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# WEAKLY $\lambda$-CONTINUOUS FUNCTIONS

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**Abstract.** It is the objective of this paper to introduce a new class of generalizations of continuous functions via $\lambda$-open sets called weakly $\lambda$-continuous functions. Moreover, we study some of its fundamental properties. It turns out that weak $\lambda$-continuity is weaker than $\lambda$-continuity [1].

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## 1. Introduction

Maki [13] offered a new and useful notion in the field of topology which he called a $\Lambda$-set. A $\Lambda$-set is a set $A$ which is equal to its kernel (= saturated set), i.e. to the intersection of all open supersets of $A$. Arenas et al. [1] introduced and investigated the notion of $\lambda$-closed sets by involving $\Lambda$-sets and closed sets. By utilizing $\lambda$-closed sets, they introduced and to some extent investigated the notion of $\lambda$-continuity. Quite recently, several authors investigated some new maps and notions via $\lambda$-open and $\lambda$-closed sets (see for example [2], [3], [4], [10], [5] and [7]).

In this paper, we establish a new class of functions called weakly $\lambda$-continuous functions which is weaker than $\lambda$-continuous functions. We also investigate some of the fundamental properties of this type of functions.

Throughout the paper a space will always mean a topological space on which no separation axioms are assumed unless explicitly stated.

**Definition 1.** A subset $A$ of a space $(X, \tau)$ is called

(1) a $\Lambda$-set [13] if it is equal to its kernel (= saturated set), i.e. to the intersection of all open supersets of $A$.

(2) $\lambda$-closed [1] if $A = B \cap C$, where $B$ is a $\Lambda$-set and $C$ is a closed set.

(3) $\lambda$-open [2] if $X \backslash A$ is $\lambda$-closed.

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The family of all $\lambda$-open subsets of a space $(X, \tau)$ shall be denoted by $\lambda O(X)$.

A point $x \in X$ is called $\lambda$-cluster point of a subset $A \subset X$ [2] if for every $\lambda$-open set $B$ of $X$ containing $x$, $A \cap B \neq \emptyset$. The set of all $\lambda$-cluster points is called the $\lambda$-closure of $A$ [3] and is denoted by $Cl_\lambda(A)$. A point $x \in X$ is said to be a $\lambda$-interior point of a subset $A \subset X$ [2] if there exists a $\lambda$-open set $B$ containing $x$ such that $B \subset A$. The set of all $\lambda$-interior points of $A$ is said to be $\lambda$-interior of $A$ and is denoted by $Int_\lambda(A)$.

**Definition 2.** A subset $A$ is said to be

(1) preopen [14] if $A \subset Int(Cl(A))$.

(2) semiopen [11] if $A \subset Cl(Int(A))$.

(3) regular open [17] (resp. regular closed) if $A = Int(Cl(A))$ (resp. $A = Cl(Int(A))$).

**Lemma 1.1.** *([2]) Let $A$ be a subset of a space $X$. Then*

*(1) $A$ is $\lambda$-closed in $X$ if and only if $A = Cl_\lambda(A)$.*

*(2) $Cl_\lambda(X \backslash A) = X \backslash Int_\lambda(A)$.*

*(3) $Cl_\lambda(A)$ is $\lambda$-closed in $X$.*

**Definition 3.** A function $f : X \to Y$ is said to be $\lambda$-continuous [1, 2] if $f^{-1}(A) \in \lambda O(X)$ for each open set $A$ of $Y$.

**Definition 4.** A subset $A$ of a space $X$ is called a generalized closed set (briefly $g$-closed) [12] if $Cl(A) \subset B$ whenever $A \subset B$ and $B$ is open. $A$ is called $g$-open if its complement is $g$-closed.

A space $X$ is called locally indiscrete [15] if every open set is closed. Recall that a space is rim-compact if it has a basis of open sets with compact boundaries. The graph of a function $f : X \to Y$, denoted by $G(f)$, is the subset $\{(x, f(x)) : x \in X\}$ of the product space $X \times Y$. A subset $A$ of a space $X$ is said to be $N$-closed relative to $X$ [6] if for each cover $\{B_i : i \in I\}$ of $A$ by open sets of $X$, there exists a finite subfamily $I_0 \subset I$ such that $A \subset \cup_{i \in I_0} Int(Cl(B_i))$.

