Novi Sad J. Math.
Vol. 41, No. 2, 2011, 159-160

ON THE NUMBER OF EVEN BLOCKS IN WORDS

Mirko Budinčević[1] and Ratko Tošić[2]

Abstract. In an earlier article the authors proved that any sequence of eight digits has at least four successive 'even blocks', where the even block is any even digit or a sequence which starts and ends with an odd digits, with even digits between them. The proof was done by brute force - by direct checking. The aim of this note is to generalize such a statement.

AMS Mathematics Subject Classification (2010): 68R15

Key words and phrases: finite word, block.

We are concerned with $A = \{0,1\}^*$, the set of all finite words with letters from the set $\{0,1\}$. Such a word with m letters we denote by

$$\alpha = a_1 a_2 \ldots a_m,$$

where m is its length. We define subwords in the usual way, i.e. α' is a subword of α if

$$\alpha' = a_i a_{i+1} \ldots a_r, \qquad 1 \le i \le r \le m.$$

Some words are of special interest to us. Those are the word 0 and all the words which start and end with the letter 1 and have only letters 0 in between. The set of all such words we denote by B and call its elements *blocks*. Also we introduce the set C consisting of all words with starting letter 1 and all other letters (if any) 0.

Now we are ready to state our theorem.

Theorem. *Let $\alpha = a_1 a_2 \ldots a_{2n}$ be any word from the set A with length $2n$. Then there exists its subword α' consisting only of k blocks such that $k \ge n$.*

Proof. The proof is obvious if all the letters are zeros. Suppose that at least one letter is 1 and that the first such letter is a_i. We can represent α in a unique way as

$$\alpha = \beta_1 \beta_2 \ldots \beta_p \tag{1}$$

or

$$\alpha = \beta_1 \beta_2 \ldots \beta_p \gamma \tag{2}$$

[1]Department of Mathematics and Informatics, University of Novi Sad, Trg Dositeja Obradovića 4, 21000 Novi Sad, Serbia, e-mail: mirkob@dmi.uns.ac.rs
[2]e-mail: ratosic@dmi.uns.ac.rs

where $\beta_l,\ l = 1, 2, \ldots, p$, are blocks and $\gamma \in C$. The first $i-1$ are 0-blocks, β_i (if it exists) is a block with letter 1 at the beginning and end and so on. Consider now the word

$$\alpha' = a_{i+1}a_{i+2}\ldots a_{2n},$$

a subword of α. Then

$$\alpha' = \beta'_1\beta'_2\ldots\beta'_q\gamma' \tag{3}$$

if α is of the form (1) or

$$\alpha' = \beta'_1\beta'_2\ldots\beta'_q \tag{4}$$

if α is of the form (2). Again, every $\beta'_l,\ l = 1, 2, \ldots, q$, is a block, while $\gamma' \in C$.

Every block which starts with 1 has the next letter 1 at its end, while every letter 0 in α is exactly one of the blocks $\beta_1, \beta_2, \ldots, \beta_p, \beta'_1, \ldots, \beta'_q$. So, we can conclude that every letter of α, with the exception of a_i is the last letter of some block among $\beta_1, \beta_2, \ldots, \beta_p, \beta'_1, \ldots, \beta'_q$. This implies that $p + q = 2n - 1$, and so $max(p, q) \geq n$. □

Remark. The authors tried to give a proof by induction but have not succeeded. Does such a proof exists?

Received by the editors December 6, 2010