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ON UPPER AND LOWER CONTRA-ω-CONTINUOUS MULTIFUNCTIONS

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Abstract. In this paper, we define contra-ω-continuous multifunctions between topological spaces and obtain some characterizations and some basic properties of such multifunctions.

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1. Introduction

Various types of functions play a significant role in the theory of classical point set topology. A great number of papers dealing with such functions have appeared, and a good many of them have been extended to the setting of multifunction [13],[3],[4],[5],[6]. A. Al-Omari et. al. introduced the concept of contra-ω-continuous functions between topological spaces. In this paper, we define contra-ω-continuous multifunctions and obtain some characterizations and some basic properties of such multifunctions.

2. Preliminaries

Throughout this paper, (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces in which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X. For a subset A of (X, τ), $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$ denote the closure of A with respect to τ and the interior of A with respect to τ, respectively. Recently, as generalization of closed sets, the notion of ω-closed sets were introduced and studied by Hdeib [8]. A point $x \in X$ is called a condensation point of A if for each $U \in \tau$ with $x \in U$, the set $U \cap A$ is uncountable. A is said to be ω-closed [8] if it contains all its condensation points. The complement of an ω-closed set is said to be ω-open. It is well known that a subset W of a space (X, τ) is ω-open if and only if for each $x \in W$, there exists $U \in \tau$ such that $x \in U$ and $U \backslash W$ is countable. The

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family of all ω-open subsets of a topological space (X,τ), denoted by $\omega O(X)$, forms a topology on X finer than τ. The family of all ω-closed subsets of a topological space (X,τ) is denoted by $\omega C(X)$. The ω-closure and the ω-interior, that can be defined in the same way as $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively, will be denoted by $\omega\,\mathrm{Cl}(A)$ and $\omega\,\mathrm{Int}(A)$, respectively. We set $\omega O(X,x) = \{A : A \in \omega O(X) \text{ and } x \in A\}$ and $\omega C(X,x) = \{A : A \in \omega C(X) \text{ and } x \in A\}$. By a multifunction $F : (X,\tau) \to (Y,\sigma)$, following [3], we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is, $F^+(B) = \{x \in X : F(x) \subset B\}$ and $F^-(B) = \{x \in X : F(x) \cap B \neq \emptyset\}$. In particular, $F^-(Y) = \{x \in X : y \in F(x)\}$ for each point $y \in Y$ and for each $A \subset X$, $F(A) = \cup_{x\in A} F(x)$. Then F is said to be surjection if $F(X) = Y$ and injection if $x \neq y$ implies $F(x) \cap F(y) = \emptyset$.

Definition 2.1. A multifunction $F : (X,\tau) \to (Y,\sigma)$ is said to be [13]:

(i) upper ω-continuous if for each point $x \in X$ and each open set V containing $F(x)$, there exists $U \in \omega O(X,x)$ such that $F(U) \subset V$;

(ii) lower ω-continuous if for each point $x \in X$ and each open set V such that $F(x) \cap V \neq \emptyset$, there exists $U \in \omega O(X,x)$ such that $U \subset F^-(V)$.

Definition 2.2. A function $f : (X,\tau) \to (Y,\sigma)$ is said to be [2] contra-ω-continuous if for each point $x \in X$ and each open set V containing $f(x)$, there exists $U \in \omega O(X,x)$ such that $f(U) \subset V$.

3. On upper and lower contra-ω-continuous multifunctions

Definition 3.1. A multifunction $F : (X,\tau) \to (Y,\sigma)$ is said to be:

(i) upper contra-ω-continuous if for each point $x \in X$ and each closed set V containing $F(x)$, there exists $U \in \omega O(X,x)$ such that $F(U) \subset V$;

(ii) lower contra-ω-continuous if for each point $x \in X$ and each closed set V such that $F(x)\cap V \neq \emptyset$, there exists $U \in \omega O(X,x)$ such that $U \subset F^-(V)$.

The following examples show that the concepts of upper ω-continuity (resp. lower ω-continuity) and upper contra-ω-continuity (resp. lower contra-ω-continuity) are independent of each other.

Example 3.2. Let $X = \Re$ with the topology $\tau = \{\emptyset, \Re, \Re - Q\}$. Define a multifunction $F : (\Re,\tau) \to (\Re,\tau)$ as follows:

$$F(x) = \begin{cases} Q & \text{if } x \in \Re - Q \\ \Re - Q & \text{if } x \in Q. \end{cases}$$

Then F is upper contra-ω-continuous but is not upper ω-continuous.

