Novi Sad J. Math.
Vol. 43, No. 1, 2013, 99-112

FROM QUADRATIC HAMILTONIANS OF POLYMOMENTA TO ABSTRACT GEOMETRICAL MAXWELL-LIKE AND EINSTEIN-LIKE EQUATIONS[1]

Alexandru Oană[2] and Mircea Neagu[3]

Abstract. The aim of this paper is to create a large geometrical background on the dual 1-jet space $J^{1*}(\mathcal{T}, M)$ for a multi-time Hamiltonian approach of the electromagnetic and gravitational physical fields. Our geometric-physical construction is achieved starting only from the given quadratic Hamiltonian function

$$H = h_{ab}(t)g^{ij}(t,x)p_i^a p_j^b + U_{(a)}^{(i)}(t,x)p_i^a + \mathcal{F}(t,x)$$

which naturally produces a canonical nonlinear connection N, a canonical Cartan N-linear connection $C\Gamma(N)$ and their corresponding local distinguished (d-) torsions and curvatures. In such a context, we construct some geometrical electromagnetic-like and gravitational-like field theories which are characterized by some natural geometrical Maxwell-like and Einstein-like equations. Some abstract and geometrical conservation laws for the multi-time Hamiltonian gravitational physical field are also given.

AMS Mathematics Subject Classification (2010): 70S05, 53C07, 53C80

Key words and phrases: dual 1-jet spaces, nonlinear connections, canonical Cartan N-linear connection, d-torsions and d-curvatures, geometrical Maxwell-like and Einstein-like equations

1. Distinguished Riemannian geometrization of metrical multi-time Hamilton spaces

Recently, the studies of Atanasiu and Neagu (see the papers [2, 3, 4]) initiated the new way of distinguished Riemannian geometrization for Hamiltonians depending on polymomenta, which represents in fact a natural "multi-time" extension of the already classical Hamiltonian geometry on cotangent bundles (synthesized in the book by Miron et al. [13]). In what follows, we expose the main geometrical ideas which characterize the distinguished Riemannian geometrical approach of Hamiltonians depending on polymomenta (see for details Oană and Neagu's papers [15, 16]).

---

[1]This research paper is dedicated to Academician Radu Miron at his 85-th year.

[2]Department of Mathematics-Informatics, Faculty of Mathematics and Informatics, University Transilvania of Braşov, Romania, e-mail: alexandru.oana@unitbv.ro

[3]Department of Mathematics-Informatics, Faculty of Mathematics and Informatics, University Transilvania of Braşov, Romania, e-mail: mircea.neagu@unitbv.ro

Let us consider that $h = (h_{ab}(t))$ is a semi-Riemannian metric on the "multi-time" (*temporal*) manifold $\mathcal{T}^m$, where $m = \dim \mathcal{T}$. Let $g = (g^{ij}(t^c, x^k, p^c_k))$ be a symmetric d-tensor on the dual 1-jet space $E^* = J^{1*}(\mathcal{T}, M)$, which has the rank $n = \dim M$ and a constant signature. At the same time, let us consider a smooth multi-time Hamiltonian function $E^* \ni (t^a, x^i, p^a_i) \to H(t^a, x^i, p^a_i) \in \mathbb{R}$, which yields the *fundamental vertical metrical d-tensor*

$$G^{(i)(j)}_{(a)(b)} = \frac{1}{2}\frac{\partial^2 H}{\partial p^a_i \partial p^b_j},$$

where $a, b = 1, ..., m$ and $i, j = 1, ..., n$.

Definition 1.1. A multi-time Hamiltonian function $H : E^* \to \mathbb{R}$, having the fundamental vertical metrical d-tensor of the form

$$G^{(i)(j)}_{(a)(b)}(t^c, x^k, p^c_k) = h_{ab}(t^c) g^{ij}(t^c, x^k, p^c_k),$$

is called a *Kronecker h-regular multi-time Hamiltonian function.*

Definition 1.2. A pair $MH^n_m = (E^*, H)$, where $m = \dim \mathcal{T}$ and $n = \dim M$, consisting of the dual 1-jet space and a Kronecker h-regular multi-time Hamiltonian function $H : E^* \to \mathbb{R}$, is called a *multi-time Hamilton space.*

Example 1.3. If we consider on E^* a symmetric d-tensor field $g^{ij}(t, x)$, having the rank n and a constant signature, we can define the Kronecker h-regular multi-time Hamiltonian function $H : E^* \to \mathbb{R}$, by setting

$$H = h_{ab}(t) g^{ij}(t, x) p^a_i p^b_j + U^{(i)}_{(a)}(t, x) p^a_i + \mathcal{F}(t, x), \tag{1.1}$$

where $U^{(i)}_{(a)}(t, x)$ is a d-tensor field on E^*, and $\mathcal{F}(t, x)$ is a function on E^*. Then, the multi-time Hamilton space $\mathcal{NEDM}H^n_m = (E^*, H)$ is called the *non-autonomous multi-time Hamilton space of electrodynamics.*

An important role for the subsequent development of our distinguished Riemannian geometrical theory for multi-time Hamilton spaces is represented by the following result (proved in the paper [2]):

Theorem 1.4. *The following statements are equivalent:*

(i) *H is a Kronecker h-regular multi-time Hamiltonian function on E^*.*
(ii) *The multi-time Hamiltonian function H reduces to the form*

$$H = \begin{cases} H(t, x^k, p^1_k), & m = \dim \mathcal{T} = 1 \\ h_{ab}(t) g^{ij}(t, x) p^a_i p^b_j + U^{(i)}_{(a)}(t, x) p^a_i + \mathcal{F}(t, x), & m = \dim \mathcal{T} \geq 2, \end{cases} \tag{1.2}$$

whose fundamental vertical metrical d-tensor has the form

$$G^{(i)(j)}_{(a)(b)} = \begin{cases} h_{11}(t) g^{ij}(t, x^k, p^1_k), & m = \dim \mathcal{T} = 1 \\ h_{ab}(t^c) g^{ij}(t^c, x^k), & m = \dim \mathcal{T} \geq 2. \end{cases} \tag{1.3}$$

Following now the Miron's geometrical ideas from [13], the paper [2] proves that any Kronecker h-regular multi-time Hamiltonian function H produces a natural nonlinear connection on the dual 1-jet space E^*, which depends only on the given Hamiltonian function H. Using the *generalized spatial Christoffel symbols* of the d-tensor g_{ij}, which are given by

$$\Gamma^k_{ij} = \frac{g^{kl}}{2}\left(\frac{\partial g_{li}}{\partial x^j} + \frac{\partial g_{lj}}{\partial x^i} - \frac{\partial g_{ij}}{\partial x^l}\right),$$

together with Theorem 1.4, we have (see [2]):

