$, T is continuous.

Taking $x = 0, y = 0$ and $z = 1$ in (2.1) we get,

$D(1, 1, 0) \leq \alpha D(0, 0, 0), \forall \alpha \in (0, \infty)$, which is not true. Hence (S, T) is not D-compatible.

(ii) Now, $Sx_n, Tx_n \to 0$ as $n \to \infty$, $TSx_n = T(0) \to 0 \neq S(0)$. Therefore (T, S) is not semi-compatible. By (i), $STx_n \to T(0)$. Therefore (S, T) is semi-compatible.

(iii) Also, we note that as $T = I, ST = TS$. Thus (S, T) is commuting yet (T, S) is not semi-compatible.

Proposition 2.2. *Let S and T be two self-maps of a D-metric space (X, D) such that $S(X) \subseteq T(X)$. For $x_0 \in X$ define sequences $\{x_n\}$ and $\{y_n\}$ in X by $Sx_{n-1} = Tx_n = y_n, \forall n \in N$. Then*

- $O(T^{-1}S, x_0) = \{x_0, x_1, x_2, \cdots, x_n, \cdots\}$,
- $O(ST^{-1}, Sx_0) = \{y_1, y_2, y_3, , \cdots, y_n, \cdots\}$.

Proof. As $Sx_0 = Tx_1$ implies $x_1 \in T^{-1}Sx_0$ and $Sx_1 = Tx_2$ gives $x_2 \in T^{-1}Sx_1 = (T^{-1}S)^2x_0$. Similarly, $Sx_{n-1} = Tx_n$ gives $x_n \in T^{-1}Sx_{n-1} = (T^{-1}S)^n x_0$. Again,

$$y_1 = Sx_0,\ y_2 = Sx_1 \in S(T^{-1}Sx_0) = (ST^{-1})Sx_0,$$
$$y_3 = Sx_2 \in S(T^{-1}ST^{-1}Sx_0) = (ST^{-1})^2Sx_0.$$
$$\cdots$$

Similarly, $y_n \in (ST^{-1})^{n-1}Sx_0$. □

In [5] Dhage introduces the following family of functions:
Let Φ denote the class of all functions $\phi : R^+ \to R^+$ satisfying:

- ϕ is continuous;
- ϕ is non-decreasing;
- $\phi(t) < t$, for $t > 0$;
- $\sum_{\infty}^{n=1} \phi^n(t) < \infty, \quad \forall t \in R^+$.

Before proving the main results we need the following lemmas :

Lemma 2.1. ([5]) *Let $\{x_n\} \subseteq X$ be bounded with D-bound M satisfying*

$$D(x_n, x_{n+1}, x_m) \le \phi^n(M), \qquad \forall \quad m > n+1,$$

where $\phi \in \Phi$. Then $\{x_n\}$ is a D-Cauchy sequence in X.

Lemma 2.2. *Let S and T be two self-maps of a D-metric space (X, D) such that:*

(I) $S(X) \subseteq T(X)$;
(II) Some orbit $\{y_n\} = O(ST^{-1}, Sx_0)$ is bounded;
(III) For all $x, y, z \in O(T^{-1}S, x_0)$ and for some $\phi \in \Phi$

$$D(Sx, Sy, Sz) \le \phi Max \left\{ \begin{array}{l} D(Tx, Ty, Tz), D(Sx, Tx, Tz), D(Sy, Ty, Tz), \\ D(Sx, Ty, Tz), D(Sy, Tx, Tz) \end{array} \right\}.$$

Then $\{y_n\}$ is a D-Cauchy sequence in $O(ST^{-1}, Sx_0)$.

Proof. Let $x_0 \in X$. As $S(X) \subseteq T(X)$, we can define sequences $\{x_n\}$ and $\{y_n\}$ in X by $Sx_{n-1} = Tx_n = y_n, \forall n \in N$. Then

$$D(y_n, y_{n+1}, y_{n+p}) = D(Sx_{n-1}, Sx_n, Sx_{n+p-1}),$$

$$\le \phi Max \left\{ \begin{array}{l} D(y_n, y_{n-1}, y_{n+p-1}), D(y_{n-1}, y_n, y_{n+p-1}), D(y_{n+1}, y_n, y_{n+p-1}), \\ D(y_n, y_n, y_{n+p-1}), D(y_{n-1}, y_{n+1}, y_{n+p-1}) \end{array} \right\}$$

i.e.
(2.2)

$$D(y_n, y_{n+1}, y_{n+p}) \le \phi Max \left\{ \begin{array}{l} D(y_n, y_{n-1}, y_{n+p-1}), D(y_{n+1}, y_n, y_{n+p-1}), \\ D(y_n, y_n, y_{n+p-1}), D(y_{n-1}, y_{n+1}, y_{n+p-1}) \end{array} \right\}$$