## 2. Weakly $\lambda$-continuous functions

**Definition 5.** A function $f : X \to Y$ is said to be weakly $\lambda$-continuous at $x \in X$ if for each open set $V$ of $Y$ containing $f(x)$, there exists a $\lambda$-open set $U$ containing $x$ such that $f(U) \subset Cl(V)$. If for each $x \in X$, $f$ is weakly $\lambda$-continuous at $x \in X$, $f$ is said to be weakly $\lambda$-continuous

**Theorem 2.1.** *For a function $f : X \to Y$, the following are equivalent:*

*(1) $f$ is weakly $\lambda$-continuous at $x \in X$,*

*(2) $x \in Int_\lambda(f^{-1}(Cl(U)))$ for each neighborhood $U$ of $f(x)$.*

*Proof.* (1) $\Rightarrow$ (2) : Let $U$ be any neighborhood of $f(x)$. Then there exists a $\lambda$-open set $G$ containing $x$ such that $f(G) \subset Cl(U)$. Since $G \subset f^{-1}(Cl(U))$ and $G$ is $\lambda$-open, then $x \in G \subset Int_\lambda(G))) \subset Int_\lambda(f^{-1}(Cl(U)))$.

(2) $\Rightarrow$ (1) : Let $x \in Int_\lambda(f^{-1}(Cl(U)))$ for each neighborhood $U$ of $f(x)$. Take $V = Int_\lambda(f^{-1}(Cl(U)))$. This implies that $f(V) \subset Cl(U)$ and $V$ is $\lambda$-open. Hence, $f$ is weakly $\lambda$-continuous at $x \in X$. □

**Definition 6.** A function $f : X \to Y$ is said to be weakly $g$-continuous if for each $x \in X$ and each open set $V$ of $Y$ containing $f(x)$, there exists a $g$-open set $U$ containing $x$ such that $f(U) \subset Cl(V)$.

**Theorem 2.2.** *For a function $f : X \to Y$ the following are equivalent:*
*(1) $f$ is weakly continuous,*
*(2) $f$ is weakly $g$-continuous and weakly $\lambda$-continuous.*

*Proof.* It follows directly from Theorem 2.4 of [1]. □

**Theorem 2.3.** *For a function $f : X \to Y$, the following are equivalent:*
*(1) $f$ is weakly $\lambda$-continuous,*
*(2) $Cl_\lambda(f^{-1}(Int(Cl(V)))) \subset f^{-1}(Cl(V))$ for every subset $V \subset Y$,*
*(3) $Cl_\lambda(f^{-1}(Int(F))) \subset f^{-1}(F)$ for every regular closed subset $F \subset Y$,*
*(4) $Cl_\lambda(f^{-1}(U)) \subset f^{-1}(Cl(U))$ for every open subset $U \subset Y$,*
*(5) $f^{-1}(U) \subset Int_\lambda(f^{-1}(Cl(U)))$ for every open subset $U \subset Y$,*
*(6) $Cl_\lambda(f^{-1}(U)) \subset f^{-1}(Cl(U))$ for each preopen subset $U \subset Y$,*
*(7) $f^{-1}(U) \subset Int_\lambda(f^{-1}(Cl(U)))$ for each preopen subset $U \subset Y$.*

*Proof.* (1) $\Rightarrow$ (2) : Let $V \subset Y$ and $x \in X \backslash f^{-1}(Cl(V))$. Then $f(x) \in Y \backslash Cl(V)$ and there exists an open set $U$ containing $f(x)$ such that $U \cap V = \emptyset$. We have $Cl(U) \cap Int(Cl(V)) = \emptyset$. Since $f$ is weakly $\lambda$-continuous, then there exists a $\lambda$-open set $W$ containing $x$ such that $f(W) \subset Cl(U)$. Then $W \cap f^{-1}(Int(Cl(V))) = \emptyset$ and $x \in X \backslash Cl_\lambda(f^{-1}(Int(Cl(V))))$. Hence, $Cl_\lambda(f^{-1}(Int(Cl(V)))) \subset f^{-1}(Cl(V))$.

(2) $\Rightarrow$ (3) : Let $F$ be any regular closed set in $Y$. Then $Cl_\lambda(f^{-1}(Int(F))) = Cl_\lambda(f^{-1}(Int(Cl(Int(F))))) \subset f^{-1}(Cl(Int(F))) = f^{-1}(F)$.

(3) $\Rightarrow$ (4) : Let $U$ be an open subset of $Y$. Since $Cl(U)$ is regular closed in $Y$, then $Cl_\lambda(f^{-1}(U)) \subset Cl_\lambda(f^{-1}(Int(Cl(U)))) \subset f^{-1}(Cl(U))$.