Theorem 1.5. *The pair of local functions* $N = \left(\underset{1}{N}{}^{(a)}_{(i)b}, \underset{2}{N}{}^{(a)}_{(i)j}\right)$ *on* E^* *defined by*

$$\underset{1}{N}{}^{(a)}_{(i)b} = \chi^a_{bc} p^c_i, \tag{1.4}$$

$$\underset{2}{N}{}^{(a)}_{(i)j} = \begin{cases} \dfrac{h^{11}}{4}\left[\dfrac{\partial g_{ij}}{\partial x^k}\dfrac{\partial H}{\partial p^1_k} - \dfrac{\partial g_{ij}}{\partial p^1_k}\dfrac{\partial H}{\partial x^k} + \right. \\ \left. + g_{ik}\dfrac{\partial^2 H}{\partial x^j \partial p^1_k} + g_{jk}\dfrac{\partial^2 H}{\partial x^i \partial p^1_k}\right], & m = \dim \mathcal{T} = 1 \\ -\Gamma^k_{ij} p^a_k + T^{(a)}_{(i)j}, & m = \dim \mathcal{T} \geq 2, \end{cases} \tag{1.5}$$

where χ^a_{bc} *are the Christoffel symbols of the semi-Riemannian temporal metric* h_{ab}*, and, if* $U_{ib} = g_{ik} U^{(k)}_{(b)}$*, then*

$$T^{(a)}_{(i)j} = \frac{h^{ab}}{4}\left(\frac{\partial U_{ib}}{\partial x^j} + \frac{\partial U_{jb}}{\partial x^i} - 2U_{sb}\Gamma^s_{ij}\right), \tag{1.6}$$

represents a nonlinear connection on E^**. This is called the* ***canonical nonlinear connection of the multi-time Hamilton space*** $MH^n_m = (E^*, H)$.

The canonical nonlinear connection N on E^* allows us the construction of the *adapted bases*

$$\left\{\frac{\delta}{\delta t^a}, \frac{\delta}{\delta x^i}, \frac{\partial}{\partial p^a_i}\right\} \subset \chi(E^*), \qquad \left\{dt^a, dx^i, \delta p^a_i\right\} \subset \chi^*(E^*),$$

where

$$\begin{gathered} \frac{\delta}{\delta t^a} = \frac{\partial}{\partial t^a} - \underset{1}{N}{}^{(b)}_{(j)a}\frac{\partial}{\partial p^b_j}, \qquad \frac{\delta}{\delta x^i} = \frac{\partial}{\partial x^i} - \underset{2}{N}{}^{(b)}_{(j)i}\frac{\partial}{\partial p^b_j}, \\ \delta p^a_i = dp^a_i + \underset{1}{N}{}^{(a)}_{(i)b} dt^b + \underset{2}{N}{}^{(a)}_{(i)j} dx^j. \end{gathered} \tag{1.7}$$

The main result of the metrical multi-time Hamilton geometry is the Theorem of existence of the Cartan canonical h-normal N-linear connection $C\Gamma(N)$ (this result is proved in the paper [16]), which allows the subsequent development of our metrical multi-time Hamilton theory of physical fields.

Theorem 1.6 (the Cartan canonical N-linear connection)**.** *On the metrical multi-time Hamilton space $MH^n_m = (J^{1*}(\mathcal{T}, M), H)$, endowed with the canonical nonlinear connection N, there exists a unique h-normal N-linear connection*

$$C\Gamma(N) = \left(\chi^a_{bc},\ A^i_{jc},\ H^i_{jk},\ C^{i(k)}_{j(c)}\right), \tag{1.8}$$

having the properties:

$$g^{ij}{}_{|k} = 0, \quad g^{ij}|^{(k)}_{(c)} = 0, \quad A^i_{jc} = \frac{g^{il}}{2}\frac{\delta g_{lj}}{\delta t^c}, \quad H^i_{jk} = H^i_{kj}, \quad C^{i(k)}_{j(c)} = C^{k(i)}_{j(c)},$$

where "$_{/a}$", "$_{|k}$" and "$|^{(k)}_{(c)}$" represent the local covariant derivatives of $C\Gamma(N)$.

Moreover, the adapted local coefficients H^i_{jk} and $C^{i(k)}_{j(c)}$ of the ***Cartan canonical connection*** *$C\Gamma(N)$ have the expressions*

$$H^i_{jk} = \frac{g^{ir}}{2}\left(\frac{\delta g_{jr}}{\delta x^k} + \frac{\delta g_{kr}}{\delta x^j} - \frac{\delta g_{jk}}{\delta x^r}\right), \qquad C^{j(k)}_{i(c)} = -\frac{g_{ir}}{2}\left(\frac{\partial g^{jr}}{\partial p^c_k} + \frac{\partial g^{kr}}{\partial p^c_j} - \frac{\partial g^{jk}}{\partial p^c_r}\right).$$

Remark 1.7. The Cartan connection $C\Gamma(N)$ of the multi-time Hamilton space MH^n_m verifies also the metrical properties $h_{ab/c} = h_{ab|k} = h_{ab}|^{(k)}_{(c)} = 0$, $g^{ij}{}_{/c} = 0$.

Corollary 1.8. *In the particular case when $m = \dim \mathcal{T} \geq 2$, the adapted coefficients of the Cartan canonical connection $C\Gamma(N)$ of the multi-time Hamilton space MH^n_m reduce to*

$$A^a_{bc} = \chi^a_{bc}, \quad A^i_{jc} = \frac{g^{il}}{2}\frac{\partial g_{lj}}{\partial t^c}, \quad H^i_{jk} = \Gamma^i_{jk}, \quad C^{i(k)}_{j(c)} = 0. \tag{1.9}$$

Applying the formulas that determine the local d-torsions and d-curvatures of an h-normal N-linear connection $D\Gamma(N)$ (see [15]) to the Cartan canonical connection $C\Gamma(N)$, we obtain (see [16]):

Theorem 1.9. *The torsion tensor $\mathbb{T}$ of the Cartan connection $C\Gamma(N)$ of the multi-time Hamilton space MH^n_m is determined by the adapted local d-components*

