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WEAK CONGRUENCES OF ALGEBRAS WITH CONSTANTS[1]

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Abstract. The paper deals with weak congruences of algebras having at least two constants in the similarity type. The presence of constants is a necessary condition for complementedness of the weak congruence lattices of non-trivial algebras. Some sufficient conditions for the same property are also given. In particular, so called 0,1-algebras have complemented weak congruence lattices if and only if their subalgebra lattices are complemented. In this context we also investigate relations among algebras with balanced congruences, balanced weak congruences, consistent and strongly consistent algebras. We prove that an algebra has balanced weak congruences if and only if it is strongly consistent and has balanced congruences on all subalgebras. For a variety, strong consistency of algebras is equivalent with having balanced weak congruences. Finally, we prove that for a class of algebras which additionally are Hamiltonian, there is a homomorphism from the congruence lattice onto the subalgebra lattice.

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0. Introduction

If $\mathcal{A} = (A, F)$ is an algebra, then we denote by $Cw\mathcal{A}$ its **weak congruence lattice** (i.e., the lattice of all symmetric, transitive and compatible relations on $\mathcal{A}$).

We have $Cw\mathcal{A} = Sub(A \times A, F \cup \{s, t\})$, the subalgebra lattice of the algebra $(A \times A, F \cup \{s, t\})$, where $(A \times A, F)$ is the direct product $\mathcal{A} \times \mathcal{A}$ and s resp. t is the unary, resp. binary operation on $A \times A$ defined by

$$s(x, y) := (y, x),$$

$$t((x, y), (u, v)) := \begin{cases} (x, v), & \text{if } y = u, \\ (x, y) & \text{otherwise.} \end{cases}$$

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This shows that $Cw\mathcal{A}$ is an algebraic lattice.

Let Δ be the diagonal relation on $\mathcal{A}$:

$$\Delta := \{(x,x) \mid x \in A\}.$$

The mapping $m_\Delta : \rho \mapsto \rho \wedge \Delta (= \rho \cap \Delta)$ is a homomorphism from $Cw\mathcal{A}$ onto the principal ideal $\downarrow\Delta$ in the same lattice, which is isomorphic with the subalgebra lattice $Sub\mathcal{A}$ of $\mathcal{A}$, under

$$B \mapsto \{(x,x) \mid x \in B\} = \Delta_B, \;\; \mathcal{B} \in Sub\mathcal{A}.$$

Obviously, the kernel of m_Δ is a congruence relation on $Cw\mathcal{A}$. The quotient lattice consists of the blocks of this congruence, which are the intervals $[\Delta_B, B^2]$ in $Cw\mathcal{A}$, and these are isomorphic to congruence lattices $Con\mathcal{B}$ of subalgebras $\mathcal{B}$.

We put $M_\Delta := \{B^2 \mid \mathcal{B} \in Sub\mathcal{A}\}$.

In this paper we give various results concerning the weak congruence lattice of an algebra, demanding only one condition throughout the paper, namely that the algebra has at least two different constants (i.e., nullary operations) in the similarity type. A trivial fact for such algebras is that $\emptyset$ is not a weak congruence. For algebras with nullary operations we can read directly from the weak congruence lattice whether such an algebra has a one-element minimal subalgebra (which is a constant at the same time) or it has a minimal subalgebra with more than one element. In the latter case we have that in the weak congruence lattice there is no element $\Delta_B \in \downarrow\Delta$, such that $\Delta_B = B^2$ (i.e., that $|B| = 1$).

If θ is a congruence on a subalgebra $\mathcal{B}$ of $\mathcal{A}$, then we denote by θ_A the smallest congruence on $\mathcal{A}$ containing θ:

$$\theta_A := \bigcap \{\rho \in Con\mathcal{A} \mid \theta \subseteq \rho\}.$$

In the weak congruence lattice we have $\theta_A = \theta \vee \Delta$. Furthermore, we put $\Delta_\theta := \theta \wedge \Delta$.

In the weak congruence lattice Δ is a codistributive element, i.e., for all $\rho, \theta \in Cw\mathcal{A}$,

$$\Delta \wedge (\rho \vee \theta) = (\Delta \wedge \rho) \vee (\Delta \wedge \theta).$$

(Note that this condition just reflects the property that m_Δ is a homomorphism from $Cw\mathcal{A}$ to $\downarrow \Delta$.)