(4) $\Rightarrow$ (5) : Let $U$ be any open set of $Y$. Since $Y \backslash Cl(U)$ is open in $Y$, then $X \setminus Int_\lambda(f^{-1}(Cl(U))) = Cl_\lambda(f^{-1}(Y \setminus Cl(U))) \subset f^{-1}(Cl(Y \setminus Cl(U))) \subset X \setminus f^{-1}(U)$. Hence, $f^{-1}(U) \subset Int_\lambda(f^{-1}(Cl(U)))$.

(5) $\Rightarrow$ (1) : Let $x \in X$ and $U$ be any open subset of $Y$ containing $f(x)$. Then $x \in f^{-1}(U) \subset Int_\lambda(f^{-1}(Cl(U)))$. Take $W = Int_\lambda(f^{-1}(Cl(U)))$. Thus $f(W) \subset Cl(U)$ and hence $f$ is weakly $\lambda$-continuous at $x$ in $X$.

(1) $\Rightarrow$ (6) : Let $U$ be any preopen set of $Y$ and $x \in X \backslash f^{-1}(Cl(U))$. There exists an open set $G$ containing $f(x)$ such that $G \cap U = \emptyset$. We have $Cl(G \cap U) = \emptyset$. Since $U$ is preopen, then $U \cap Cl(G) \subset Int(Cl(U)) \cap Cl(G) \subset Cl(Int(Cl(U)) \cap$

$G) \subset Cl(Int(Cl(U) \cap G)) \subset Cl(Int(Cl(U \cap G))) \subset Cl(U \cap G) = \emptyset$. Since $f$ is weakly $\lambda$-continuous and $G$ is an open set containing $f(x)$, there exists a $\lambda$-open set $W$ in $X$ containing $x$ such that $f(W) \subset Cl(G)$. Then $f(W) \cap U = \emptyset$ and $W \cap f^{-1}(U) = \emptyset$. This implies that $x \in X \backslash Cl_\lambda(f^{-1}(U))$ and then $Cl_\lambda(f^{-1}(U)) \subset f^{-1}(Cl(U))$.

(6) $\Rightarrow$ (7) : Let $U$ be any preopen set of $Y$. Since $Y \backslash Cl(U)$ is open in $Y$, then $X \setminus Int_\lambda(f^{-1}(Cl(U))) = Cl_\lambda(f^{-1}(Y \setminus Cl(U))) \subset f^{-1}(Cl(Y \setminus Cl(U))) \subset X \setminus f^{-1}(U)$. This shows that $f^{-1}(U) \subset Int_\lambda(f^{-1}(Cl(U)))$.

(7) $\Rightarrow$ (1) : Let $x \in X$ and $U$ any open set of $Y$ containing $f(x)$. We have $x \in f^{-1}(U) \subset Int_\lambda(f^{-1}(Cl(U)))$. Take $W = Int_\lambda(f^{-1}(Cl(U)))$. Then $f(W) \subset Cl(U)$ and hence $f$ is weakly $\lambda$-continuous at $x$ in $X$. □

**Theorem 2.4.** *If $f : X \to Y$ is a weakly $\lambda$-continuous function and $Y$ is Hausdorff, then $f$ has $\lambda$-closed point inverses.*

*Proof.* Let $y \in Y$ and $x \in \{x \in X : f(x) \neq y\}$. Since $f(x) \neq y$ and $Y$ is Hausdorff, there exist disjoint open sets $G_1$, $G_2$ such that $f(x) \in G_1$ and $y \in G_2$. Since $G_1 \cap G_2 = \varnothing$, then $Cl(G_1) \cap G_2 = \varnothing$. We have $y \notin Cl(G_1)$. Since $f$ is weakly $\lambda$-continuous, there exists a $\lambda$-open set $U$ containing $x$ such that $f(U) \subset Cl(G_1)$. Assume that $U$ is not contained in $\{x \in X : f(x) \neq y\}$. There exists a point $u \in U$ such that $f(u) = y$. Since $f(U) \subset Cl(G_1)$, we have $y = f(u) \in Cl(G_1)$. This is a contradiction. Hence, $U \subset \{x \in X : f(x) \neq y\}$ and $U$ is $\lambda$-open in $X$. This shows that $\{x \in X : f(x) \neq y\}$ is $\lambda$-open in $X$, equivalently $f^{-1}(y) = \{x \in X : f(x) = y\}$ is $\lambda$-closed in $X$.