	$h_{\mathcal{T}}$	h_M		v	
	$m \geq 1$	$m = 1$	$m \geq 2$	$m = 1$	$m \geq 2$
$h_{\mathcal{T}}h_{\mathcal{T}}$	0	0	0	0	$R^{(f)}_{(r)ab}$
$h_M h_{\mathcal{T}}$	0	T^r_{1j}	T^r_{aj}	$R^{(1)}_{(r)1j}$	$R^{(f)}_{(r)aj}$
$vh_{\mathcal{T}}$	0	0	0	$P^{(1)\ (j)}_{(r)1(1)}$	$P^{(f)\ (j)}_{(r)a(b)}$
$h_M h_M$	0	0	0	$R^{(1)}_{(r)ij}$	$R^{(f)}_{(r)ij}$
vh_M	0	$P^{r(j)}_{i(1)}$	0	$P^{(1)\ (j)}_{(r)i(1)}$	0
vv	0	0	0	0	0

where

(i) *for* $m = \dim \mathcal{T} = 1$, *we have*

$$T^r_{1j} = -A^r_{j1}, \quad P^{(1)\ (j)}_{(r)1(1)} = \frac{\partial \underset{1}{N}{}^{(1)}_{(r)1}}{\partial p^1_j} + A^j_{r1} - \delta^j_r \chi^1_{11}, \quad P^{(1)\ (j)}_{(r)i(1)} = \frac{\partial \underset{2}{N}{}^{(1)}_{(r)i}}{\partial p^1_j} + H^j_{ri},$$

$$P^{r(j)}_{i(1)} = C^{r(j)}_{i(1)}, \quad R^{(1)}_{(r)1j} = \frac{\delta \underset{1}{N}{}^{(1)}_{(r)1}}{\delta x^j} - \frac{\delta \underset{2}{N}{}^{(1)}_{(r)j}}{\delta t}, \quad R^{(1)}_{(r)ij} = \frac{\delta \underset{2}{N}{}^{(1)}_{(r)i}}{\delta x^j} - \frac{\delta \underset{2}{N}{}^{(1)}_{(r)j}}{\delta x^i};$$

(ii) *for* $m = \dim \mathcal{T} \geq 2$, *using the equality* (1.6) *and the notations*

$$\chi^c_{fab} = \frac{\partial \chi^c_{fa}}{\partial t^b} - \frac{\partial \chi^c_{fb}}{\partial t^a} + \chi^d_{fa}\chi^c_{db} - \chi^d_{fb}\chi^c_{da},$$

$$\mathfrak{R}^r_{kij} = \frac{\partial \Gamma^r_{ki}}{\partial x^j} - \frac{\partial \Gamma^r_{kj}}{\partial x^i} + \Gamma^p_{ki}\Gamma^r_{pj} - \Gamma^p_{kj}\Gamma^r_{pi},$$

we have

$$T^r_{aj} = -A^r_{ja}, \quad P^{(f)\ (j)}_{(r)a(b)} = \delta^f_b A^j_{ra}, \quad R^{(f)}_{(r)ab} = \chi^f_{gab} p^g_r,$$

$$R^{(f)}_{(r)aj} = -\frac{\partial \underset{2}{N}{}^{(f)}_{(r)j}}{\partial t^a} - \chi^f_{ca} T^{(c)}_{(r)j}, \qquad R^{(f)}_{(r)ij} = -\mathfrak{R}^k_{rij} p^f_k + \left[T^{(f)}_{(r)i|j} - T^{(f)}_{(r)j|i}\right].$$

Theorem 1.10. *The curvature tensor* $\mathbb{R}$ *of the Cartan connection* $C\Gamma(N)$ *of the multi-time Hamilton space* MH^n_m *is determined by the following adapted local curvature d-tensors:*

	$h_\mathcal{T}$	h_M		v	
	$m \geq 1$	$m = 1$	$m \geq 2$	$m = 1$	$m \geq 2$
$h_\mathcal{T} h_\mathcal{T}$	χ^d_{abc}	0	R^l_{ibc}	0	$-R^{(d)(i)}_{(l)(a)bc}$
$h_M h_\mathcal{T}$	0	R^l_{i1k}	R^l_{ibk}	$-R^{(1)(l)}_{(i)(1)1k} = -R^l_{i1k}$	$-R^{(d)(i)}_{(l)(a)bk}$
$v h_\mathcal{T}$	0	$P^{l\ (k)}_{i1(1)}$	0	$-P^{(1)(l)\ (k)}_{(i)(1)1(1)} = -P^{l\ (k)}_{i1(1)}$	0
$h_M h_M$	0	R^l_{ijk}	$\mathfrak{R}^l_{ijk}$	$-R^{(1)(l)}_{(i)(1)jk} = -R^l_{ijk}$	$-R^{(d)(i)}_{(l)(a)jk}$
$v h_M$	0	$P^{l\ (k)}_{ij(1)}$	0	$-P^{(1)(l)\ (k)}_{(i)(1)j(1)} = -P^{l\ (k)}_{ij(1)}$	0
vv	0	$S^{l(j)(k)}_{i(1)(1)}$	0	$-S^{(1)(l)(j)(k)}_{(i)(1)(1)(1)} = -S^{l(j)(k)}_{i(1)(1)}$	0

where the following formulas are true:

(i) *for* $m = \dim \mathcal{T} = 1$, *we have* $\chi^1_{111} = 0$,

$$R^l_{i1k} = \frac{\delta A^l_{i1}}{\delta x^k} - \frac{\delta H^l_{ik}}{\delta t} + A^r_{i1} H^l_{rk} - H^r_{ik} A^l_{r1} + C^{l(r)}_{i(1)} R^{(1)}_{(r)1k},$$

$$R^l_{ijk} = \frac{\delta H^l_{ij}}{\delta x^k} - \frac{\delta H^l_{ik}}{\delta x^j} + H^r_{ij} H^l_{rk} - H^r_{ik} H^l_{rj} + C^{l(r)}_{i(1)} R^{(1)}_{(r)jk},$$

$$P^{l\ (k)}_{i1(1)} = \frac{\partial A^l_{i1}}{\partial p^1_k} - C^{l(k)}_{i(1)/1} + C^{l(r)}_{i(1)} P^{(1)\ (k)}_{(r)1(1)},$$

$$P^{l\ (k)}_{ij(1)} = \frac{\partial H^l_{ij}}{\partial p^1_k} - C^{l(k)}_{i(1)|j} + C^{l(r)}_{i(1)} P^{(1)\ (k)}_{(r)j(1)},$$

$$S^{l(j)(k)}_{i(1)(1)} = \frac{\partial C^{l(j)}_{i(1)}}{\partial p^1_k} - \frac{\partial C^{l(k)}_{i(1)}}{\partial p^1_j} + C^{r(j)}_{i(1)} C^{l(k)}_{r(1)} - C^{r(k)}_{i(1)} C^{l(j)}_{r(1)};$$

