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# ON GENERALIZED $\phi$- RECURRENT AND GENERALIZED CONCIRCULARLY $\phi$-RECURRENT P-SASAKIAN MANIFOLDS

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**Abstract.** The object of the present paper is to study generalized $\phi-$ recurrent and generalized concircular $\phi-$ recurrent P-Sasakian manifold.

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## 1. Introduction

The notion of local symmetry in a Riemannian manifold has been weakened by many authors in several ways to different extent. As a weaker version of local symmetry, Takahashi [11] introduced the the notion of locally $\phi-$ symmetry on a Sasakian manifold. Some authors like De and Pathak [6], Venkatesha and Wagewadi [13], Shaikh and De [7] have extended this notion to 3-dimensional Kenmotsu, Trans-Sasakian and LP-Sasakian manifolds respectively. Recently Jaiswal and Ojha [8] studied generalized $\phi-$ recurrent LP-Sasakian manifold and obtained some interesting results. A space form (i.e. complete simply connected Riemannian manifold of constant curvature) is said to be elliptic, hyperbolic or euclidean accordingly as the sectional curvature tensor is positive, negative or zero [4].

In this paper we studied some properties of generalized $\phi-$ recurrent and generalized concircular $\phi-$ recurrent P-Sasakian manifold. The paper is organized as follows: Section 2 consist the basic definitions of P-Sasakian and $\eta-$ Einstein manifolds. In section 3, we studied generalized $\phi-$ recurrent P-Sasakian manifold and proved that a generalized $\phi-$ recurrent P-Sasakian manifold is an Einstein manifold. In section 4, we studied generalized concircularly $\phi-$ recurrent P-Sasakian manifold. At first it is shown that a generalized concircularly $\phi-$ recurrent P-Sasakian manifold is an $\eta-$Einstein manifold. Then we have shown that in a generalized concircularly $\phi-$ recurrent P-Sasakian manifold the characteristic vector field $\xi$ and the vector fields $\rho_1, \rho_2$ associated to the 1-forms $A, B$ respectively are co-directional. Finally in the last section, we have shown that a 3- dimensional locally generalized concircularly $\phi-$ recurrent P-Sasakian manifold is of constant curvature.

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## 2. Preliminaries

An n-dimensional differentiable manifold $M^n$ is a Para-Sasakian (briefly P-Sasakian) manifold if it admits a (1,1) tensor field $\phi$, a contravariant vector field $\xi$, a covariant vector field $\eta$, and a Riemannian metric g, which satisfy

$$\phi^2 X = X - \eta(X)\xi, \quad g(X,\xi) = \eta(X), \quad \phi\xi = 0, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \tag{2.2}$$

$$(D_X\phi)Y = -g(X,Y)\xi - \eta(Y)X + 2\,\eta(X)\eta(Y)\xi \tag{2.3}$$

$$D_X\xi = \phi X, \tag{2.4}$$

$$(D_X\eta)(Y) = g(\phi X, Y) = g(\phi Y, X), \tag{2.5}$$

for any vector fields X and Y, where D denotes covariant differentiation with respect to g ([1], [2]).
It can be seen that in a P-Sasakian manifold $M^n$ with the structure $(\phi, \xi, \eta, g)$, the following hold:

$$(a)\ \ \eta(\xi) = 1 \quad (b) \quad \eta(\phi X) = 0, \tag{2.6}$$

$$rank(\phi) = (n-1). \tag{2.7}$$

Further in a P-Sasakian manifold the following relations also hold ([1], [2])

$$\eta(K(X,Y)Z) = g(X,Z)\eta(Y) - g(Y,Z)\eta(X), \tag{2.8}$$

$$K(X,Y)\xi = \eta(X)Y - \eta(Y)X, \tag{2.9}$$

$$S(X,\xi) = -(n-1)\eta(X), \tag{2.10}$$

$$K(\xi,X)Y = \eta(Y)X - g(X,Y)\xi, \tag{2.11}$$

$$S(\phi X, \phi Y) = S(X,Y) + (n-1)g(X,Y), \tag{2.12}$$

for any vector fields $X, Y, Z$, where $K$ and $S$ are the Riemannian curvature tensor and Ricci tensor of the manifold respectively .
A P-Sasakian manifold $M^n$ is said to be $\eta$- Einstein if its Ricci tensor $S$ is of the form

$$S(X,Y) = \alpha\, g(X,Y) + \beta\, \eta(X)\eta(Y), \tag{2.13}$$

for any vector fields $X$ and $Y$, where $\alpha,\ \beta$ are smooth functions on $M^n$ [3]. In particular if $\beta = 0$ in above equation then $\eta$ - Einstein manifold becomes an Einstein manifold.