Recall that a point $x \in X$ is said to be in the $\theta$-closure [18] of a subset $A$ of $X$, denoted by $\theta$-$Cl(G)$, if $Cl(G) \cap A \neq \emptyset$ for each open set $G$ of $X$ containing $x$. $A$ is called $\theta$-closed if $A = \theta$-$Cl(A)$. The complement of a $\theta$-closed set is called $\theta$-open.

**Theorem 2.5.** *For a function $f : X \to Y$, the following equivalent:*

*(1) $f$ is weakly $\lambda$-continuous,*

*(2) $f(Cl_\lambda(V)) \subset \theta$-$Cl(f(V))$ for each subset $V \subset X$,*

*(3) $Cl_\lambda(f^{-1}(G)) \subset f^{-1}(\theta$-$Cl(G))$ for each subset $G \subset Y$,*

*(4) $Cl_\lambda(f^{-1}(Int(\theta$-$Cl(G)))) \subset f^{-1}(\theta$-$Cl(G))$ for every subset $G \subset Y$.*

*Proof.* (1) $\Rightarrow$ (2) : Let $V \subset X$, $x \in Cl_\lambda(V)$ and $U$ be any open set of $Y$ containing $f(x)$. There exists a $\lambda$-open set $W$ containing $x$ such that $f(W) \subset Cl(U)$. Since $x \in Cl_\lambda(V)$, then $W \cap V \neq \emptyset$. This implies that $\emptyset \neq f(W) \cap f(V) \subset Cl(U) \cap f(V)$ and $f(x) \in \theta$-$Cl(f(V))$. Hence, $f(Cl_\lambda(V)) \subset \theta$-$Cl(f(V))$.

(2) $\Rightarrow$ (3) : Let $G \subset Y$. Then $f(Cl_\lambda(f^{-1}(G))) \subset \theta$-$Cl(G)$ and hence $Cl_\lambda(f^{-1}(G)) \subset f^{-1}(\theta$-$Cl(G))$.

(3) $\Rightarrow$ (4) : Let $G \subset Y$. Since $\theta$-$Cl(G)$ is closed in $Y$, then $Cl_\lambda(f^{-1}(Int(\theta$-$Cl(G)))) \subset f^{-1}(\theta$-$Cl(Int(\theta$-$Cl(G))))) = f^{-1}(Cl(Int(\theta$-$Cl(G))))) \subset f^{-1}(\theta$-$Cl(G))$.

(4) $\Rightarrow$ (1) : Let $U$ be any open set of $Y$. We have $U \subset Int(Cl(U)) = Int(\theta$-$Cl(U))$. Thus, $Cl_\lambda(f^{-1}(U)) \subset Cl_\lambda(f^{-1}(Int(\theta$-$Cl(U)))) \subset f^{-1}(\theta$-$Cl(U)) =$

$f^{-1}(Cl(U))$. This implies from Theorem 2.3 that $f$ is weakly $\lambda$-continuous. □

**Theorem 2.6.** *If $f^{-1}(\theta\text{-}Cl(V))$ is $\lambda$-closed in $X$ for every subset $V \subset Y$, then $f$ is weakly $\lambda$-continuous.*

*Proof.* Let $V \subset Y$. Since $f^{-1}(\theta\text{-}Cl(V))$ is $\lambda$-closed in $X$, then $Cl_\lambda(f^{-1}(V)) \subset Cl_\lambda(f^{-1}(\theta\text{-}Cl(V))) = f^{-1}(\theta\text{-}Cl(V))$. This implies from Theorem 2.5 that $f$ is weakly $\lambda$-continuous. □

**Theorem 2.7.** *Let $f : X \to Y$ be a function. If $f$ is weakly $\lambda$-continuous, then $f^{-1}(V)$ is $\lambda$-closed in $X$ for every $\theta$-closed subset $V \subset Y$.*

*Proof.* Follows from Theorem 2.5. □

**Corollary 2.8.** *Let $f : X \to Y$ be a function. If $f$ is weakly $\lambda$-continuous, then $f^{-1}(V)$ is $\lambda$-open in $X$ for every $\theta$-open subset $V \subset Y$.*

## 3. The related functions

**Definition 7.** A function $f : X \to Y$ is said to be almost $\lambda$-continuous [10] if for each $x \in X$ and each open set $A$ of $Y$ containing $f(x)$, there exists a $\lambda$-open set $B$ containing $x$ such that $f(B) \subset Int(Cl(A))$.