Example 3.3. Let $X = \Re$ with the topology $\tau = \{\emptyset, \Re, \Re - Q\}$. Define a multifunction $F : (\Re,\tau) \to (\Re,\tau)$ as follows:

$$F(x) = \begin{cases} Q & \text{if } x \in Q \\ \Re - Q & \text{if } x \in \Re - Q. \end{cases}$$

Then F is upper ω-continuous but is not upper contra-ω-continuous.

In a similar form, we can find examples in order to show that lower contra-ω-continuity and lower ω-continuity are independent of each other.

Theorem 3.4. *The following statements are equivalent for a multifunction $F : (X, \tau) \to (Y, \sigma)$:*

(i) F is upper contra-ω-continuous;

(ii) $F^+(V) \in \omega O(X)$ for every closed subset V of Y;

(iii) $F^-(V) \in \omega C(X)$ for every open subset V of Y;

(iv) for each $x \in X$ and each closed set K containing $F(x)$, there exists $U \in \omega O(X, x)$ such that if $y \in U$, then $F(y) \subset K$.

Proof. $(i) \Leftrightarrow (ii)$: Let V be a closed subset in Y and $x \in F^+(V)$. Since F is upper contra-ω-continuous, there exists $U \in \omega O(X, x)$ such that $F(U) \subset V$. Hence, $F^+(V)$ is ω-open in X. The converse is similar.
$(ii) \Leftrightarrow (iii)$: It follows from the fact that $F^+(Y \backslash V) = X \backslash F^-(V)$ for every subset V of Y.
$(iii) \Leftrightarrow (iv)$: This is obvious. □

Theorem 3.5. *The following statements are equivalent for a multifunction $F : (X, \tau) \to (Y, \sigma)$:*

(i) F is lower contra-ω-continuous;

(ii) $F^-(V) \in \omega O(X)$ for every closed subset V of Y;

(iii) $F^+(K) \in \omega C(X)$ for every open subset K of Y;

(iv) for each $x \in X$ and each closed set K such that $F(x) \cap K \neq \emptyset$, there exists $U \in \omega O(X, x)$ such that if $y \in U$, then $F(y) \subset K \neq \emptyset$.

Proof. The proof is similar to that of Theorem 3.4. □

Corollary 3.6. *[2] The following statements are equivalent for a function $f : X \to Y$:*

(i) f is contra-ω-continuous;

(ii) $f^{-1}(V) \in \omega O(X)$ for every closed subset V of Y;

(iii) $f^{-1}(U) \in \omega C(X)$ for every open subset U of Y;

(iv) for each $x \in X$ and each closed set K containing $f(x)$, there exists $U \in \omega O(X, x)$ such that $f(U) \subset K$.

Definition 3.7. A topological space (X, τ) is said to be semi-regular [11] if for each open set U of X and for each point $x \in U$, there exists a regular open set V such that $x \in V \subset U$.

Definition 3.8. [12] Let (X, τ) be a topological space and A a subset of X and x a point of X. Then

(i) x is called δ-cluster point of A if $A \cap \mathrm{Int}(\mathrm{Cl}(U)) \neq \emptyset$, for each open set U containing x.

(ii) the family of all δ-cluster points of A is called the δ-closure of A and is denoted by $\mathrm{Cl}_\delta(A)$.

(iii) A is said to be δ-closed if $\mathrm{Cl}_\delta(A) = A$. The complement of a δ-closed set is said to be a δ-open set.

Theorem 3.9. *For a multifunction $F : (X, \tau) \to (Y, \sigma)$, where Y is semi-regular, the following are equivalent:*

(i) F is upper contra-ω-continuous;

(ii) $F^+(\mathrm{Cl}_\delta(B)) \in \omega O(X)$ for every subset B of Y;

(iii) $F^+(K) \in \omega O(X)$ for every δ-closed subset K of Y;

(iv) $F^-(V) \in \omega C(X)$ for every δ-open subset V of Y.