## 3. Generalized $\phi$ - recurrent P - Sasakian manifold

Analogous of consideration of generalized recurrent manifolds [5], we give the following definition

**Definition 3.1.** A P-Sasakian manifold is said to be a generalized $\phi$ - recurrent if its curvature tensor $K$ satisfies the condition

$$\begin{aligned} \phi^2((D_W K)(X,Y)Z) &= A(W)K(X,Y)Z \\ &+ B(W)[g(Y,Z)X - g(X,Z)Y], \end{aligned} \tag{3.1}$$

where $A$ and $B$ are two 1-forms, $B$ is non zero and they are defined by

$$A(X) = g(X,\rho_1),\ B(X) = g(X,\rho_2), \tag{3.2}$$

and $\rho_1,\ \rho_2$ are vector fields associated with 1-forms $A,\ B$ respectively.

If the 1-form $B$ in (3.1) becomes zero, then the manifold reduces to a $\phi$ - recurrent P-Sasakian manifold which is studied in [10].
By the virtue of (2.1),the equation (3.1) becomes

$$\begin{aligned} (D_W K)(X,Y)Z &= \eta((D_W K)(X,Y)Z)\xi + A(W)K(X,Y)Z \\ &+ B(W)[g(Y,Z)X - g(X,Z)Y] \end{aligned} \tag{3.3}$$

from which it follows that

$$\begin{aligned} g((D_W K)(X,Y)Z,U) &= \eta((D_W K)(X,Y)Z)\eta(U) + A(W)g(K(X,Y)Z,U) \\ &+ B(W)[g(Y,Z)g(X,U) - g(X,Z)g(Y,U)]. \end{aligned} \tag{3.4}$$

Let $e_i,\ i = 1,2,....,n$ be an orthonormal basis of the tangent space at any point of the manifold.Then putting $X = U = e_i$ in (3.4) and taking summation over $i,\ 1 \leq i \leq n$, we get

$$\begin{aligned} (D_W S)(Y,Z) &- \sum_{i=1}^{n} \eta((D_W K)(e_i,Y)Z)\eta(e_i) \\ &= A(W)S(Y,Z) + (n-1)B(W)g(Y,Z). \end{aligned} \tag{3.5}$$

The second term in of L.H.S. of (3.5) by putting $Z = \xi$ assumes the form

$$\sum_{i=1}^{n}[g((D_W K)(e_i,Y)\xi,\xi)g(e_i,\xi)],$$

which is denoted by $E$. In this case $E$ vanishes. Namely, we have

$$\begin{aligned} g((D_W K)(e_i,Y)\xi,\xi) &= g(D_W K(e_i,Y)\xi,\xi) - g(K(D_W e_i,Y)\xi,\xi) \\ &- g(K(e_i,D_W Y)\xi,\xi) - g(K(e_i,Y)D_W\xi,\xi), \end{aligned}$$

at $p \in M^n$. Since $\{e_i\}$ is an orthonormal basis so $D_W e_i = 0$ at $p$, using (2.8),we get

$$\begin{aligned} g(K(e_i,D_W Y)\xi,\xi) &= g(e_i,\xi)g(D_W Y,\xi) \\ &- g(D_W Y,\xi)g(e_i,\xi) = 0. \end{aligned} \tag{3.6}$$

Thus we obtain

$$g((D_W K)(e_i, Y)\xi, \xi) = g((D_W K(e_i, Y)\xi, \xi) - g(K(e_i, Y)D_W\xi, \xi). \tag{3.7}$$

Taking account of $g(K(e_i, Y)\xi, \xi) = g(K(\xi, \xi)Y, e_i) = 0$, we get

$$g((D_W K(e_i, Y)\xi, \xi) + g(K(e_i, Y)\xi, D_W\xi) = 0. \tag{3.8}$$

In view of (3.8), (3.7) becomes

$$g((D_W K)(e_i, Y)\xi, \xi) = -g(K(e_i, Y)\xi, D_W\xi) - g(K(e_i, Y)D_W\xi, \xi). \tag{3.9}$$

Hence finally we have

$$\begin{aligned} E &= -\sum_{i=1}^{n}[g(K(\phi W, \xi)Y, e_i)g(\xi, e_i) + g(K(\xi, \phi W)Y, e_i)g(\xi, e_i)] \\ &= -g(K(\phi W, \xi)Y, \xi) - g(K(\xi, \phi W)Y, \xi) = 0. \end{aligned}$$