**Remark 3.1.** Every weakly continuous and almost $\lambda$-continuous function is weakly $\lambda$-continuous but this implication is not reversible as shown in the following example.

**Example 3.2.** Let $X = \{a, b, c\}$, $Y = \{a, b, c, d\}$ and $\tau = \{X, \varnothing, \{a\}, \{a, b\}\}$, $\sigma = \{Y, \varnothing, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, d\}\}$. Then the function $f : (X, \tau) \to (Y, \sigma)$ defined by $f(a) = a$, $f(b) = b$, $f(c) = d$ is weakly $\lambda$-continuous but it is neither weakly continuous nor almost $\lambda$-continuous.

**Lemma 3.3.** *([3]) A space $X$ is locally indiscrete if and only if every $\lambda$-open set of $X$ is open in $X$.*

**Theorem 3.4.** *Let $f : X \to Y$ be a function and $X$ is locally indiscrete. Then the following are equivalent:*
*(1) $f$ is weakly continuous,*
*(2) $f$ is weakly $\lambda$-continuous.*

*Proof.* It follows immediately from Lemma 3.3. □

**Theorem 3.5.** *Let $f : X \to Y$ be a function with the closed graph and $Y$ be a rim-compact space. Suppose that $\lambda O(X)$ is closed under finite intersections. Then $f$ is weakly $\lambda$-continuous if and only if $f$ is $\lambda$-continuous.*

*Proof.* It is an immediate consequence of [16]. $\square$

**Definition 8.** A function $f : X \to Y$ is said to be

(1) $(\lambda, s)$-open if $f(A)$ is semiopen for every $\lambda$-open subset $A \subset X$.

(2) neatly weak $\lambda$-continuous if for each $x \in X$ and each open set $V$ of $X$ containing $f(x)$, there exists a $\lambda$-open set $U$ containing $x$ such that $Int(f(U)) \subset Cl(V)$.

**Theorem 3.6.** *If a function $f : X \to Y$ is neatly weak $\lambda$-continuous and $(\lambda, s)$-open, then $f$ is weakly $\lambda$-continuous.*

*Proof.* Let $x \in X$ and $V$ be an open subset of $Y$ containing $f(x)$. Since $f$ is neatly weak $\lambda$-continuous, there exists a $\lambda$-open set $U$ of $X$ containing $x$ such that $Int(f(U)) \subset Cl(V)$. Since $f$ is $(\lambda, s)$-open, then $f(U)$ is semiopen in $Y$. Then $f(U) \subset Cl(Int(f(U))) \subset Cl(V)$. Thus, $f$ is weakly $\lambda$-continuous. $\square$

**Theorem 3.7.** *If $f : X \to Y$ is weakly $\lambda$-continuous and $Y$ is Hausdorff, then for each $(x, y) \notin G(f)$, there exist a $\lambda$-open set $V \subset X$ and an open set $U \subset Y$ containing $x$ and $y$, respectively, such that $f(V) \cap Int(Cl(U)) = \varnothing$.*

*Proof.* Let $(x, y) \notin G(f)$. We have $y \neq f(x)$. Since $Y$ is Hausdorff, there exist disjoint open sets $U$ and $V$ containing $y$ and $f(x)$, respectively. We have $Int(Cl(U)) \cap Cl(V) = \varnothing$. Since $f$ is weakly $\lambda$-continuous, there exists an $\lambda$-open set $G$ containing $x$ such that $f(G) \subset Cl(V)$. Hence, $f(G) \cap Int(Cl(U)) = \varnothing$. $\square$

**Definition 9.** A function $f : X \to Y$ is said to be faintly $\lambda$-continuous if for each $x \in X$ and each $\theta$-open set $V$ of $Y$ containing $f(x)$, there exists a $\lambda$-open set $U$ containing $x$ such that $f(U) \subset V$.

**Theorem 3.8.** *Let $f : X \to Y$ be a function. The following are equivalent:*

*(1) $f$ is faintly $\lambda$-continuous,*

*(2) $f^{-1}(V)$ is $\lambda$-open in $X$ for every $\theta$-open subset $V \subset Y$,*

*(3) $f^{-1}(V)$ is $\lambda$-closed in $X$ for every $\theta$-closed subset $V \subset Y$.*

*Proof.* Obvious. $\square$

**Theorem 3.9.** *Let $f : X \to Y$ be a function, where $Y$ is regular. The following are equivalent:*

*(1) $f$ is $\lambda$-continuous,*

*(2) $f^{-1}(\theta\text{-}Cl(V))$ is $\lambda$-closed in $X$ for every subset $V \subset Y$,*
*(3) $f$ is weakly $\lambda$-continuous,*
*(4) $f$ is faintly $\lambda$-continuous.*

*Proof.* (1) $\Rightarrow$ (2) : Let $V \subset Y$. Since $\theta\text{-}Cl(V)$ is closed, then $f^{-1}(\theta\text{-}Cl(V))$ is $\lambda$-closed in $X$.