(ii) *for* $m = \dim \mathcal{T} \geq 2$, *we have*

$$\chi^d_{abc} = \frac{\partial \chi^d_{ab}}{\partial t^c} - \frac{\partial \chi^d_{ac}}{\partial t^b} + \chi^f_{ab}\chi^d_{fc} - \chi^f_{ac}\chi^d_{fb},$$

$$R^l_{ibc} = \frac{\partial A^l_{ib}}{\partial t^c} - \frac{\partial A^l_{ic}}{\partial t^b} + A^r_{ib}A^l_{rc} - A^r_{ic}A^l_{rb},$$

$$R^l_{ibk} = \frac{\partial A^l_{ib}}{\partial x^k} - \frac{\partial \Gamma^l_{ik}}{\partial t^b} + A^r_{ib}\Gamma^l_{rk} - \Gamma^r_{ik}A^l_{rb},$$

$$\mathfrak{R}^l_{ijk} = \frac{\partial \Gamma^l_{ij}}{\partial x^k} - \frac{\partial \Gamma^l_{ik}}{\partial x^j} + \Gamma^r_{ij}\Gamma^l_{rk} - \Gamma^r_{ik}\Gamma^l_{rj},$$

together with the relations

$$-R^{(d)(i)}_{(l)(a)bc} = \delta^i_l \chi^d_{abc} - \delta^d_a R^i_{lbc}, \quad -R^{(d)(i)}_{(l)(a)bk} = -\delta^d_a R^i_{lbk}, \quad -R^{(d)(l)}_{(i)(a)jk} = -\delta^d_a \mathfrak{R}^l_{ijk}.$$

In the next sections, following the physical and geometrical ideas of the already classical Lagrangian geometry of physical fields (see the works [12, 13, 14, 18]), we construct a possible multi-time Hamiltonian approach of the electromagnetic and gravitational physical fields, which is characterized by some natural geometrical Maxwell-like and Einstein-like equations. To reach this aim, we consider a multi-time Hamilton space $MH^n_m = \left(J^{1*}(T, M), H\right)$ endowed with its canonical nonlinear connection N (see formulas (1.4) and (1.5)) and we also consider the Cartan canonical N-linear connection of the space MH^n_m, which is locally expressed by the coefficients (1.8) or, in the case $m = \dim \mathcal{T} \geq 2$, by (1.9).

2. Multi-time Hamilton electromagnetism. Geometrical Maxwell-like equations

Let us consider the *canonical Liouville-Hamilton d-tensor field of polymomenta* $\mathbb{C}^* = p^a_i \left(\partial/\partial p^a_i\right)$, together with the fundamental vertical metrical d-tensor $G^{(i)(j)}_{(a)(b)}$ of the multi-time Hamilton spaces MH^n_m. These geometrical objects allow us to construct the *metrical deflection d-tensors*

$$\Delta^{(i)}_{(a)b} = G^{(i)(k)}_{(a)(c)} \Delta^{(c)}_{(k)b} = p^{(i)}_{(a)/b}, \qquad \Delta^{(i)}_{(a)j} = G^{(i)(k)}_{(a)(c)} \Delta^{(c)}_{(k)j} = p^{(i)}_{(a)|j},$$

$$\vartheta^{(i)(j)}_{(a)(b)} = G^{(i)(k)}_{(a)(c)} \vartheta^{(c)(j)}_{(k)(b)} = p^{(i)}_{(a)}|^{(j)}_{(b)},$$

where $p^{(i)}_{(a)} = G^{(i)(k)}_{(a)(c)} p^c_k$ and "$_{/b}$", "$_{|j}$" and "$|^{(j)}_{(b)}$" are the local covariant derivatives induced by the Cartan connection $C\Gamma(N)$.

Taking into account the expressions of the local covariant derivatives of the Cartan connection $C\Gamma(N)$ (see the paper [15]), by a direct calculation, we obtain

Proposition 2.1. *The metrical deflection d-tensors of the multi-time Hamilton spaces MH_m^n have the expresions:*

(i) *for* $m = \dim \mathcal{T} = 1$, *we have*

$$\Delta_{(1)1}^{(i)} = -h_{11} g^{ik} A_{k1}^r p_r^1, \qquad \Delta_{(1)j}^{(i)} = h_{11} g^{ik} \left[-\underset{2}{N}{}_{(k)j}^{(1)} - H_{kj}^r p_r^1 \right], \tag{2.1}$$
$$\vartheta_{(1)(1)}^{(i)(j)} = h_{11} g^{ij} - h_{11} g^{ik} C_{k(1)}^{r(j)} p_r^1;$$

(ii) *for* $m = \dim \mathcal{T} \geq 2$, *we have*

$$\Delta_{(a)b}^{(i)} = -h_{ac} g^{ik} A_{kb}^r p_r^c, \qquad \vartheta_{(a)(b)}^{(i)(j)} = h_{ab} g^{ij}, \tag{2.2}$$
$$\Delta_{(a)j}^{(i)} = -\frac{g^{ik}}{4} \left(\frac{\partial U_{ka}}{\partial x^j} + \frac{\partial U_{ja}}{\partial x^k} - 2 U_{sa} \Gamma_{jk}^s \right).$$

In order to construct our metrical multi-time Hamiltonian theory of electromagnetism, we introduce the following concept:

Definition 2.2. The distinguished 2-form on $J^{1*}(\mathcal{T}, M)$, locally defined by

$$\mathbb{F} = F_{(a)j}^{(i)} \delta p_i^a \wedge dx^j + f_{(a)(b)}^{(i)(j)} \delta p_i^a \wedge \delta p_j^b, \tag{2.3}$$

where

$$F_{(a)j}^{(i)} = \frac{1}{2} \left[\Delta_{(a)j}^{(i)} - \Delta_{(a)i}^{(j)} \right], \qquad f_{(a)(b)}^{(i)(j)} = \frac{1}{2} \left[\vartheta_{(a)(b)}^{(i)(j)} - \vartheta_{(a)(b)}^{(j)(i)} \right], \tag{2.4}$$

is called the *multi-time electromagnetic field of the metrical multi-time Hamilton space* MH_m^n.

