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UNSTEADY MAGNETOHYDRODYNAMIC FLOW OF A DUSTY FLUID BETWEEN TWO OSCILLATING PLATES UNDER VARYING CONSTANT PRESSURE GRADIENT

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Abstract. In this paper we have studied the unsteady flow of an electrically conducting viscous, incompressible dusty fluid flowing between two oscillating plates. The fluid is acted upon by a constant magnetic field perpendicular to the plates. Exact velocities of fluid and dust particles are derived by using differential geometry techniques and Laplace transforms.

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1. Introduction

The phenomenon of the flow of dusty fluids has been studied by a number of researchers. The flow of a dusty and electrically conducting fluid through the channels in the presence of a transverse magnetic field is encountered in a variety of applications such as magnetohydrodynamic (MHD) generators, pumps, accelators and flowmeters. In these devices the solid particles in the form of ash or soot are suspended in the conducting fluid as a result of corrosion and wear activities and or combustion process in the MHD generators and plasma MHD accelators. The consequent effect of the presence of solid particles on the performance of such devices has led to the studies of particulate suspensions in a conducting fluid in the presence of externally applied magnetic field.

P.G Saffman [10] has discussed the stability of the laminar flow of a dusty gas in which the dust particles are uniformly distributed. Liu. [7] has studied the flow induced by an oscillating infinite flat plate in a dusty gas. Michael and Miller [8] investigated the motion of dusty gas with uniform distribution of the dust particles placed in the semi–infinite space above a rigid plane boundary. Later, Samba Siva Rao [11] has obtained the analytical solutions for the dusty fluid flow through a circular tube under the influence of constant pressure gradient, using appropriate boundary conditions.

To investigate the kinematical properties of fluid flows in the field of fluid mechanics some researchers like Kanwal [6], Truesdell [13], Indrasena [5], Purushotham [9]. Bagewadi, Shantharajappa and Gireesha [1, 2, 3] have applied

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differential geometry techniques. In this paper we study the flow of an unsteady viscous, incompressible dusty fluid bounded by two oscillating plates. A uniform magnetic field of small magnetic Reynolds number is applied perpendicular to the plates, and a constant pressure gradient is applied to the fluid.

2. Equations of Motion

The equations of motion of an unsteady viscous incompressible fluid with uniform distribution of dust particles are given as:

For fluid phase

$$\nabla.\vec{u} = 0 \quad \text{(continuity)} \tag{1}$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\,\vec{u} = -\rho^{-1}\nabla p + \nu\nabla^2\vec{u} + \frac{kN}{\rho}(\vec{v}-\vec{u}) - \frac{\sigma B_0^2}{\rho}\vec{u} \quad \text{(Linear Momentum Equation)} \tag{2}$$

For dust phase

$$\nabla.\vec{v} = 0 \quad \text{(continuity)} \tag{3}$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}.\nabla)\,\vec{v} = \frac{k}{m}(\vec{u}-\vec{v}) \quad \text{(Linear Momentum Equation)} \tag{4}$$

where $\vec{u}, \vec{v}, \rho, N$ and ν are velocity of the fluid, velocity of the dust particle, density of the gas, number of density of dust particles and kinematic viscosity, $k = 6\pi a\mu$ is the Stoke's resistance (drag coefficient), where a is a spherical radius of dust particle and μ is the coefficient of viscosity of fluid particles, σ and B_0 respectively denote the electrical conductivity of the fluid and the magnetic field, p is the pressure of the fluid and t is the time.