The dual is not always satisfied. If it is, then $\mathcal{A}$ is said to possess the **congruence intersection property**, the **CIP**. So, $\mathcal{A}$ has the CIP if for all $\rho, \theta \in Cw\mathcal{A}$

$$\Delta \vee (\rho \wedge \theta) = (\Delta \vee \rho) \wedge (\Delta \vee \theta).$$

Recall also that an algebra $\mathcal{A}$ is **simple** if it has no non-trivial congruences.

1. Complemented weak congruence lattices

Having nullary operations is a necessary condition for a non-trivial algebra to have a complemented weak congruence lattice:

Lemma 1. *If an algebra $\mathcal{A}$ has more than one element and Δ has a complement in the weak congruence lattice, then $\mathcal{A}$ has constants and its smallest subuniverse has more than one element.*

Proof. The complement ρ of the diagonal relation Δ in the weak congruence lattice belongs to the bottom class of the quotient lattice, i.e., to $Con\mathcal{B}_m$, where $\mathcal{B}_m$ denotes the smallest subalgebra of $\mathcal{A}$. It cannot be Δ_{B_m}, because $\Delta_{B_m} \leq \Delta$ and $A^2 \neq \Delta$. Moreover, we can easily conclude that a complement for Δ is B_m^2. Hence, $B_m^2 \vee \Delta = A^2$. Therefore, for any square C^2 of a subuniverse C, we have that $C^2 \vee \Delta = A^2$. Since a one-element or empty subuniverse coincides with its square, it obviously cannot be a complement of Δ. $\square$

Another necessary condition for the complementedness of the weak congruence lattice is the complementedness of the subalgebra lattice.

Proposition 1. *If the weak congruence lattice of $\mathcal{A}$ is complemented, then the subalgebra lattice of $\mathcal{A}$ is also complemented.*

Proof. This is a consequence of the fact that $\rho \mapsto \rho \wedge \Delta$ is a homomorphism from $Cw\mathcal{A}$ onto $Sub\mathcal{A}$: The quotient lattice is isomorphic to the subalgebra lattice. If $Cw\mathcal{A}$ is complemented, then this quotient lattice and hence also $Sub\mathcal{A}$ must be complemented. $\square$

Lemma 2. *If $\mathcal{B} \in Sub\mathcal{A}$, $B \neq \emptyset$, $\theta \in Cw\mathcal{A}$ and $B^2 \leq \theta$, then the block of θ containing B is also a subalgebra of $\mathcal{A}$.*

Proof. Straightforward. $\square$

In the following theorem we give a sufficient condition for the complementedness of weak congruence lattices.

Theorem 1. *Let $\mathcal{A}$ be an algebra with constants. If $Sub\mathcal{A}$ and each $Con\mathcal{B}$ for $\mathcal{B} \in Sub\mathcal{A}$ are complemented lattices and no congruence $\neq A^2$ has a block which is a subalgebra, then $Cw\mathcal{A}$ is complemented.*

Proof. Let $\rho \in Con\mathcal{C}$, where $\mathcal{C} \in Sub\mathcal{A}$. We will prove that a complement for ρ is given by a complement of the element $(\rho \vee \Delta) \wedge D^2$ in $Con\mathcal{D}$, where $\mathcal{D}$ is a complement of $\mathcal{C}$ in $Sub\mathcal{A}$.

Let θ be a complement of the element $(\rho \vee \Delta) \wedge D^2$ in $Con\mathcal{D}$, i.e.,

$$\theta \wedge (\rho \vee \Delta) \wedge D^2 = \Delta_D, \tag{1}$$

$$\theta \vee ((\rho \vee \Delta) \wedge D^2) = D^2. \tag{2}$$

Now, we prove that θ is a complement of ρ in $Cw\mathcal{A}$.

By (1),

$\rho \wedge \theta = \rho \wedge \theta \wedge (\rho \vee \Delta) \wedge D^2 = \rho \wedge \Delta_D = \Delta_C \wedge \Delta_D = \Delta_{B_m}$, where $\mathcal{B}_m$ denotes the smallest subalgebra of $\mathcal{A}$.