Again
(2.3)

$$D(y_{n-1}, y_n, y_{n+p-1}) \le \phi Max \left\{ \begin{array}{l} D(y_{n-2}, y_{n-1}, y_{n+p-2}), D(y_{n-1}, y_n, y_{n+p-2}), \\ D(y_{n-1}, y_{n-1}, y_{n+p-2}), D(y_n, y_{n-2}, y_{n+p-2}) \end{array} \right\}$$

(2.4)

$$D(y_{n+1}, y_n, y_{n+p-1}) \leq \phi Max \left\{ \begin{array}{l} D(y_n, y_{n-1}, y_{n+p-2}), D(y_{n+1}, y_n, y_{n+p-2}), \\ D(y_n, y_{n-1}, y_{n+p-2}), D(y_{n-1}, y_{n+1}, y_{n+p-2}), \\ D(y_n, y_n, y_{n+p-2}) \end{array} \right\}$$

(2.5)

$$D(y_n, y_n, y_{n+p-1}) \leq \phi Max \{D(y_{n-1}, y_{n-1}, y_{n+p-2}), D(y_n, y_{n-1}, y_{n+p-2})\}$$

(2.6)

$$D(y_{n-1}, y_{n+1}, y_{n+p-1}) \leq \phi Max \left\{ \begin{array}{l} D(y_{n-2}, y_n, y_{n+p-2}), D(y_{n-1}, y_{n-2}, y_{n+p-2}), \\ D(y_{n+1}, y_n, y_{n+p-2}), D(y_{n-1}, y_n, y_{n+p-2}), \\ D(y_{n-2}, y_{n+1}, y_{n+p-2}) \end{array} \right\}$$

Substituting (2.3)-(2.6) into (2.2) we get,

$$D(y_n, y_{n+1}, y_{n+p}) \leq \phi^2 Max_{a,b,c}\{D(y_a, y_b, y_c)\},$$

for all a, b, c such that $n-2 \leq a \leq n, n-1 \leq b \leq n+1, c = n+p-1$.

Continuing this process it follows that

$$D(y_n, y_{n+1}, y_{n+p}) \leq \phi^n Max_{a,b,c}\{D(y_a, y_b, y_c)\}, \tag{2.7}$$

for all a, b, c such that $0 \leq a \leq n, 1 \leq b \leq n+1, c = p$. Let M be the bound of $O(ST^{-1}, Sx_0)$. Then it follows from (2.7) that

$$D(y_n, y_{n+1}, y_{n+p}) \leq \phi^n(M).$$

Therefore, by Lemma 2.1, $\{y_n\}$ is a D-Cauchy sequence in $O(ST^{-1}, Sx_0)$. □

3. Main results

Theorem 3.1. *Let S and T be self-maps of a D-metric space (X, D) such that*

(3.11) $S(X) \subseteq T(X)$;

(3.12) *The pair (S, T) is semi-compatible and T is continuous;*

(3.13) *For some $x_0 \in X$, some orbit $\{y_n\} = O(ST^{-1}, Sx_0)$ is bounded and complete;*

(3.14) *For some $\phi \in \Phi$ and for all $x, y \in O(T^{-1}S, x_0) \cup O(ST^{-1}, Sx_0)$ and for all $z \in X$*

$$D(Sx, Sy, Sz) \leq \phi Max \left\{ \begin{array}{l} D(Tx, Ty, Tz), D(Sx, Tx, Tz), D(Sy, Ty, Tz), \\ D(Sx, Ty, Tz), D(Sy, Tx, Tz) \end{array} \right\}.$$

Then S and T have a unique common fixed point in X.