By a straightforward calculation, Proposition 2.1 implies

Proposition 2.3. *The components* $F_{(a)j}^{(i)}$ *and* $f_{(a)(b)}^{(i)(j)}$ *of the multi-time electromagnetic field* $\mathbb{F}$, *associated to the multi-time Hamilton space* MH_m^n, *have the following expressions:*

(i) *in the case* $m = \dim \mathcal{T} = 1$, *we have*

$$F_{(1)j}^{(i)} = \frac{h^{11}}{2} \left[g^{jk} \underset{2}{N}{}_{(k)i}^{(1)} - g^{ik} \underset{2}{N}{}_{(k)j}^{(1)} + \left(g^{jk} H_{ki}^r - g^{ik} H_{kj}^r \right) p_r^1 \right], \quad f_{(1)(1)}^{(i)(j)} = 0;$$

(ii) *in the case* $m = \dim \mathcal{T} \geq 2$, *we have* $f_{(a)(b)}^{(i)(j)} = 0$ *and*

$$F_{(a)j}^{(i)} = \frac{1}{8} \left[g^{jk} \left(\frac{\partial U_{ka}}{\partial x^i} + \frac{\partial U_{ia}}{\partial x^k} - 2 U_{sa} \Gamma_{ik}^s \right) - g^{ik} \left(\frac{\partial U_{ka}}{\partial x^j} + \frac{\partial U_{ja}}{\partial x^k} - 2 U_{sa} \Gamma_{jk}^s \right) \right].$$

The main result of our abstract geometrical Hamilton multi-time electromagnetism is given by

Theorem 2.4. *The electromagnetic components $F^{(i)}_{(a)j}$ of the multi-time Hamilton space MH^n_m are governed by the following* ***geometrical Maxwell-like equations:***

(i) *for $m = \dim \mathcal{T} = 1$, we have*

$$\begin{cases} F^{(i)}_{(1)k/1} = \dfrac{1}{2}\mathcal{A}_{\{i,k\}}\left\{\Delta^{(i)}_{(1)1|k} + \Delta^{(i)}_{(1)r}T^r_{1k} + \vartheta^{(i)(r)}_{(1)(1)}R^{(1)}_{(r)1k} + R^i_{r1k}p^{(r)}_{(1)}\right\} \\ \sum_{\{i,j,k\}} F^{(i)}_{(1)j|k} = -\dfrac{1}{2}\sum_{\{i,j,k\}}\left\{\vartheta^{(i)(r)}_{(1)(1)}R^{(1)}_{(r)jk} + R^i_{rjk}p^{(r)}_{(1)}\right\} \\ F^{(i)}_{(1)j}|^{(k)}_{(1)} = \dfrac{1}{2}\mathcal{A}_{\{i,j\}}\left\{\vartheta^{(i)(k)}_{(1)(1)|j} - P^{i\ \ (k)}_{rj(1)}p^{(r)}_{(1)} - \Delta^{(i)}_{(1)r}C^{r(k)}_{j(1)} - \vartheta^{(i)(r)}_{(1)(1)}P^{(1)\ \ (k)}_{(r)j(1)}\right\}; \end{cases}$$

(ii) *for $m = \dim \mathcal{T} \geq 2$, we have*

$$\begin{cases} F^{(i)}_{(a)k/b} = \dfrac{1}{2}\mathcal{A}_{\{i,k\}}\left\{\Delta^{(i)}_{(a)b|k} + \Delta^{(i)}_{(a)r}T^r_{bk} + \vartheta^{(i)(r)}_{(a)(f)}R^{(f)}_{(r)bk} + R^i_{rbk}p^{(r)}_{(a)}\right\} \\ \sum_{\{i,j,k\}} F^{(i)}_{(a)j|k} = -\dfrac{1}{2}\sum_{\{i,j,k\}}\left\{\vartheta^{(i)(r)}_{(a)(f)}R^{(f)}_{(r)jk} + \mathfrak{R}^i_{rjk}p^{(r)}_{(a)}\right\} \\ \sum_{\{i,j,k\}} F^{(i)}_{(a)j}|^{(k)}_{(c)} = 0, \end{cases}$$

where $\mathcal{A}_{\{i,j\}}$ means an alternate sum, $\sum_{\{i,j,k\}}$ means a cyclic sum, and we used the notations $p^{(i)}_{(1)} = G^{(i)(j)}_{(1)(1)}p^1_j$ and $p^{(i)}_{(a)} = G^{(i)(j)}_{(a)(b)}p^b_j$.

Proof. The general Ricci identities (see [15, 16]) applied to g^{ij} lead us to the equalities:

$$\begin{gathered} g^{ir}R^j_{rbk} + g^{jr}R^i_{rbk} = 0, \qquad g^{ir}R^j_{rkl} + g^{jr}R^i_{rkl} = 0, \\ g^{ir}P^{j\ \ (l)}_{rk(c)} + g^{jr}P^{i\ \ (l)}_{rk(c)} = 0. \end{gathered} \tag{2.5}$$

Let us consider the following non-metrical deflection d-tensor identities ([15]):

(d_1) $\Delta^{(d)}_{(p)b|k} - \Delta^{(d)}_{(p)k/b} = p^d_r R^r_{pbk} - \Delta^{(d)}_{(p)r}T^r_{bk} - \vartheta^{(d)(r)}_{(p)(f)}R^{(f)}_{(r)bk}$,

(d_2) $\Delta^{(d)}_{(p)j|k} - \Delta^{(d)}_{(p)k|j} = p^d_r R^r_{pjk} - \vartheta^{(d)(r)}_{(p)(f)}R^{(f)}_{(r)jk}$,

(d_3) $\Delta^{(d)}_{(p)j}|^{(k)}_{(c)} - \vartheta^{(d)(k)}_{(p)(c)|j} = p^d_r P^{r\ \ (k)}_{pj(c)} - \Delta^{(d)}_{(p)r}C^{r(k)}_{j(c)} - \vartheta^{(d)(r)}_{(p)(f)}P^{(f)\ \ (k)}_{(r)j(c)}$,

where $\Delta^{(a)}_{(i)b} = p^a_{i/b}$, $\Delta^{(a)}_{(i)j} = p^a_{i|j}$, $\vartheta^{(a)(j)}_{(i)(b)} = p^a_i|^{(j)}_{(b)}$.