Further, $\rho \vee \theta \geq \Delta_C \vee \Delta_D = \Delta$, hence

$\rho \vee \theta = \rho \vee \theta \vee \Delta \geq \rho \vee \theta \vee ((\rho \vee \Delta) \wedge D^2) = \rho \vee D^2$ by (2).

Hence,

$\rho \vee \theta = \rho \vee D^2 = \rho \vee \Delta \vee D^2$.

Thus $\rho \vee \theta \in Con\mathcal{A}$, $D^2 \leq \rho \vee \theta$, and $\mathcal{D}$ is a (non-empty) subalgebra of $\mathcal{A}$. By Lemma 2, $\rho \vee \theta$ contains a block which is a subalgebra, thus $\rho \vee \theta = A^2$. $\square$

Remark For a variety $\mathcal{V}$, the following are equivalent (see [4]):

(i) For any $\mathcal{A} \in \mathcal{V}$, no congruence $\neq A^2$ has a block which is a subalgebra.

(ii) $\mathcal{V}$ is "semidegenerated", i. e. non-singleton algebras of $\mathcal{V}$ have no singleton subalgebras.

2. 0,1-algebras

In the following we consider algebras $\mathcal{A}$ with two constants $\mathbf{0}$ and $\mathbf{1}$ satisfying:

(i) The set $\{\mathbf{0}, \mathbf{1}\}$ is a subuniverse of $\mathcal{A}$.

(ii) If $\theta \neq A^2$ is a congruence on $\mathcal{A}$, then $[\mathbf{0}]_\theta \neq [\mathbf{1}]_\theta$.

Such algebras $\mathcal{A}$ are said to be **$\mathbf{0}, \mathbf{1}$-algebras**.

A variety $\mathcal{V}$ is called a **$\mathbf{0}, \mathbf{1}$-variety** if every algebra of $\mathcal{V}$ is a $\mathbf{0}, \mathbf{1}$-algebra.

Examples of $\mathbf{0}, \mathbf{1}$-algebras (varieties) are bounded lattices, ortholattices, orthomodular lattices, Boolean algebras and Boolean rings with unit.

The following proposition is a direct consequence of the fact that $\{\mathbf{0}, \mathbf{1}\}$ is the smallest subalgebra.

Proposition 2. *If $\mathcal{A}$ is a $\mathbf{0}, \mathbf{1}$-algebra, then the smallest m_Δ-block (the bottom block) in $Cw\mathcal{A}$ is a two-element chain.*

Proposition 3. *For a $\mathbf{0}, \mathbf{1}$-algebra $\mathcal{A}$, the set M_Δ is a sublattice of $Cw\mathcal{A}$.*

Proof. If $\mathcal{B}, \mathcal{C}$ are two subalgebras of $\mathcal{A}$, then $B \cap C \neq \emptyset$ (the constants $\mathbf{0}, \mathbf{1}$ are in $B \cap C$), and hence $B^2 \cap C^2 = (B \cap C)^2$ and $B^2 \vee C^2 = (B \vee C)^2$. The first equation is obvious (and holds also for $B \cap C = \emptyset$), the second one follows from Lemma 2 and the fact that $B \vee C$ is the smallest subuniverse containing the union $B \cup C$. $\square$

Observe that the equation $B^2 \vee C^2 = (B \vee C)^2$ need not hold in case that $\mathcal{B}, \mathcal{C}$ are subalgebras with an empty intersection.

Proposition 4. *If $\mathcal{A}$ is a $\mathbf{0},\mathbf{1}$-algebra, then for every subalgebra $\mathcal{B}$ of $\mathcal{A}$,*

$$B^2 \vee \Delta = A^2.$$

Proof. Every subalgebra of $\mathcal{A}$ contains the constants $\mathbf{0},\mathbf{1}$ and, by condition (ii), no non-trivial congruence on $\mathcal{A}$ has a block containing both $\mathbf{0}$ and $\mathbf{1}$. □

As an obvious consequence of Proposition 4, we have also the following

Corollary 1. *No congruence distinct from A^2 in a $\mathbf{0},\mathbf{1}$-algebra $\mathcal{A}$ has a block which is a subalgebra of $\mathcal{A}$.*