Proof. For $x_0 \in X$, construct sequences $\{x_n\}$ and $\{y_n\}$ in X as $Sx_{n-1} = Tx_n = y_n, \forall n \in N$. Then by Lemma 2.2, $\{y_n\}$ is a D-Cauchy sequence in $O(ST^{-1}, Sx_0)$, which is complete. Therefore,

$$y_n (= Tx_n = Sx_{n-1}) \to u \in X \tag{3.1}$$

As T is continuous and (S,T) is semi-compatible we get,

$$T^2x_n \to Tu, STx_n \to Tu \tag{3.2}$$

Step 1: Putting $x = Tx_n, y = Tx_n$ and $z = u$ in (3.14) we get

$$D(STx_n, STx_n, Su) \leq \phi Max \left\{ \begin{array}{l} D(TTx_n, TTx_n, Tu), D(STx_n, TTx_n, Tu) \\ D(STx_n, TTx_n, Tu), D(STx_n, TTx_n, Tu), \\ D(STx_n, TTx_n, Tu) \end{array} \right\}.$$

Taking limit as $n \to \infty$, using (3.2) we get,

$$D(Tu, Tu, Su) = 0,$$

which gives

$$Tu = Su. \tag{3.3}$$

Step 2: Putting $x = x_n, y = x_n$ and $z = u$ in (3.14) we get,

$$D(Sx_n, Sx_n, Su) \leq \phi Max \left\{ \begin{array}{l} D(Tx_n, Tx_n, Tu), D(Sx_n, Tx_n, Tu), \\ D(Sx_n, Tx_n, Tu), D(Sx_n, Tx_n, Tu), \\ D(Sx_n, Tx_n, Tu) \end{array} \right\}.$$

Letting $n \to \infty$ using (3.1) and (3.3) we get,

$$D(u, u, Su) \leq \phi\{D(u, u, Su)\} < D(u, u, Su), \text{ if } D(u, u, Su) > 0,$$

which is a contradiction. Therefore $D(u, u, Su) = 0$, which gives $u = Su$. Hence $u = Su = Tu$ i.e. u is a common fixed point of S and T .

Step 3: (Uniqueness) Let w be another common fixed point of S and T, then $w = Sw = Tw$. Putting $x = x_n, y = x_n$ and $z = w$ in (3.14) we get,

$$D(Sx_n, Sx_n, Sw) \leq \phi Max \left\{ \begin{array}{l} D(Tx_n, Tx_n, Tw), D(Sx_n, Tx_n, Tw), \\ D(Sx_n, Tx_n, Tw), D(Sx_n, Tx_n, Tw), \\ D(Sx_n, Tx_n, Tw) \end{array} \right\}.$$

Taking limit as $n \to \infty$ we get,

$$D(u, u, w) \leq \phi\{D(u, u, w)\} < D(u, u, w), \text{ if } D(u, u, w) > 0,$$

which is a contradiction. Therefore $D(u, u, w) = 0$, which gives $u = w$. Hence u is the unique common fixed point of S and T. □

Remark 3.1. *By (i) of Remark (2.1) it follows that there are semi-compatible maps (S,T) which are not D-compatible even if T is continuous. The above theorem investigates the common fixed points of such semi-compatible maps (S,T) in D-metric spaces.*

In [8], Ume and Kim have proved the following result using contraction of Rhoades [7] :

Corollary 3.1. ([8]) *Let X be a complete D-metric space and S and T be self maps on X satisfying :*
- $\delta_d(O_S(Tx_0)) < \infty$;
- $S(X) \subseteq T(X)$;
- *The pair (S,T) is D-compatible and T is continuous;*
- *For some $q \in [0,1)$ and for all $x, y, z \in X$,*

$$D(Sx,Sy,Sz) \leq qMax\left\{ \begin{array}{l} D(Tx,Ty,Tz), D(Sx,Tx,Tz), D(Sy,Ty,Tz), \\ D(Sx,Ty,Tz), D(Sy,Tx,Tz) \end{array} \right\}.$$

Then S and T have a unique common fixed point.

The following corollary is a generalization of it.

Corollary 3.2. *Let S and T be self-maps of a D-metric space (X,D) satisfying* (3.11), (3.13), (3.14) *and*
(3.31) *The pair (S,T) is D-compatible and T is continuous.*
Then S and T have a unique common fixed point in X.

Proof. Result follows by using Theorem 3.1 and proposition 2.1. □

Remark 3.2. *The above result of [8] is a particular case of Corollary 3.2.*

The following theorem is a counterpart of Theorem 3.1, in which the continuity of S is assumed instead of that of T. This also improves the result of [8].