Let $\vec{s},\ \vec{n},\ \vec{b}$ be orthogonal triad of unit tangent, principal normal and binormal vectors respectively for a space curve formed by the fluid phase velocity and dust phase velocity lines respectively. By using the Frenet formulae [4]

$$\begin{array}{rllll}
(i) & \dfrac{\partial \vec{s}}{\partial s} = k_s\vec{n}, & \dfrac{\partial \vec{n}}{\partial s} = \tau_s\vec{b} - k_s\vec{s}, & \dfrac{\partial \vec{b}}{\partial s} = -\tau_s\vec{n} \\
(ii) & \dfrac{\partial \vec{n}}{\partial n} = k'_n\vec{s}, & \dfrac{\partial \vec{b}}{\partial n} = -\sigma'_n\vec{s}, & \dfrac{\partial \vec{s}}{\partial n} = \sigma'_n\vec{b} - k'_n\vec{n} \\
(iii) & \dfrac{\partial \vec{b}}{\partial b} = k''_b\,\vec{s}, & \dfrac{\partial \vec{n}}{\partial b} = -\sigma''_b\,\vec{s}, & \dfrac{\partial \vec{s}}{\partial b} = \sigma''_b\,\vec{n} - k''_b\,\vec{b} \\
(iv) & \nabla.\vec{s} = \theta_{ns} + \theta_{bs} \ :\ \nabla.\vec{n} = \theta_{bn} - k_s \ :\ \nabla.\vec{b} = \theta_n b
\end{array} \tag{5}$$

where $\frac{\partial}{\partial s}$, $\frac{\partial}{\partial n}$ and $\frac{\partial}{\partial b}$ are the intrinsic differential operators of fluid phase velocity (or dust phase velocity) lines along tangential, principal normal and binormal, respectively. The functions $(k_s,\ k'_n,\ k''_b)$ and $(\tau_s,\ \sigma'_n,\ \sigma''_b)$ are the curvatures and torsions of the above curves and θ_{ns} and θ_{bs} are normal deformations of these spatial curves along their principal normal and binormal respectively.

3. Formulation and Solution of the Problem

Let the viscous, incompressible, dusty fluid be bounded between two oscillating plates. The flow is due to the influence of oscillation of the plates and the constant pressure gradient. Both the fluid and dust particles are supposed to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size. The number density of the dust particles is taken as a constant throughout the flow. A uniform magnetic field is applied perpendicular to the plates. The magnetic Reynolds number is assumed very small, so that the induced magnetic field is neglected. Under these assumptions the flow will be a parallel flow in which the streamlines are along the tangential direction and the velocities vary along the binormal direction with time t, since we have extended the fluid to infinity in the principal normal direction and we have assumed a constant pressure gradient. We can write

$$-\frac{1}{\rho}\frac{\partial p}{\partial s} = a_0$$

where a_0 is a constant.

By virtue of the system of equations (5) the continuity and linear momentum equations for the fluid phase and dust particle phase become,

$$\frac{\partial u_s}{\partial t} = \nu\left[\frac{\partial^2 u_s}{\partial b^2} - C_r u_s\right] + \frac{kN}{\rho}(v_s - u_s) + a_0 - \frac{\sigma B_0^2}{\rho} u_s \tag{6}$$

$$2u_s^2 k_s = \nu\left[2\sigma_b'' \frac{\partial u_s}{\partial b} - u_s k_s^2\right] \tag{7}$$

$$0 = \nu\left[u_s k_s \tau_s - 2k_b'' \frac{\partial u_s}{\partial b}\right] \tag{8}$$

$$\frac{\partial v_s}{\partial t} = \frac{k}{m}(u_s - v_s) \tag{9}$$

$$2v_s^2 k_s = 0 \tag{10}$$

where $\left(C_r = {\sigma'_n}^2 + {k'_b}^2 + {k''_b}^2 + {\sigma''_b}^2\right)$ is called the curvature number [3].

From equation (10), we see that $v_s^2 k_s = 0$, which implies either $v_s = 0$ or $k_s = 0$. The choice $v_s = 0$ is impossible, since if it happens then $u_s = 0$, which shows that the flow does not exist. Hence $k_s = 0$, it suggests that the curvature of the streamline along the tangential direction is zero. Thus, no radial flow exists.