Corollary 2. *$\{\mathbf{0},\mathbf{1}\}^2$ is a complement of the diagonal relation Δ in the weak congruence lattice of any $\mathbf{0},\mathbf{1}$-algebra.*

Proposition 5. *No non-simple $\mathbf{0},\mathbf{1}$-algebra possesses the CIP.*

Proof. Denote by $\mathcal{B}_m$ the smallest subalgebra of $\mathcal{A}$ ($B_m = \{\mathbf{0},\mathbf{1}\}$), and let θ be a non-trivial congruence of $\mathcal{A}$. Then B_m^2 and θ are not comparable (under inclusion), otherwise $\mathbf{0}$ and $\mathbf{1}$ would be in the same block of θ. Hence,

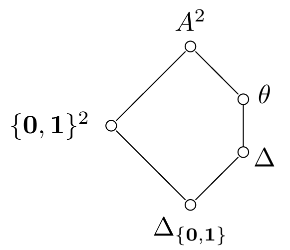
$$\Delta \vee (B_m^2 \wedge \theta) = \Delta \vee \Delta_{B_m} = \Delta.$$

On the other hand

$$(\Delta \vee B_m^2) \wedge (\Delta \vee \theta) = A^2 \wedge \theta = \theta,$$

and the CIP is not satisfied. □

The proof of Proposition 5 shows that for a non-simple $\mathbf{0},\mathbf{1}$-algebra $\mathcal{A}$, its weak congruence lattice contains the following sublattice:



Therefore, we have proved:

Proposition 6. *Non-simple algebras in $\mathbf{0},\mathbf{1}$-varieties have non-modular weak congruence lattices.*

Complementedness of weak congruence lattices of $\mathbf{0},\mathbf{1}$-algebras is equivalent to complementedness of their subalgebra lattices, as proved in the sequel:

Theorem 2. *A* $\mathbf{0},\mathbf{1}$*-algebra* $\mathcal{A}$ *has a complemented weak congruence lattice if and only if its subalgebra lattice is complemented.*

Proof. Suppose that the subalgebra lattice $Sub\mathcal{A}$ is complemented. Let $\theta \in Cw\mathcal{A}$, more precisely, let $\theta \in Con\mathcal{C}$, $\mathcal{C} \in Sub\mathcal{A}$. Further, let $\mathcal{C}'$ be a complement of $\mathcal{C}$ in the lattice $Sub\mathcal{A}$. Hence,

$$C \wedge C' = B_m = \{\mathbf{0}, \mathbf{1}\}, \;\; C \vee C' = A.$$

Case 1: θ does not contain B_m^2. We claim that ${C'}^2$ is a complement of θ in $Cw\mathcal{A}$.

Indeed,

$\theta \wedge {C'}^2 \in Con\mathcal{B}_m$, since $\Delta_\theta = \Delta_C$ and $C \wedge C' = B_m$.

Now, since θ does not contain $\{\mathbf{0},\mathbf{1}\}^2 = B_m^2$, it follows that $\theta \wedge {C'}^2$ must be equal to Δ_{B_m} (the only remaining element in $Con\mathcal{B}_m$).

Further,

$\theta \vee C'^2 \geq \Delta_C \vee \Delta_{C'} \vee C'^2 = \Delta \vee C'^2 = A^2$ by Proposition 4, thus

$\theta \vee C'^2 = A^2$.

Case 2: $\theta \geq B_m^2$. We claim that $\Delta_{C'}$ is a complement of θ in $Cw\mathcal{A}$. $\theta \wedge \Delta_{C'} = \Delta_C \wedge \Delta_{C'} = \Delta_{B_m}$, furthermore $\theta \vee \Delta_{C'} \geq B_m^2 \vee \Delta_C \vee \Delta_{C'} = B_m^2 \vee \Delta = A^2$, again by Proposition 4.

Conversely, suppose that $Cw\mathcal{A}$ is a complemented lattice. Then Proposition 1 shows that also $Sub\mathcal{A}$ is complemented. □

3. Balanced weak congruences and strongly consistent algebras

An algebra $\mathcal{A}$ with $\mathbf{0}$ and $\mathbf{1}$ has **balanced congruences** (cf. [2] or [4]) if for every $\rho, \theta \in Con\mathcal{A}$ the following holds:

$[\mathbf{0}]_\rho = [\mathbf{0}]_\theta$ if and only if $[\mathbf{1}]_\rho = [\mathbf{1}]_\theta$.