Putting $Z = \xi$ in (3.5) and using (2.10), we obtain

$$(D_W S)(Y, \xi) = -(n-1)A(W)\eta(Y) + (n-1)B(W)\eta(Y). \tag{3.10}$$

We know that

$$(D_W S)(Y, \xi) = D_W S(Y, \xi) - S(D_W Y, \xi) - S(Y, D_W\xi). \tag{3.11}$$

By the virtue of (2.10) and (2.4) the above relation takes the form as

$$(D_W S)(Y, \xi) = -(n-1)g(\phi W, Y) - S(Y, \phi W). \tag{3.12}$$

Comparing equations (3.10) and (3.12) we obtain

$$-(n-1)g(\phi W, Y) - S(Y, \phi W) = -(n-1)A(W)\eta(Y) + (n-1)B(W)\eta(Y). \tag{3.13}$$

Replacing $Y$ by $\phi Y$ and then using (2.2),(2.6) and (2.12) in above, we obtain

$$S(Y, W) = -(n-1)g(Y, W), \tag{3.14}$$

for vector fields $Y, W$. This leads to the following theorem:

**Theorem 3.2.** *A generalized $\phi-$ recurrent P-Sasakian manifold is an Einstein manifold.*

Making use of (2.4) and (2.9) it can be easily seen that in a P-Sasakian manifold the following result holds

$$(D_W K)(X, Y)\xi = g(W, \phi Y)X - g(W, \phi X)Y - K(X, Y, \phi W). \tag{3.15}$$

By the virtue of (2.8), it follows from (3.15) that

$$\eta((D_W K)(X,Y)\xi) = 0. \tag{3.16}$$

Now assume that $X, Y, Z$ are (local) vector fields such that $(DX)_p = (DY)_p = (DZ)_p = 0$ for a fixed point $p$ of $M^n$. By Ricci identity for $\phi$ [12]

$$-(K(X,Y).\phi Z) = (D_X D_Y \phi)Z - (D_Y D_X \phi)Z.$$

We have at the point $p$,

$$-K(X,Y,\phi Z) + \phi K(X,Y)Z = D_X((D_Y \phi)Z) - D_Y((D_X \phi)Z).$$

Using (2.3) in above we get

$$\begin{aligned} -K(X,Y,\phi Z) &+ \phi K(X,Y)Z \\ &= D_X\{-g(Y,Z)\xi - \eta(Z)Y + 2\,\eta(Y)\eta(Z)\xi\} \\ &- D_Y\{-g(X,Z)\xi - \eta(Z)X + 2\,\eta(Z)\eta(X)\xi\} \\ &= -g(Y,Z)D_X\xi - (D_X\eta)(Z)Y + 2\,(D_X\eta)(Z)\eta(Y)\xi \\ &+ 2\,\eta(Z)(D_X\eta)(Y)\xi + 2\,\eta(Z)\eta(Y)(D_X\xi) \\ &+ g(X,Z)D_X\xi + (D_Y\eta)(Z)X - 2\,(D_Y\eta)\eta(X)\xi \\ &- 2\eta(Z)(D_Y\eta)(X)\xi - 2\,\eta(Z)\eta(X)(D_Y\xi). \end{aligned}$$

Using (2.4) and (2.5), we obtain

$$\begin{aligned} -K(X,Y,\phi Z) &+ \phi K(X,Y)Z \\ &= g(Y,Z)\phi X + g(X,Z)\phi Y - g(\phi X,Z)Y + g(\phi Y,Z)X \\ &+ 2\,g(\phi X,Z)\eta(Y)\xi + 2\,g(\phi Y,Z)\eta(X)\xi \\ &+ 2\,\eta(Y)\eta(Z)\phi X - 2\,\eta(Z)\eta(X)\phi Y. \end{aligned}$$

Making use of (3.15) the above relation yields

$$\begin{aligned} (D_W K)(X,Y)\xi &= -g(Y,W)\phi X + g(X,W)\phi Y + 2\,g(\phi X,W)\eta(Y)\xi \\ &+ 2\,g(\phi Y,W)\eta(X)\xi + 2\,\eta(Y)\eta(W)\phi X \\ &- 2\,\eta(W)\eta(X)\phi Y - \phi K(X,Y)W. \end{aligned} \tag{3.17}$$

In view of (3.3) and (3.16) above equation gives

$$\begin{aligned} A(W)K(X,Y)\xi &+ B(W)\{\eta(Y)X - \eta(X)Y\} \\ &= g(X,W)\phi Y - g(Y,W)\phi X + 2\,g(\phi X,W)\eta(Y)\xi \\ &+ g(\phi Y,W)\eta(X)\xi + 2\,\eta(Y)\eta(W)\phi X \\ &- 2\,\eta(X)\eta(W)\phi Y - \phi K(X,Y)W. \end{aligned} \tag{3.18}$$