Equations (6) and (9) are to be solved subject to the initial and boundary conditions:

$$\left\{\begin{array}{ll} \text{Initial condition:} & \text{at } t = 0: \; u_s = 0, \;\; v_s = 0 \\ \text{Boundary condition:} & \text{for } t > 0: \; u_s = u_0 \sin t, \text{ at } b = 0 \text{ and } b = h \end{array}\right\} \tag{11}$$

We define the Laplace transformations of u_s and v_s as

$$U = \int_0^\infty e^{-st}.u_s\, dt \text{ and } V = \int_0^\infty e^{-st}.v_s\ dt \tag{12}$$

Applying the Laplace transform to equations (6), (9) and to the boundary conditions, then by using the initial conditions, we have

$$sU = \nu \left[\frac{\partial^2 U}{\partial b^2} - C_r U \right] + \frac{l}{\tau}(V - U) + \frac{a_0}{s} - \frac{\sigma B_0^2}{\rho} U \tag{13}$$

$$sV = \frac{1}{\tau}(\mathrm{U} - V) \tag{14}$$

$$U = \frac{u_0}{(1+s^2)} \text{ at } b = 0 \text{ and } b = h \tag{15}$$

where $l = \frac{mN}{\rho}$ and $\tau = \frac{m}{k}$. Equation (14) implies

$$V = \frac{U}{(1+s\tau)} \tag{16}$$

Eliminating V from (13) and (16) we obtain the following equation

$$\frac{d^2U}{db^2} - Q^2 U = -\frac{a_0}{s\nu} \tag{17}$$

where $Q^2 = \left(C_r + \frac{s}{\nu} + \frac{sl}{\nu(1+s\tau)} + M \right)$ and $M = \frac{\sigma B_0^2}{\mu}$.

The velocities of fluid and dust particles are obtained by solving the equation (17) under the boundary conditions (15) as follows

$$\begin{aligned} U &= \frac{u_0}{(1+s^2)} \left\{ \frac{\sinh(Qb) - \sinh(Q(b-h))}{\sinh(Qh)} \right\} \\ &\quad + \frac{a_0}{Q^2 \nu s} \left[\frac{\sinh(Q(b-h)) - \sinh(Qb)}{\sinh(Qh)} + 1 \right] \end{aligned}$$

using U in (16) we obtain V as

$$\begin{aligned} V &= \frac{u_0}{(1+s^2)(1+s\tau)} \left\{ \frac{\sinh(Qb) - \sinh(Q(b-h))}{\sinh(Qh)} \right\} \\ &\quad + \frac{a_0}{Q^2 \nu s(1+s\tau)} \left[\frac{\sinh(Q(b-h)) - \sinh(Qb)}{\sinh(Qh)} + 1 \right] \end{aligned}$$

By taking the inverse Laplace transform to U and V, one can obtain

$$\begin{aligned}
u_s &= \frac{u_0}{E^2+F^2}\left((AE-BF)\sin t+(BE+AF)\cos t\right) \\
&+\frac{a_0}{\nu\,(M+C_r)}\left(\frac{\sinh\left(\sqrt{(M+C_r)}\,(b-h)\right)-\sinh\left(\sqrt{(M+C_r)}\,b\right)}{\sinh\left(\sqrt{(M+C_r)}\,h\right)}+1\right) \\
&+u_0\pi\nu\frac{2}{h^2}\sum_{n=0}^{\infty}(-1)^n\,(2n+1)\sin\left(\frac{2n+1}{h}\pi b\right) \\
&\times\left[\frac{(1+x_1\tau)^2\,e^{x_1t}}{(1+x_1^2)\left[(1+x_1\tau)^2+l\right]}+\frac{(1+x_2\tau)^2\,e^{x_2t}}{(1+x_2^2)\left[(1+x_2\tau)^2+l\right]}\right] \\
&-\frac{2a_0}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)}.\sin\left(\frac{2n+1}{h}\pi b\right) \\
&\left[\frac{(1+x_1\tau)^2\,e^{x_1t}}{x_1\left[(1+x_1\tau)^2+l\right]}+\frac{(1+x_2\tau)^2\,e^{x_2t}}{x_2\left[(1+x_2\tau)^2+l\right]}\right]
\end{aligned} \tag{18}$$