Contracting the above deflection d-tensor identities with the fundamental vertical metrical d-tensor $G^{(i)(p)}_{(a)(d)}$, and using the equalities (2.5), we obtain the following *metrical deflection d-tensor identities:*

(d'_1) $\Delta^{(i)}_{(a)b|k} - \Delta^{(i)}_{(a)k/b} = -p^{(r)}_{(a)}R^i_{rbk} - \Delta^{(i)}_{(a)r}T^r_{bk} - \vartheta^{(i)(r)}_{(a)(f)}R^{(f)}_{(r)bk}$,

(d'_2) $\Delta^{(i)}_{(a)j|k} - \Delta^{(i)}_{(a)k|j} = -p^{(r)}_{(a)}R^i_{rjk} - \vartheta^{(i)(r)}_{(a)(f)}R^{(f)}_{(r)jk}$,

(d'_3) $\Delta^{(i)}_{(a)j}|^{(k)}_{(c)} - \vartheta^{(i)(k)}_{(a)(c)|j} = -p^{(r)}_{(a)}P^{i\ \ (k)}_{rj(c)} - \Delta^{(i)}_{(a)r}C^{r(k)}_{j(c)} - \vartheta^{(i)(r)}_{(a)(f)}P^{(f)\ \ (k)}_{(r)j(c)}$.

To obtain the first (respectively, the third) geometrical Maxwell-like equation, we permute the indices i and k in the identity (d_1') (respectively, the indices i and j in the identity (d_3')), and we subtract this new identity from the initial one. For $m = \dim \mathcal{T} \geq 2$ we find the following new identity:

$$F_{(a)j}^{(i)}|_{(c)}^{(k)} = \frac{1}{2}\left[\vartheta_{(a)(c)|j}^{(i)(k)} - \vartheta_{(a)(c)|i}^{(j)(k)}\right].$$

Consequently, doing a cyclic sum by $\{i, j, k\}$ for $m \geq 2$, we obtain what we were looking for.

Doing a cyclic sum after the indices $\{i, j, k\}$ in the identity (d_2'), it follows the second geometrical Maxwell-like equation. □

3. Multi-time Hamilton gravitational field. Geometrical Einstein-like equations

Let us consider that $h = (h_{ab}(t))$ is a fixed semi-Riemannian metric on the temporal manifold $\mathcal{T}$ and let

$$N = \left(\underset{1}{N}{}_{(i)b}^{(a)},\ \underset{2}{N}{}_{(i)j}^{(a)}\right)$$

be an "*a priori*" given nonlinear connection on the dual 1-jet space $J^{1*}(\mathcal{T}, M)$.

Let

$$\delta p_i^a = dp_i^a + \underset{1}{N}{}_{(i)b}^{(a)} dt^b + \underset{2}{N}{}_{(i)j}^{(a)} dx^j$$

be the vertical distinguished 1-forms adapted to the nonlinear connection N.

An essential element in the development of our abstract geometrical multi-time Hamilton gravitational theory is given by the following definition:

Definition 3.1. From an abstract physical point of view, an adapted metrical d-tensor $\mathbb{G}$ on the dual 1-jet space $E^* = J^{1*}(\mathcal{T}, M)$, locally expressed by

$$\mathbb{G} = h_{ab} dt^a \otimes dt^b + g_{ij} dx^i \otimes dx^j + h_{ab} g^{ij} \delta p_i^a \otimes \delta p_j^b,$$

where $g_{ij} = g_{ij}(t^c, x^k, x_c^k)$ is a symmetric d-tensor field of rank $n = \dim M$ having a constant signature on $E^* = J^{1*}(\mathcal{T}, M)$, is called a *multi-time gravitational h-potential* on E^*.

Now, taking a multi-time Hamilton space $MH_m^n = (E^*, H)$, via its fundamental vertical metrical d-tensor $G_{(a)(b)}^{(i)(j)}$ (which is given by (1.3)) and its canonical nonlinear connection N, we naturally construct a corresponding multi-time gravitational h-potential on E^*, setting

$$\mathbb{G} = h_{ab} dt^a \otimes dt^b + g_{ij} dx^i \otimes dx^j + h_{ab} g^{ij} \delta p_i^a \otimes \delta p_j^b.$$

At the same time, let us consider that

$$C\Gamma(N) = \left(\chi_{ab}^c, A_{jc}^k, H_{jk}^i, C_{j(c)}^{i(k)}\right)$$

is the Cartan canonical connection of the multi-time Hamilton space MH_m^n.

Postulate. *We postulate that the* ***geometrical Einstein-like equations*** *which govern the multi-time gravitational h-potential* $\mathbb{G}$ *of the multi-time Hamilton space* MH^n_m*, are the abstract geometrical Einstein equations attached to the Cartan canonical connection* $C\Gamma(N)$ *and to the adapted metric* $\mathbb{G}$ *on* E^**, namely*

$$\operatorname{Ric}(C\Gamma) - \frac{\operatorname{Sc}(C\Gamma)}{2}\mathbb{G} = \mathcal{K}\mathbb{T}, \tag{3.1}$$

where $\operatorname{Ric}(C\Gamma)$ *represents the* ***Ricci tensor*** *of the Cartan connection,* $\operatorname{Sc}(C\Gamma)$ *is the* ***scalar curvature****,* $\mathcal{K}$ *is the* ***Einstein constant*** *and* $\mathbb{T}$ *is an intrinsic d-tensor of matter, which is called the* ***stress-energy d-tensor of polymomenta****.*

In the adapted basis of vector fields

$$(X_A) = \left(\frac{\delta}{\delta t^a}, \frac{\delta}{\delta x^i}, \frac{\partial}{\partial p^a_i}\right),$$

which is produced by the canonical nonlinear connection N of the multi-time Hamilton space MH^n_m, the curvature tensor $\mathbb{R}$ of the Cartan canonical connection $C\Gamma(N)$ is locally expressed by $\mathbb{R}(X_C, X_B)X_A = \mathbf{R}^D_{ABC}X_D$. It follows that we have

$$R_{AB} = \operatorname{Ric}(X_A, X_B) = \mathbf{R}^D_{ABD}, \qquad \operatorname{Sc}(C\Gamma) = G^{AB}R_{AB},$$

where

$$G^{AB} = \begin{cases} h^{ab}, & \text{for } A = a,\ B = b \\ g_{ij}, & \text{for } A = i,\ B = j \\ h^{ab}g_{ij}, & \text{for } A = {(a) \atop (i)},\ B = {(b) \atop (j)} \\ 0, & \text{otherwise.} \end{cases} \tag{3.2}$$