Proof. $(i) \Rightarrow (ii)$: Let B be any subset of Y. Then $\mathrm{Cl}_\delta(B)$ is closed and by Theorem 3.4, $F^+(\mathrm{Cl}_\delta(B)) \in \omega O(X)$. $(ii) \Rightarrow (iii)$: Let K be a δ-closed set of Y. Then $\mathrm{Cl}_\delta(K) = K$. By (ii), $F^+(K)$ is ω-open. $(iii) \Rightarrow (iv)$: Let V be a δ-open set of Y. Then $Y \backslash V$ is δ-closed. By (iii), $F^+(Y \backslash V) = X \backslash F^-(V)$ is ω-open. Hence, $F^-(V)$ is ω-closed. $(iv) \Rightarrow (i)$: Let V be any open set of Y. Since Y is semi-regular, V is δ-open. By (iv), $F^-(V)$ is ω-closed and by Theorem 3.4, F is upper contra-ω-continuous. □

Theorem 3.10. *For a multifunction $F : (X, \tau) \to (Y, \sigma)$, where Y is semi-regular, the following are equivalent:*

(i) F is lower contra-ω-continuous;

(ii) $F^-(\mathrm{Cl}_\delta(B)) \in \omega O(X)$ for every subset B of Y;

(iii) $F^-(K) \in \omega O(X)$ for every δ-closed subset K of Y;

(iv) $F^+(V) \in \omega C(X)$ for every δ-open subset V of Y.

Proof. The proof is similar to that of Theorem 3.9. □

Remark 3.11. By Theorems 3.9 and 3.10, we obtain the following new characterization for contra-ω-continuous functions.

Corollary 3.12. *For a function $f : X \to Y$, where Y is semi-regular, the following are equivalent:*

(i) f is contra-ω-continuous;

(ii) $f^{-1}(\mathrm{Cl}_\delta(B)) \in \omega O(X)$ for every subset B of Y;

(iii) $f^{-1}(K) \in \omega O(X)$ for every δ-closed subset K of Y;

(iv) $f^{-1}(V) \in \omega C(X)$ for every δ-open subset V of Y.

Definition 3.13. A subset K of a space X is said to be strongly S-closed [7] (resp. ω-compact [2]) relative to X if every cover of K by closed (resp. ω-open) sets of X has a finite subcover. A space X is said to be strongly S-closed (resp. ω-compact) if X is strongly S-closed (resp. ω-compact) relative to X.

Theorem 3.14. *Let $F : (X, \tau) \to (Y, \sigma)$ be an upper contra-ω-continuous surjective multifunction and $F(x)$ is strongly S-closed relative to Y for each $x \in X$. If A is a ω-compact relative to X, then $F(A)$ is strongly S-closed relative to Y.*

Proof. Let $\{V_i : i \in \Delta\}$ be any cover of $F(A)$ by closed sets of Y. For each $x \in A$, there exists a finite subset $\Delta(x)$ of Δ such that $F(x) \subset \cup\{V_i : i \in \Delta(x)\}$. Put $V(x) = \cup\{V_i : i \in \Delta(x)\}$. Then $F(x) \subset V(x)$ and there exists $U(x) \in \omega O(X, x)$ such that $F(U(x)) \subset V(x)$. Since $\{U(x) : x \in A\}$ is a cover of A by ω-open sets in X, there exists a finite number of points of A, say, x_1, x_2,....x_n such that $A \subset \cup\{U(x_i) : 1 = 1, 2, ...n\}$. Therefore, we obtain $F(A) \subset F(\overset{n}{\underset{i=1}{\cup}} U(x_i)) \subset \overset{n}{\underset{i=1}{\cup}} F(U(x_i)) \subset \overset{n}{\underset{i=1}{\cup}} V(x_i) \subset \overset{n}{\underset{i=1}{\cup}} \underset{i=\Delta(x_i)}{\cup} V_i$. This shows that $F(A)$ is strongly S-closed relative to Y. □

Corollary 3.15. *Let $F : (X, \tau) \to (Y, \sigma)$ be an upper contra-ω-continuous surjective multifunction and $F(x)$ is ω-compact relative to Y for each $x \in X$. If X is ω-compact, then Y is strongly S-closed.*

Corollary 3.16. *If $f : (X, \tau) \to (Y, \sigma)$ is contra-ω-continuous surjective and A is ω-compact relative to X, then $f(A)$ is strongly S-closed relative to Y.*

Lemma 3.17. *[1] Let A and B be subsets of a topological space (X, τ).*

(i) If $A \in \omega O(X)$ and $B \in \tau$, then $A \cap B \in \omega O(B)$;

(ii) If $A \in \omega O(B)$ and $B \in \omega O(X)$, then $A \in \omega O(X)$.