and

$$\begin{aligned}
v_s &= \frac{u_0}{(E^2+F^2)\,(1+\tau^2)} \\
&\quad\left((AE-BF)\,(\sin t-\tau\cos t)+(BE+AF)\,(\cos t+\tau\sin t)\right) \\
&+\frac{a_0}{\nu\,(M+C_r)}\left(\frac{Sinh\left(\sqrt{(M+C_r)}\,(b-h)\right)-\sinh\left(\sqrt{(M+C_r)}\,b\right)}{\sinh\left(\sqrt{(M+C_r)}\,h\right)}+1\right) \\
&+u_0\pi\nu\frac{2}{h^2}\sum_{n=0}^{\infty}(-1)^n\;(2n+1)\sin\left(\frac{2n+1}{h}\pi b\right) \\
&\times\left[\frac{(1+x_1\tau)\,e^{x_1t}}{(1+x_1^2)\left[(1+x_1\tau)^2+l\right]}+\frac{(1+x_2\tau)\,e^{x_2t}}{(1+x_2^2)\left[(1+x_2\tau)^2+l\right]}\right] \\
&-\frac{2a_0}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)}.\sin\left(\frac{2n+1}{h}\pi b\right) \\
&\left[\frac{(1+x_1\tau)\,e^{x_1t}}{x_1\left[(1+x_1\tau)^2+l\right]}+\frac{(1+x_2\tau)\,e^{x_2t}}{x_2\left[(1+x_2\tau)^2+l\right]}\right]
\end{aligned} \tag{19}$$

where

$$\begin{aligned}
x_1 &= -\frac{1}{2\tau}\left(1+l+\nu C_r\tau+\nu M\tau+\nu\tau\frac{n^2\pi^2}{h^2}\right) \\
&+\frac{1}{2\tau}\sqrt{\left(1+l+\nu C_r\tau+\nu M\tau+\nu\tau\frac{n^2\pi^2}{h^2}\right)^2-4\tau\nu\left(C_r+M+\frac{n^2\pi^2}{h^2}\right)}
\end{aligned}$$

$$
\begin{aligned}
x_2 &= -\frac{1}{2\tau}\left(1+l+\nu C_r\tau+\nu M\tau+\nu\tau\frac{n^2\pi^2}{h^2}\right)\\
&\quad -\frac{1}{2\tau}\sqrt{\left(1+l+\nu C_r\tau+\nu M\tau+\nu\tau\frac{n^2\pi^2}{h^2}\right)^2-4\tau\nu\left(C_r+M+\frac{n^2\pi^2}{h^2}\right)}\\
y_1 &= -\frac{1}{2\tau}\left(1+l+\nu C_r\tau+M\nu\tau\right)\\
&\quad +\frac{1}{2\tau}\sqrt{\left(1+l+\nu C_r\tau+M\nu\tau\right)^2-4\nu\tau\left(M+C_r\right)}\\
y_2 &= -\frac{1}{2\tau}\left(1+l+\nu C_r\tau+M\nu\tau\right)\\
&\quad -\frac{1}{2\tau}\sqrt{\left(1+l+\nu C_r\tau+M\nu\tau\right)^2-4\nu\tau\left(M+C_r\right)}\\
A &= \sinh\left(\alpha b\right).\cos\left(\beta b\right)-\sinh\left(\alpha\left(b-h\right)\right).\cos\left(\beta\left(b-h\right)\right)\\
B &= \cosh\left(\alpha\left(b-h\right)\right).\sin\left(\beta\left(b-h\right)\right)-\cosh\left(\alpha b\right).\sin\left(\beta b\right)\\
E &= \sinh\left(\alpha h\right).\cos\left(\beta h\right),\quad F=\sin\left(\beta h\right).\cosh\left(\alpha h\right)\\
\alpha &= \sqrt{\frac{\left(y_1y_2-1\right)+\sqrt{\left(y_1y_2-1\right)^2+\left(y_1+y_2\right)^2}}{2}}\\
\beta &= \sqrt{\frac{\left(1-y_1y_2\right)+\sqrt{\left(y_1y_2-1\right)^2+\left(y_1+y_2\right)^2}}{2}}
\end{aligned}
$$

Fluid velocity and velocity of dust particle can be calculated by equations (18) and (19).

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