(2) $\Rightarrow$ (3) : Follows from Theorem 2.6.

(3) $\Rightarrow$ (4) : Let $V$ be a $\theta$-closed subset of $Y$. By Theorem 2.5, we have $Cl_\lambda(f^{-1}(V)) \subset f^{-1}(\theta\text{-}Cl(V)) = f^{-1}(V)$. This shows that $f^{-1}(V)$ is $\lambda$-closed and hence $f$ is faintly $\lambda$-continuous.

(4) $\Rightarrow$ (1) : Let $V$ be an open subset of $Y$. Since $Y$ is regular, $V$ is $\theta$-open in $Y$. Since $f$ is faintly $\lambda$-continuous, then $f^{-1}(V)$ is $\lambda$-open in $X$. Thus, $f$ is $\lambda$-continuous. □

**Definition 10.** A space $X$ is said to be almost regular [16] if for each point $x \in X$ and each regular closed set $A \subset X$ not containing $x$, there exist disjoint open sets $U$ and $V$ such that $x \in U$ and $A \subset V$.

**Theorem 3.10.** *If $f : X \to Y$ is a function such that $Y$ is almost regular. Then the following are equivalent:*
*(1) $f$ is almost $\lambda$-continuous,*
*(2) $f$ is weakly $\lambda$-continuous.*

*Proof.* (1) $\Rightarrow$ (2) : Obvious.

(2) $\Rightarrow$ (1) : Let $V$ be a regular open set of $Y$ and $x \in f^{-1}(V)$. Then $f(x) \in V$. Since $Y$ is almost regular, by Theorem 2.2 of [17], there exists a regular open set $W$ such that $f(x) \in W \subset Cl(W) \subset V$. Since $f$ is weakly $\lambda$-continuous, there exists a $\lambda$-open set $U_x$ containing $x$ such that $f(U_x) \subset Cl(W)$. We have $x \in U_x \subset f^{-1}(V)$. This shows that $f^{-1}(V)$ is $\lambda$-open in $X$ and hence $f$ is almost $\lambda$-continuous.

## 4. Properties

**Definition 11.** A space $X$ is called $\lambda$-$T_2$ [2] if for $x$, $y \in X$ such that $x \neq y$ there exist disjoint $\lambda$-open sets $U$ and $V$ such that $x \in U$ and $y \in V$.

It should be noticed that Ganster et al. [8] have shown that $\lambda$-$T_2$ is equivalent with $T_0$.

**Theorem 4.1.** *If for each pair of distinct points $x_1$ and $x_2$ in a space $X$, there exist a function $f$ of $X$ into $(Y, \sigma)$ such that $Y$ is Urysohn, $f(x_1) \neq f(x_2)$ and $f$ is weakly $\lambda$-continuous at $x_1$ and $x_2$, then $X$ is $\lambda$-$T_2$.*

*Proof.* Let $x_1$ and $x_2$ be any distinct points in $X$. Then there exists a function $f : X \to Y$ such that $Y$ is Urysohn, $f(x_1) \neq f(x_2)$ and $f$ is weakly $\lambda$-continuous at $x_1$ and $x_2$. Let $y_i = f(x_i)$ for $i = 1, 2$. We have $y_1 \neq y_2$. Since $Y$ is Urysohn, then there exist open sets $V_1$ and $V_2$ containing $y_1$ and $y_2$, respectively, such that $Cl(V_1) \cap Cl(V_2) = \varnothing$. Since $f$ is weakly $\lambda$-continuous at $x_1$ and $x_2$, then there exist $\lambda$-open sets $U_i$ for $i = 1, 2$ containing $x_i$ such that $f(U_i) \subset Cl(V_i)$. This shows that $U_1 \cap U_2 = \varnothing$ and hence $X$ is $\lambda$-$T_2$. □

**Theorem 4.2.** *If $f : X \to Y$ is weakly $\lambda$-continuous and $g : Y \to Z$ is continuous, then the composition $gof : X \to Z$ is weakly $\lambda$-continuous.*