Taking into account, on the one hand, the form of the metrical d-tensor $\mathbb{G} = (G_{AB})$ of the multi-time Hamilton space MH^n_m, and, on the other hand, taking into account the expressions of the local curvature d-tensors attached to the Cartan canonical connection $C\Gamma\,(N)$, by direct computations, we find

Proposition 3.2. *The Ricci tensor* $\operatorname{Ric}(C\Gamma)$ *of the Cartan canonical connection* $C\Gamma\,(N)$ *of the multi-time Hamilton space* MH^n_m *is determined by the following adapted components:*

(i) *for* $m = \dim \mathcal{T} = 1$*, we have*

$$R_{11} := \chi_{11} = 0, \qquad R_{1i} = R^1_{1i1} = 0,$$

$$R_{1{(i) \atop (1)}} = -P^{1(i)}_{1(1)1} = 0, \qquad R_{i1} = R^r_{i1r}, \qquad R_{ij} = R^r_{ijr},$$

$$R^{(i)}_{(1)1} := -P^{(i)}_{(1)1} = -P^{i\ (r)}_{r1(1)}, \qquad R_{i{(j) \atop (1)}} := -P_{i{(j) \atop (1)}} = -P^{r\ (j)}_{ir(1)},$$

$$R^{(i)(j)}_{(1)(1)} := -S^{(i)(j)}_{(1)(1)} = -S^{i(j)(r)}_{r(1)(1)}, \qquad R^{(i)}_{(1)j} := -P^{(i)}_{(1)j} = -P^{i\ (r)}_{rj(1)};$$

(ii) *for* $m = \dim \mathcal{T} \geq 2$, *we have*

$$R_{ab} := \chi_{ab} = \chi^{f}_{abf}, \qquad R_{ai} = R^{f}_{aif} = 0,$$

$$R_{a(b)}^{\;\;(j)} = -P_{af(b)}^{f\;\;(j)} = 0, \qquad R_{ia} = R^{r}_{iar}, \qquad R_{ij} = R^{r}_{ijr},$$

$$R^{(i)(j)}_{(a)(b)} := -S^{(i)(j)}_{(a)(b)} = -S^{i(j)(r)}_{r(a)(b)} = 0, \quad R_{i(b)}^{\;(j)} := -P_{i(b)}^{\;(j)} = -P_{ir(b)}^{r\;(j)} = 0,$$

$$R^{(i)}_{(a)b} := -P^{(i)}_{(a)b} = -P^{i\;(r)}_{rb(a)} = 0, \qquad R^{(i)}_{(a)j} := -P^{(i)}_{(a)j} = -P^{i\;(r)}_{rj(a)} = 0.$$

Using the notations $\chi = h^{ab}\chi_{ab}$, $R = g^{ij}R_{ij}$ and $S = h^{11}g_{ij}S^{(i)(j)}_{(1)(1)}$, we obtain the following corollary.

Corollary 3.3. *The scalar curvature* $\mathrm{Sc}(C\Gamma)$ *of the Cartan canonical connection* $C\Gamma\,(N)$ *of the multi-time Hamilton space* MH^n_m *is given by the formulas:*

(i) *for* $m = \dim \mathcal{T} = 1$, *we have* $\mathrm{Sc}(C\Gamma) = R - S$;

(ii) *for* $m = \dim \mathcal{T} \geq 2$, *we have* $\mathrm{Sc}(C\Gamma) = \chi + R$.

In this context, the main result of the Hamilton geometrical multi-time gravitational theory is offered by the following theorem.

Theorem 3.4. *The* ***geometrical Einstein-like equations*** *which govern the multi-time gravitational h-potential* $\mathbb{G}$ *of the multi-time Hamilton space* MH^n_m, *have the following adapted local form:*

(i) *for* $m = \dim \mathcal{T} = 1$, *we have*

$$(3.3)\qquad \begin{cases} -\dfrac{R-S}{2}h_{11} = \mathcal{K}\mathbb{T}_{11} \\ R_{ij} - \dfrac{R-S}{2}g_{ij} = \mathcal{K}\mathbb{T}_{ij} \\ -S^{(i)(j)}_{(1)(1)} - \dfrac{R-S}{2}h_{11}g^{ij} = \mathcal{K}\mathbb{T}^{(i)(j)}_{(1)(1)} \\ 0 = \mathbb{T}_{1i}, \qquad R_{i1} = \mathcal{K}\mathbb{T}_{i1} \\ 0 = \mathbb{T}_{1(1)}^{\;\;(i)}, \qquad -P_{i(1)}^{\;(j)} = \mathcal{K}\mathbb{T}_{i(1)}^{\;(j)} \\ -P^{(i)}_{(1)1} = \mathcal{K}\mathbb{T}^{(i)}_{(1)1}, \qquad -P^{(i)}_{(1)j} = \mathcal{K}\mathbb{T}^{(i)}_{(1)j}; \end{cases}$$

(ii) *for* $m = \dim \mathcal{T} \geq 2$, *we have*

$$(3.4)\qquad \begin{cases} \chi_{ab} - \dfrac{\chi + R}{2} h_{ab} = \mathcal{K}\mathbb{T}_{ab} \\ R_{ij} - \dfrac{\chi + R}{2} g_{ij} = \mathcal{K}\mathbb{T}_{ij} \\ -\dfrac{\chi + R}{2} h_{ab} g^{ij} = \mathcal{K}\mathbb{T}^{(i)(j)}_{(a)(b)} \\ 0 = \mathbb{T}_{ai}, \qquad R_{ia} = \mathcal{K}\mathbb{T}_{ia} \\ 0 = \mathbb{T}^{(i)}_{(a)b}, \qquad 0 = \mathbb{T}_{a(b)}^{\ \ (j)} \\ 0 = \mathbb{T}_{i(b)}^{\ \ (j)}, \qquad 0 = \mathbb{T}^{(i)}_{(a)j}, \end{cases}$$

where $\mathbb{T}_{AB},\ A, B \in \left\{a,\ i,\ {}^{(i)}_{(a)}\right\}$ *represent the adapted components of the stress-energy d-tensor of matter* $\mathbb{T}$.

Remark 3.5. In order to have the compatibility of the system of geometrical Einstein-like equations, it is necessary the "a priori" vanishing of certain adapted components of the stress-energy d-tensor of matter $\mathbb{T}$.