Theorem 3.18. *Let $F : (X, \tau) \to (Y, \sigma)$ be a multifunction and U an open subset of X. If F is an upper contra-ω-continuous (resp. lower contra-ω-continuous), then $F_{|_U} : U \to Y$ is an upper contra-ω-continuous (resp. lower contra-ω-continuous) multifunction.*

Proof. Let V be any closed set of Y. Let $x \in U$ and $x \in F^-_{|_U}(V)$. Since F is lower contra-ω-continuous multifunction, there exists a ω-open set G containing x such that $G \subset F^-(V)$. Then $x \in G \cap U \in \omega O(A)$ and $G \cap U \subset F^-_{|_U}(V)$. This shows that $F_{|_U}$ is a lower contra-ω-continuous. The proof of the upper contra-ω-continuous of $F_{|_U}$ is similar. □

Corollary 3.19. *If $f : (X, \tau) \to (Y, \sigma)$ is contra-ω-continuous and $U \in \tau$, then $f_{|U} : U \to Y$ is contra-ω-continuous.*

Theorem 3.20. *Let $\{U_i : i \in \Delta\}$ be an open cover of a topological space X. A multifunction $F : (X, \tau) \to (Y, \sigma)$ is upper contra-ω-continuous if and only if the restriction $F_{|U_i} : U_i \to Y$ is upper contra-ω-continuous for each $i \in \Delta$.*

Proof. Suppose that F is upper contra-ω-continuous. Let $i \in \Delta$ and $x \in U_i$ and V be a closed set of Y containing $F_{|U_i}(x)$. Since F is upper contra-ω-continuous and $F(x) = F_{|U_i}(x)$, there exists $G \in \omega O(X, x)$ such that $F(G) \subset V$. Set $U = G \cap U_i$, then $x \in U \in \omega O(U_i, x)$ and $F_{|U_i}(U) = F(U) \subset V$. Therefore, $F_{|U_i}$ is upper contra-ω-continuous. Conversely, let $x \in X$ and $V \in \omega O(Y)$ containing $F(x)$. There exists $i \in \Delta$ such that $x \in U_i$. Since $F_{|U_i}$ is upper contra-ω-continuous and $F(x) = F_{|U_i}(x)$, there exists $U \in \omega O(U_i, x)$ such that $F_{|U_i}(U) \subset V$. Then we have $U \in \omega O(X, x)$ and $F(U) \subset V$. Therefore, F is upper contra-ω-continuous. □

Theorem 3.21. *Let X and X_j be topological spaces for $i \in I$. If a multifunction $F : X \to \prod_{i \in I} X_i$ is an upper (lower) contra-ω-continuous multifunction, then $P_i \circ F$ is an upper (lower) contra-ω-continuous multifunction for each $i \in I$, where $P_i : \prod_{i \in I} X_i \to X_i$ is the projection for each $i \in I$.*

Proof. Let H_i be a closed subset of X_j. We have $(P_i \circ F)^+(H_j) = F^+(P_j^+(H_j)) = F^+(H_j \times \prod_{i \neq j} X_i)$. Since F an upper contra-ω-continuous multifunction, $F^+(H_j \times \prod_{i \neq j} X_i)$ is ω-open in X. Hence, $P_i \circ F$ is an upper (lower) contra-ω-continuous. □

Corollary 3.22. *Let X and X_i be topological spaces for $i \in I$. If a function $F : X \to \prod_{i \in I} X_i$ is a contra-ω-continuous, then $P_i \circ F$ is a contra-ω-continuous function for each $i \in I$, where $P_i : \prod_{i \in I} X_i \to X_i$ is the projection for each $i \in I$.*

Definition 3.23. A topological space X is said to be:

(i) ω-normal [9] if each pair of nonempty disjoint closed sets can be separated by disjoint ω-open sets.

(ii) ultranormal [10] if each pair of nonempty disjoint closed sets can be separated by disjoint clopen sets.

Theorem 3.24. *If $F : (X, \tau) \to (Y, \sigma)$ is an upper contra-ω-continuous punctually closed multifunction and Y is ultranormal, then X is ω-normal.*

Proof. The proof follows from the definitions. □

Corollary 3.25. *If $f : (X, \tau) \to (Y, \sigma)$ is a contra-ω-continuous closed multifunction and Y is ultranormal, then X is ω-normal.*

Definition 3.26. [2] Let A be a subset of a space X. The ω-frontier of A denoted by $\omega Fr(A)$, is defined as follows: $\omega Fr(A) = \omega\,\mathrm{Cl}(A) \cap \omega\,\mathrm{Cl}(X \backslash A)$.