*Proof.* Let $x \in X$ and $A$ be an open set of $Z$ containing $g(f(x))$. We have $g^{-1}(A)$ is an open set of $Y$ containing $f(x)$. Then there exists a $\lambda$-open set $B$ containing $x$ such that $f(B) \subset Cl(g^{-1}(A))$. Since $g$ is continuous, then $(gof)(B) \subset g(Cl(g^{-1}(A))) \subset Cl(A)$. Thus, $gof$ is weakly $\lambda$-continuous. □

**Remark 4.3.** Here we have an observation concerning $\lambda$-connectedness. By definition, if a space $X$ can not be written as the union of two nonempty disjoint $\lambda$-open sets, then $X$ is said to be $\lambda$-connected. It is obvious that every $\lambda$-connected space is indiscrete. Because we know that if a space is not indiscrete, then there is a nontrivial open set. This set and its complement provide a decomposition of the space into nonempty disjoint $\lambda$-open sets. Hence every $\lambda$-connected space must be indiscrete and therefore the notion is not interesting.

**Theorem 4.4.** *Let $f, g : X \to Y$ be weakly $\lambda$-continuous functions and $Y$ be Urysohn. If $\lambda O(X)$ is closed under the finite intersections, then the set $\{x \in X : f(x) = g(x)\}$ is $\lambda$-closed in $X$.*

*Proof.* Obvious. □

**Theorem 4.5.** *Let $f : X \to Y$ be a weakly $\lambda$-continuous function and $K$ be a $\theta$-closed subset of $X \times Y$. Suppose that $\lambda O(X)$ is closed under the finite intersections. Then $p(K \cap G(f))$ is $\lambda$-closed in $X$, where $p$ is the projection of $X \times Y$ onto $X$.*

*Proof.* Let $x \in Cl_\lambda(p(K \cap G(f)))$, $G$ be an open subset of $X$ containing $x$ and $H$ be an open subset of $Y$ containing $f(x)$. Since $f$ is weakly $\lambda$-continuous, then $x \in f^{-1}(H) \subset Int_\lambda(f^{-1}(Cl(H)))$. This implies that $x \in G \cap Int_\lambda(f^{-1}(Cl(H)))$. Since $x \in Cl_\lambda(p(K \cap G(f)))$, then $(G \cap Int_\lambda(f^{-1}(Cl(H)))) \cap p(K \cap G(f))$ contains a point $x_0 \in X$. We have $(x_0, f(x_0)) \in K$ and $f(x_0) \in Cl(H)$. Then $\emptyset \neq (G \times Cl(H)) \cap K \subset Cl(G \times H) \cap K$ and $(x, f(x)) \in \theta$-$Cl(K)$. Since $K$ is $\theta$-closed, $(x, f(x)) \in K \cap G(f)$ and $x \in p(K \cap G(f))$. This shows that $p(K \cap G(f))$ is $\lambda$-closed in $X$. □

**Corollary 4.6.** *Let $f : X \to Y$ be a function with the $\theta$-closed graph and $g : X \to Y$ be a weakly $\lambda$-continuous function. Suppose that $\lambda O(X)$ is closed* *under the finite intersections. Then the set $\{x \in X : f(x) = g(x)\}$ is $\lambda$-closed in $X$.*

*Proof.* Let $G(f)$ be $\theta$-closed. We have $p(G(f) \cap G(g)) = \{x \in X : f(x) = g(x)\}$. By Theorem 4.5, $\{x \in X : f(x) = g(x)\}$ is $\lambda$-closed in $X$. □

**Theorem 4.7.** *Let $f : X \to Y$ be a function, where $\lambda O(X)$ is closed under the finite intersections. If for each $(x, y) \notin G(f)$, there exist a $\lambda$-open set $U \subset X$ and an open set $V \subset Y$ containing $x$ and $y$, respectively, such that $f(U) \cap Int(Cl(V)) = \varnothing$, then inverse image of each $N$-closed set of $Y$ is $\lambda$-closed in $X$.*