Theorem 3.2. *Let S and T be self-maps of a D-metric space (X,D) satisfying* (3.11), (3.13) *and*
(3.51) *The pair (S,T) is semi-compatible and S is continuous.*
(3.52) *For some $\phi \in \Phi$, for all $x, y \in O(T^{-1}S, x_0), z \in X$,*

$$D(Sx,Sy,Sz) \leq \phi Max\left\{ \begin{array}{l} D(Tx,Ty,Tz), D(Sx,Tx,Tz), D(Sy,Ty,Tz), \\ D(Sx,Ty,Tz), D(Sy,Tx,Tz) \end{array} \right\}.$$

Then the self-maps S and T have a unique common fixed point.

Proof. For $x_0 \in X$, construct sequences $\{x_n\}$ and $\{y_n\}$ in X as in proof of theorem 3.1.Therefore (3.1) holds. As S is continuous we get $STx_n \to Su$, and as(S,T) is semi-compatible we get

$$STx_n \to Tu.$$

As the limit of a sequence is unique we get $Su = Tu$ and the rest of the proof follows from steps 2 and 3 of Theorem 3.1. □

Remark 3.3. *The above theorem is an improvement of Theorem 3.1 and also of the result of [8]. It (in view of 3.51) also underlines the exclusive importance of semi-compatibility in fixed point theory.*

In [7] Rhoades has proved the following:
Theorem 1 [7]: Let X be a complete and bounded D-metric space and let S be a self-map of X satisfying

$$D(Sx, Sy, Sz) \leq qMax \left\{ \begin{array}{l} D(x, y, z), D(Sx, x, z), D(Sy, y, z), \\ D(Sx, y, z), D(x, Sy, z) \end{array} \right\}$$

for all $x, y, z \in X$ and for $0 \leq q < 1$. Then S has a unique fixed point p in X and S is continuous at p .

The following corollary improves and generalizes it by restricting the domains of boundedness, completeness and that of variables x and y to same orbit only.

Corollary 3.3. *Let S be a self map of a D-metric spaces (X, D) satisfying*
(3.71) *For $x_0 \in X$, an orbit $O(S, x_0)$ is bounded and complete;*
(3.72) *For some $0 \leq q < 1$, for all $x, y \in O(S, x_0)$ and $z \in X$,*

$$D(Sx, Sy, Sz) \leq qMax \left\{ \begin{array}{l} D(x, y, z), D(Sx, x, z), D(Sy, y, z) \\ D(Sx, y, z), D(x, Sy, z) \end{array} \right\}.$$

Then S has a unique fixed point.

Proof. Result follows from Theorem 3.1 by taking $T = I$ and $\phi = q(< 1)$ then (3.11) and (3.12) are trivially satisfied and in this case $O(T^{-1}S, x_0) \cup O(ST^{-1}, Sx_0) = O(S, x_0)$. □

In [6] Dhage et. al prove the following:

Theorem 3.3. ([6]) *Let (X, D) be a D-metric space and S be a self map of X. Suppose that there exists $x_0 \in X$ such that $O(S, x_0)$ is D-bounded and S is orbitally complete. Suppose also that S satisfies*

$$D(Sx, Sy, Sz) \leq \lambda Max\{D(x, y, z), D(x, Sx, z)\}, \forall x, y, z \in \overline{O(S, x_0)},$$

for some $0 \leq \lambda < 1$. Then S has a unique fixed point in X.

The following corollary improves, corrects and generalizes this result.

Corollary 3.4. *Let X be a D-metric space and S be a self-map on X satisfying* (3.61) *and*
(3.81) $D(Sx, Sy, Sz) \leq \lambda Max\{D(x, y, z), D(x, Sx, z)\}$,
$\forall x, y \in O(S, x_0), \forall z \in X$. *Then S has a unique fixed point.*

Proof. Result follows from Corollary 3.3 by taking the maximum of first two factors in place of five factors of (3.72). □

Remark 3.4. *The above corollary improves the result of [6] in which x, y and z are taken in $\overline{O(S, x_0)}$ in the contractive condition whereas in the above corollary the domain of x, y is just the orbit $O(S, x_0)$, not its closure. Also, the domain of z is the whole space X not $\overline{O(S, x_0)}$, for otherwise the uniqueness of the fixed point does not follow. This is the correction required in [6].*

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