Here we introduce a similar notion for weak congruences:

An algebra $\mathcal{A}$ with $\mathbf{0}$ and $\mathbf{1}$ has **balanced weak congruences** if for every $\rho, \theta \in Cw\mathcal{A}$

$[\mathbf{0}]_\rho = [\mathbf{0}]_\theta$ if and only if $[\mathbf{1}]_\rho = [\mathbf{1}]_\theta$.

Following [1], we give the definition of $\mathbf{0},\mathbf{1}$-coherent algebras:

$\mathcal{A}$ is **$\mathbf{0},\mathbf{1}$-coherent** if for every subalgebra $\mathcal{B}$ of $\mathcal{A}$, the following holds: if B contains a block of a congruence with $\mathbf{0}$ or $\mathbf{1}$, then B is a union of blocks.

An algebra $\mathcal{A}$ with $\mathbf{0}$ and $\mathbf{1}$ is **consistent** (cf. [3] or [4]) if for each $\theta \in Con\mathcal{A}$ and every subalgebra $\mathcal{B}$ of $\mathcal{A}$ the following holds:

$[\mathbf{0}]_\theta \subseteq B$ if and only if $[\mathbf{1}]_\theta \subseteq B$.

We say that $\mathcal{A}$ is **strongly consistent** if for each $\theta \in Cw\mathcal{A}$ and every subalgebra $\mathcal{B}$ of $\mathcal{A}$

$[\mathbf{0}]_\theta \subseteq B$ if and only if $[\mathbf{1}]_\theta \subseteq B$.

In the following part we investigate connections between: consistent algebras, algebras with consistent subalgebras, and strongly consistent algebras on the one hand and algebras with balanced congruences, balanced congruences on all subalgebras and balanced weak congruences on the other.

Examples:

Every Boolean algebra has balanced weak congruences.
No non-trivial bounded lattice has balanced weak congruences.

In the rest of the paper, $\mathcal{A}$ is always an algebra with two constants $\mathbf{0}$ and $\mathbf{1}$. The following proposition is straightforward:

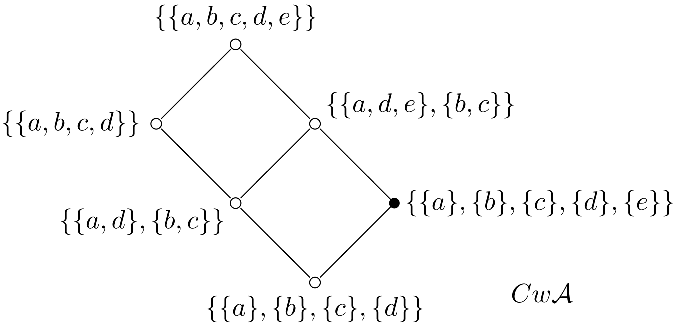
Proposition 7. *If $\mathcal{A}$ has balanced weak congruences, then $\mathcal{A}$ has balanced congruences.*

However, the opposite implication does not hold, which is illustrated by the following example. In order to present properties of congruences on all subalgebras of an algebra $\mathcal{A}$, we use the lattice $Cw\mathcal{A}$ of weak congruences of $\mathcal{A}$.

Example 1

$\mathcal{A} = (\{a, b, c, d, e\}, *, a, b)$

$*$	a	b	c	d	e
a	d	a	d	a	d
b	c	c	b	b	c
c	b	b	c	c	b
d	a	d	a	d	d
e	a	a	e	e	e



$Cw\mathcal{A}$

This algebra has balanced congruences on all subalgebras, but it does not have balanced weak congruences. The algebra possesses the CIP, but it is not coherent.

Proposition 8. *If every subalgebra of an algebra $\mathcal{A}$ has balanced congruences and is $\mathbf{0}, \mathbf{1}$-coherent, then $\mathcal{A}$ has balanced weak congruences.*

Proof. Let $\rho \in Con\mathcal{B}$, $\theta \in Con\mathcal{C}$, $\mathcal{B}, \mathcal{C} \in Sub\,\mathcal{A}$. If $[\mathbf{0}]_\rho = [\mathbf{0}]_\theta$ then $[\mathbf{0}]_\rho \subseteq B \cap C$ and $[\mathbf{0}]_\theta \subseteq B \cap C$.