Theorem 3.27. *The set of points x of X at which a multifunction $F : (X, \tau) \to (Y, \sigma)$ is not upper contra-ω-continuous (resp. upper contra-ω-continuous) is identical with the union of the ω-frontiers of the upper (resp. lower) inverse images of closed sets containing (resp. meeting) $F(x)$.*

Proof. Let x be a point of X at which F is not upper contra-ω-continuous. Then there exists a closed set V of Y containing $F(x)$ such that $U \cap (X \backslash F^+(V)) \neq \emptyset$ for each $U \in \omega O(X, x)$. Then $x \in \omega \operatorname{Cl}(X \backslash F^+(V))$. Since $x \in F^+(V)$, we have $x \in \omega \operatorname{Cl}(F^+(Y)$ and hence $x \in \omega Fr(F^+(A))$. Conversely, let V be any closed set of Y containing $F(x)$ and $x \in \omega Fr(F^+(V))$. Now, assume that F is upper contra-ω-continuous at x, then there exists $U \in \omega O(X, x)$ such that $F(U) \subset V$. Therefore, we obtain $x \in U \subset \omega \operatorname{Int}(F^+(V)$. This contradicts that $x \in \omega Fr(F^+(V))$. Thus, F is not upper contra-ω-continuous. The proof of the second case is similar. □

Corollary 3.28. *[2] The set of all points x of X at which $f : (X, \tau) \to (Y, \sigma)$ is not contra-ω-continuous is identical with the union of the ω-frontiers of the inverse images of closed sets of Y containing $f(x)$.*

Definition 3.29. A multifunction $F : (X, \tau) \to (Y, \sigma)$ is said to have a contra ω-closed graph if for each pair $(x, y) \in (X \times Y) \backslash G(F)$ there exist $U \in \omega O(X, x)$ and a closed set V of Y containing y such that $(U \times V) \cap G(F) = \emptyset$.

Lemma 3.30. *For a multifunction $F : (X, \tau) \to (Y, \sigma)$, the following holds:*

(i) $G_F^+(A \times B) = A \cap F^+(B)$;

(ii) $G_F^-(A \times B) = A \cap F^-(B)$

for any subset A of X and B of Y.

Theorem 3.31. *Let $F : (X, \tau) \to (Y, \sigma)$ be an u.ω-c. multifunction from a space X into a T_2 space Y. If $F(x)$ is α-paracompact for each $x \in X$, then $G(F)$ is ω-closed.*

Proof. Suppose that $(x_0, y_0) \notin G(F)$. Then $y_0 \notin F(x_0)$. Since Y is a T_2 space, for each $y \in F(x_0)$ there exist disjoint open sets $V(y)$ and $W(y)$ containing y and y_0, respectively. The family $\{V(y) : y \in F(x_0)\}$ is an open cover of $F(x_0)$. Thus, by α-paracompactness of $F(x_0)$, there is a locally finite open cover $\Delta = \{U_\beta : \beta \in I\}$ which refines $\{V(y) : y \in F(x_0)\}$. Therefore, there exists an open neighborhood W_0 of y_0 such that W_0 intersects only finitely many members U_{β_1}, U_{β_2},.....U_{β_n} of Δ. Choose y_1, y_2,.....y_n in $F(x_0)$ such that $U_{\beta_i} \subset V(y_i)$ for each $1 \leq i \leq n$, and set $W = W_0 \cap (\bigcap_{i=1}^{n} W(y_i))$. Then W is an open neighborhood of y_0 such that $W \cap (\bigcup_{\beta \in I} V_\beta) = \emptyset$. By the upper ω-continuity of F, there is a $U \in \omega O(X, x_0)$ such that $U \subset F^+(\bigcup_{\beta \in I} V_\beta)$. It follows that $(U \times W) \cap G(F) = \emptyset$. Therefore, $G(F)$ is ω-closed. □

Theorem 3.32. *Let $F : (X, \tau) \to (Y, \sigma)$ be a multifunction from a space X into a ω-compact space Y. If $G(F)$ is ω-closed, then F is u.ω-c..*

Proof. Suppose that F is not *u.ω-c.*. Then there exists a nonempty closed subset C of Y such that $F^-(C)$ is not ω-closed in X. We may assume $F^-(C) \neq \emptyset$. Then there exists a point $x_0 \in \omega \operatorname{Cl}(F^-(C)) \backslash F^-(C)$. Hence for each point $y \in C$, we have $(x_0, y) \notin G(F)$. Since F has a ω-closed graph, there are