From a physical point of view, it is well known that in the classical Riemannian theory of gravity, the stress-energy d-tensor of matter have to verify some conservation laws. By a natural extension of the classical Riemannian conservation laws, in our geometrical Hamiltonian context, we postulate the following *generalized conservation laws* of the stress-energy d-tensor of polymomenta $\mathbb{T}$:

$$\mathbb{T}^B_{A|B} = 0, \qquad \forall\, A \in \left\{a,\ i,\ {}^{(a)}_{(i)}\right\},$$

where $\mathbb{T}^B_A = G^{BD}\mathbb{T}_{DA}$. Consequently, by straightforward computations, we obtain the following theorem.

Theorem 3.6. *The* ***generalized conservation laws*** *of the Einstein-like equations of the multi-time Hamilton space* MH^n_m *are expressed by the following formulas:*

(i) *for* $m = \dim \mathcal{T} = 1$, *we have*

$$(3.5)\qquad \begin{cases} \left[\dfrac{R - S}{2}\right]_{/1} = R^r_{1|r} - P^{(1)}_{(r)1}\Big|^{(r)}_{(1)} \\ \left[R^r_j - \dfrac{R - S}{2}\delta^r_j\right]_{|r} = P^{(1)}_{(r)j}\Big|^{(r)}_{(1)} \\ \left[S^{(1)(j)}_{(r)(1)} + \dfrac{R - S}{2}\delta^j_r\right]\Big|^{(r)}_{(1)} = -P^{r(j)}_{\ \ (1)|r}, \end{cases}$$

where

$$R^i_1 = g^{iq}R_{q1}, \qquad P^{(1)}_{(i)1} = h^{11}g_{iq}P^{(q)}_{(1)1}, \quad R^i_j = g^{iq}R_{qj},$$

$$P^{(1)}_{(i)j} = h^{11}g_{iq}P^{(q)}_{(1)j}, \quad P^{i(j)}_{\ \ (1)} = g^{iq}P_{q(1)}^{\ \ (j)} \qquad S^{(1)(j)}_{(i)(1)} = h^{11}g_{iq}S^{(q)(j)}_{(1)(1)};$$

(ii) *for* $m = \dim \mathcal{T} \geq 2$, *we have*

$$\left\{ \begin{array}{l} \left[\chi_b^f - \dfrac{\chi + R}{2}\delta_b^f\right]_{/f} = -R^r_{b|r} \\ \left[R_j^r - \dfrac{\chi + R}{2}\delta_j^r\right]_{|r} = 0, \end{array} \right. \tag{3.6}$$

where

$$\chi_b^f = h^{fc}\chi_{cb}, \quad R_j^i = g^{iq}R_{qj}, \quad R_b^i = g^{iq}R_{qb}.$$

Open problem. The authors of this paper consider that the finding of a possible real physical meaning of the present multi-time Hamiltonian geometrical theories of gravity and electromagnetism may represent an open problem for physicists.

Acknowledgement

Many thanks go to Professor Gh. Atanasiu for his encouragements and useful suggestions concerning this research topic.

References

[1] Asanov, G. S., Jet extension of Finslerian gauge approach. Fortschritte der Physik 38, no. 8 (1990), 571-610.

[2] Atanasiu, Gh., Neagu, M., Canonical nonlinear connections in the multi-time Hamilton geometry. Balkan J. of Geom. and Its Appl. 14, no. 2 (2009), 1-12.

[3] Atanasiu, Gh., Neagu, M., Distinguished tensors and Poisson brackets in the multi-time Hamilton geometry. BSG Proceedings 16, pp. 12-27. Bucharest: Geometry Balkan Press 2009.

[4] Atanasiu, Gh., Neagu, M., Distinguished torsion, curvature and deflection tensors in the multi-time Hamilton geometry. Electronic Journal "Diff. Geom.–Dyn. Sys." 11 (2009), 20-40.

[5] Čomić, I., Multi-time Lagrange spaces. Electronic Journal "Diff. Geom.–Dyn. Sys." 7 (2005), 15-33.

[6] Giachetta, G., Mangiarotti, L., Sardanashvily, G., Covariant Hamiltonian field theory. arXiv:hep-th/9904062v1 (1999).

[7] Giachetta, G., Mangiarotti, L., Sardanashvily, G., Polysymplectic Hamiltonian formalism and some quantum outcomes. arXiv:hep-th/0411005v1 (2004).

[8] Gotay, M., Isenberg, J., Marsden, J. E., Montgomery, R., Momentum maps and classical fields. Part I. Covariant field theory. arXiv:physics/9801019v2 [math-ph] (2004).

[9] Gotay, M., Isenberg, J., Marsden, J. E., Momentum maps and classical fields. Part II. Canonical analysis of field theories. arXiv:math-ph/0411032v1 (2004).

[10] Kanatchikov, I. V., Basic structures of the covariant canonical formalism for fields based on the De Donder-Weyl theory. arXiv:hep-th/9410238v1 (1994).

[11] Kanatchikov, I. V., On quantization of field theories in polymomentum variables. AIP Conf. Proc. 453, no. 1 (1998), 356-367.

[12] Miron, R., Anastasiei, M., The geometry of Lagrange spaces: theory and applications. Dordrecht: Kluwer Academic Publishers 1994.

[13] Miron, R., Hrimiuc, D., Shimada, H., Sabău, S. V., The geometry of Hamilton and Lagrange spaces. Dordrecht: Kluwer Academic Publishers 2001.

[14] Neagu, M., Riemann-Lagrange geometry on 1-jet spaces. Bucharest: Matrix Rom 2005.

[15] Oană, A., Neagu, M., The local description of the Ricci and Bianchi identities for an h-normal N-linear connection on the dual 1-jet space $J^{1*}(\mathcal{T}, M)$. arXiv:1111.4173v1 [math.DG] (2011).

[16] Oană, A., Neagu, M., A distinguished Riemannian geometrization for quadratic Hamiltonians of polymomenta. Bull. of the Transilvania Univ. of Braşov, Ser. III: Math., Inform., Phys. 5 (54), no. 1 (2012), 35-44.

[17] Saunders, D. J., The geometry of jet bundles. New York and London: Cambridge University Press 1989.

[18] Vacaru, S., Goncharenko, Y., Yang-Mills fields and gauge gravity on generalized Lagrange and Finsler spaces. Int. J. of Theor. Phys. 34, no. 9 (1995), 1955-1980.

Received by the editors February 28, 2012