Hence, $B \cap C$ is a union of classes of ρ and it is also a union of classes of θ (by $\mathcal{B}$, $\mathcal{C}$ being $\mathbf{0}, \mathbf{1}$-coherent). Therefore, $[\mathbf{1}]_\rho \subseteq B \cap C$ and $[\mathbf{1}]_\theta \subseteq B \cap C$. Since $B \cap C$ has balanced congruences, we have that $[\mathbf{1}]_\rho = [\mathbf{1}]_\theta$ $\square$

Proposition 9. *If $\mathcal{A}$ has balanced weak congruences, then it is strongly consistent.*

Proof. Suppose that the algebra $\mathcal{A}$ is not strongly consistent, i.e., that there are $\phi \in Cw\,\mathcal{A}$, $\phi \in Con\,\mathcal{C}$, $\mathcal{C} \in Sub\,\mathcal{A}$ and $\mathcal{B} \in Sub\,\mathcal{A}$ such that $[\mathbf{0}]_\phi \subseteq B$ and $[\mathbf{1}]_\phi \not\subseteq B$.

Consider $\phi \cap (B \cap C)^2$ which is a congruence on $\mathcal{B} \cap \mathcal{C}$.

$[\mathbf{0}]_{\phi\cap(B\cap C)^2} = [\mathbf{0}]_\phi$, since $[\mathbf{0}]_\phi \subseteq B$ and $[\mathbf{0}]_\phi \subseteq C$.

On the other hand, $[\mathbf{1}]_{\phi\cap(B\cap C)^2} \neq [\mathbf{1}]_\phi$, since $[\mathbf{1}]_\phi \not\subseteq B$ and $[\mathbf{1}]_{\phi\cap(B\cap C)^2} \subseteq B$. Therefore, $\mathcal{A}$ does not have balanced weak congruences. □

Proposition 10. *If $\mathcal{A}$ is strongly consistent and if it has balanced congruences on all subalgebras, then it has balanced weak congruences.*

Proof. Let $\rho \in Con\,\mathcal{B}$, $\theta \in Con\,\mathcal{C}$, $\mathcal{B}, \mathcal{C} \in Sub\,\mathcal{A}$ and $[\mathbf{0}]_\rho = [\mathbf{0}]_\theta$. Then $[\mathbf{0}]_\rho \subseteq B \cap C$ and $[\mathbf{0}]_\theta \subseteq B \cap C$. Since $\mathcal{A}$ is strongly consistent, $[\mathbf{1}]_\rho \subseteq B \cap C$ and $[\mathbf{1}]_\theta \subseteq B \cap C$. By $[\mathbf{0}]_\rho = [\mathbf{0}]_\theta$ it follows that $[\mathbf{0}]_{\rho\cap(B\cap C)^2} = [\mathbf{0}]_{\theta\cap(B\cap C)^2}$, and since all congruences are balanced on $\mathcal{B} \cap \mathcal{C}$,

$$[\mathbf{1}]_{\rho\cap(B\cap C)^2} = [\mathbf{1}]_{\theta\cap(B\cap C)^2}.$$

Hence, $[\mathbf{1}]_\rho = [\mathbf{1}]_\theta$. □

Corollary 3. *An algebra $\mathcal{A}$ is strongly consistent and has balanced congruences on all subalgebras if and only if it has balanced weak congruences.*

A variety $\mathcal{V}$ with $\mathbf{0}$ and $\mathbf{1}$ is called (strongly) consistent resp. balanced if every algebra of $\mathcal{V}$ is (strongly) consistent resp. balanced.

Theorem 3. *A variety $\mathcal{V}$ with $\mathbf{0}$ and $\mathbf{1}$ is strongly consistent if and only if each $\mathcal{A} \in \mathcal{V}$ has balanced weak congruences.*

Proof. One direction follows by Proposition 9. "Only if" part follows by the fact that every consistent variety is balanced (see [3] or [4]) and by Proposition 10. □

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