ω-open subsets $U(y)$ and $V(y)$ containing x_0 and y, respectively such that $(U(y) \times V(y)) \cap G(F) = \emptyset$. Then $\{Y \backslash C\} \cup \{V(y) : y \in C\}$ is a ω-open cover of Y, and thus it has a subcover $\{Y \backslash C\} \cup \{V(y_i) : y_i \in C, 1 \leq i \leq n\}$. Let $U = \bigcap_{i=1}^{n} U(y_i)$ and $V = \bigcup_{i=1}^{n} V(y_i)$. It is easy to verify that $C \subset V$ and $(U \times V) \cap G(F) = \emptyset$. Since U is a ω-neighborhood of x_0, $U \cap F^{-}(C) \neq \emptyset$. It follows that $\emptyset \neq (U \times C) \cap G(F) \subset (U \times V) \cap G(F)$. This is a contradiction. Hence the proof is completed. □

Corollary 3.33. *Let $F : (X, \tau) \rightarrow (Y, \sigma)$ be a multifunction into a ω-compact T_2 space Y such that $F(x)$ is ω-closed for each $x \in X$. Then F is u.ω-c. if and only if it has a ω-closed graph.*

Theorem 3.34. *Let $F : (X, \tau) \rightarrow (Y, \sigma)$ be an u.ω-c. multifunction into a ω-T_2 space Y. If $F(x)$ is α-paracompact for each $x \in X$, then $G(F)$ is ω-closed.*

Proof. The proof is clear. □

Theorem 3.35. *Let $F : (X, \tau) \rightarrow (Y, \sigma)$ be a multifunction and X be a connected space. If the graph multifunction of F is upper contra-ω-continuous (resp. lower contra-ω-continuous), then F is upper contra-ω-continuous (resp. lower contra-ω-continuous).*

Proof. Let $x \in X$ and V be any open subset of Y containing $F(x)$. Since $X \times V$ is a ω-open set of $X \times Y$ and $G_F(x) \subset X \times V$, there exists a ω-open set U containing x such that $G_F(U) \subset X \times V$. By Lemma 3.30, we have $U \subset G_F^{+}(X \times V) = F^{+}(V)$ and $F(U) \subset V$. Thus, F is *u.ω-c.*. The proof of the *l.ω-c.* of F can be done using a similar argument. □

References

[1] Al-Zoubi, K., Al-Nashef, B., The topology of ω-open subsets. Al-Manarah. 9 (2003), 169-179.

[2] Al-Omari, A., Noorani, M. S. M., Contra-ω-continuous and almost ω-continuous functions. Int. J. Math. Math. Sci. 9 (2007), 169-179.

[3] Banzaru, T., Multifunctions and M-product spaces. Bull. Stin. Tech. Inst. Politech. Timisoara, Ser. Mat. Fiz. Mer. Teor. Apl. 17(31)(1972), 17-23.

[4] Carpintero, C., Rajesh, N., Rosas, E., Saranyasri, S., Some properties of upper/lower almost ω-continuous multifunctions. Scientific Studies and Research Series Mathematics and Informatics 23 (2) (2013), 35-55.

[5] , C., Rajesh, N., Rosas, E., Saranyasri, S., Properties of faintly ω-continuous functions. Boletin de Matematicas 20(2) (2013), 135-143.

[6] Carpintero, C., Rajesh, N., Rosas, E., Saranyasri, S., On slightly ω-continuous multifunctions. Punjab University Journal of Mathematics. 46 (1) (2014), 51-57.

[7] Dontchev, J., Contra-continuous functions and strongly S-closed spaces. Internat. J. Math. Math. Sci. 19 (1996), 303-310.

[8] Hdeib, H. Z., ω-closed mappings. Revista Colombiana Mat. 16 (1982), 65-78.

[9] Noiri, T., Al-omari, A., Noorani, M. S. M., Slightly ω-continuous functions. Fasc. Math. 41 (2009), 97-106.

[10] Staum, R., The algebra of bounded continuous fuctions into a nonarchimedean field. Pacific J. Math. 50 (1974), 169-185.

[11] Stone, M., Applications of the theory of boolean rings to general topology. Trans. Amer. Math. Soc. 41 (1937), 374-381.

[12] Velicko, N. V., H-closed topological spaces. Amer. Math. Soc. Transl. 78 (1968), 103-118.

[13] Zorlutuna, I., ω-continuous multifunctions. Filomat. 27(1) (2013), 155-162.

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