*Proof.* Suppose that there exists an $N$-closed set $W \subset Y$ such that $f^{-1}(W)$ is not $\lambda$-closed in $X$. We have a point $x \in Cl_\lambda(f^{-1}(W)) \backslash f^{-1}(W)$. Since $x \notin f^{-1}(W)$, then $(x, y) \notin G(f)$ for each $y \in W$. There exist $\lambda$-open sets $U_y(x) \subset X$ and an open set $V(y) \subset Y$ containing $x$ and $y$, respectively, such that $f(U_y(x)) \cap Int(Cl(V(y))) = \varnothing$. The family $\{V(y) : y \in W\}$ is a cover of $W$ by open sets of $Y$. Since $W$ is $N$-closed, there exit a finite number of points $y_1$, $y_2$, ..., $y_n$ in $W$ such that $W \subset \cup_{i=1}^{n} Int(Cl(V(y_i)))$. Take $U = \cap_{i=1}^{n} U_{y_i}(x)$. We have $f(U) \cap W = \varnothing$. Since $x \in Cl_\lambda(f^{-1}(W))$, then $f(U) \cap W \neq \varnothing$. This is a contradiction. □

For a function $f : X \to Y$, the graph function $g : X \to X \times Y$ of $f$ is defined by $g(x) = (x, f(x))$ for each $x \in X$.

**Theorem 4.8.** *If the graph function $g$ of a function $f : X \to Y$ is weakly $\lambda$-continuous, then $f$ is weakly $\lambda$-continuous.*

*Proof.* Let $g$ be weakly $\lambda$-continuous and $x \in X$ and $U$ be an open set of $Y$ containing $f(x)$. Then $X \times U$ is an open set containing $g(x)$. There exists a $\lambda$-open set $V$ containing $x$ such that $g(V) \subset Cl(X \times U) = X \times Cl(U)$. This implies that $f(V) \subset Cl(U)$ and hence $f$ is weakly $\lambda$-continuous.

## References

[1] Arenas, F. G., Dontchev, J., Ganster, M., On $\lambda$-sets and dual of generalized continuity. Questions Answers Gen. Topology 15 (1997), 3-13.

[2] Caldas, M., Jafari, S., On some low separation axioms via $\lambda$-open and $\lambda$-closure operator. Rend. Circ. Mat. Palermo 54 (2005), 195-208.

[3] Caldas, M., Jafari, S., Navalagi, G., More on $\lambda$-closed sets in topological spaces. Revista Colombiana (to appear)

[4] Caldas, M., Ekici, E., Jafari, S., Noiri, T., On the class of contra $\lambda$-continuous functions. Ann. Univ. Sci. Budapest Sec. Math. 49 (2006), 75-86.

[5] Caldas, M., Hatir, E., Jafari, S., Noiri, T., A new Kupka type continuity, $\lambda$-compactness and multifunctions (submitted)

[6] Carnahan, D., Locally nearly compact spaces. Boll. Un. Mat. Ital. (4) 6 (1972), 146-153.

[7] Ganster, M., Jafari, S., Steiner, M., On some very strong compactness conditions (submitted)

[8] Ganster, M., Jafari, S., Steiner, M., Some observations on $\lambda$-closed sets (submitted)

[9] Isbell, J. R., Uniform spaces. Amer. Math. Soc. Math. Surveys No. 12 (1962).

[10] Jafari, S., Moshokoa, S. P., Nailana, K. R., Noiri, T.,On almost $\lambda$-continuous functions (submitted).

[11] Levine, N., Semi-open sets and semi-continuity in topological spaces. Amer. Math. Monthly 70 (1963), 36-41.

[12] Levine, N., Generalized closed sets in topology. Rend. Circ. Mat. Palermo (2) 19(1970), 89-96.

[13] Maki, H., Generalized $\lambda$-sets and the associated closure operator. The Special Issue in Commemoration of Prof. Kazusada IKEDA' Retirement, 1. Oct. 1986, 139-146.

[14] Mashhour, A. S., Abd El-Monsef, M. E., El-Deeb, S. N., On precontinuous and weak precontinuous mappings. Proc. Math. Phys. Soc. Egypt 53 (1982), 47-53.

[15] Nieminen, T., On ultrapseudocompact and related topics. Ann. Acad. Sci. Fenn. Ser. A I Math. 3 (1977), 185-205.

[16] Noiri, T., Weak continuity and closed graphs. Casopis. Pest. Mat. 101 (1976), 379-382.

[17] Singal, M. K., Arya, S. P., On almost regular spaces. Glasnik Mat. Ser. III 4 (24) (1969), 89-99.

[18] Stone, M. H., Applications of the theory of Boolean rings to general topology. Trans. Amer. Math. Soc. 41 (1937), 375-381.

[19] Veličko, N. V., $H$-closed topological spaces. Amer. Math. Soc. Transl. 78 (1968), 102-118.

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