Using (2.9) in equation (3.18) we get

$$\begin{aligned} \{A(W) &- B(W)\}\{\eta(X)Y - \eta(Y)X\} \\ &= g(X,W)\phi Y - g(Y,W)\phi X + 2\,g(\phi X,W)\eta(Y)\xi \\ &+ g(\phi Y,W)\eta(X)\xi + 2\,\eta(Y)\eta(W)\phi X \\ &- 2\,\eta(X)\eta(W)\phi Y - \phi K(X,Y)W. \end{aligned} \tag{3.19}$$

Hence if $X, Y$ are orthogonal to $\xi$ then the equation (3.19) becomes

$$\phi K(X,Y)W = g(X,W)\phi Y - g(Y,W)\phi X. \tag{3.20}$$

Operating $\phi$ on both sides of (3.20), we get

$$K(X,Y)W = g(X,W)Y - g(Y,W)X. \tag{3.21}$$

This leads to the following theorem:

**Theorem 3.3.** *A generalized $\phi$ - recurrent P-Sasakian manifold is locally isomorphic to the hyperbolic space $H^n(-1)$ provided that $X$ and $Y$ are orthogonal to $\xi$.*

## 4. Generalized concircular $\phi$ - recurrent P - Sasakian manifold

Analogously to the consideration of generalized recurrent manifolds in [5], we give the following definition

**Definition 4.1.** A P-Sasakian manifold is called a generalized concircular $\phi$ - recurrent if its concircular curvature tensor $C$

$$C(X,Y)Z = K(X,Y)Z - \frac{\tau}{n(n-1)}[g(Y,Z)X - g(X,Z)Y \tag{4.1}$$

satisfies the condition

$$\begin{aligned} \phi^2((D_W C)(X,Y)Z) &= A(W)C(X,Y)Z \\ &+ B(W)[g(Y,Z)X - g(X,Z)Y], \end{aligned} \tag{4.2}$$

where $A$ and $B$ are defined as (3.2) and $\tau$ is the scalar curvature.

If the 1-form $B$ in (4.2) becomes zero, then the manifold reduces to a concircular $\phi$ - recurrent P-Sasakian manifold which is studied in [10].
Let us consider a generalized concircular $\phi$ - recurrent P-Sasakian manifold. Then in consequence of (2.1) the equation (4.2) gives

$$\begin{aligned} ((D_W C)(X,Y)Z) &= \eta((D_W C)(X,Y)Z)\xi + A(W)C(X,Y)Z \\ &+ B(W)[g(Y,Z)X - g(X,Z)Y]. \end{aligned} \tag{4.3}$$

Taking inner product of above with $U$, we obtain

$$\begin{aligned} g((D_W C)(X,Y)Z), U) &= \eta((D_W C)(X,Y)Z)\eta(U) + A(W)g(C(X,Y)Z,U) \\ &+ B(W)[g(Y,Z)g(X,U) - g(X,Z)g(Y,U)]. \end{aligned} \tag{4.4}$$

Let $e_i,\ i = 1, 2, ...., n$ be an orthonormal basis of the tangent space at any point of the manifold. Then putting $Y = Z = e_i$ (4.4) and taking summation over $i,\ 1 \le i \le n$, we get

$$\begin{aligned} (D_W S)(X,U) - \frac{d\tau(W)}{n} g(X,U) &= (D_W S)(X,\xi)\eta(U) - \frac{d\tau(W)}{n}\eta(X)\eta(U) \\ &+ A(W)[S(X,U) - \frac{\tau}{n} g(X,U)] \\ &+ (n-1)B(W)g(X,U). \end{aligned} \tag{4.5}$$

Replacing $U$ by $\xi$ in (4.5) then using (2.1) and (2.10), we get

$$A(W)[(n-1)+\frac{\tau}{n}]\eta(X)-(n-1)B(W)\eta(X)=0. \tag{4.6}$$

By the virtue of $X=\xi$ the above equation gives

$$A(W)[(n-1)+\frac{\tau}{n}]-(n-1)B(W)=0. \tag{4.7}$$

Now, putting $X=U=e_i$ in (4.4) and taking summation over $i,\ 1\leq i\leq n$, we get

$$\begin{aligned}(D_WS)(Y,Z) &- \sum_{i=1}^{n} g((D_WK)(e_i,Y)Z,\xi)g(e_i,\xi)\\ &= \frac{d\tau(W)}{n}g(Y,Z)-\frac{d\tau(W)}{n(n-1)}[g(Y,Z)-\eta(Y)\eta(Z)]\\ &+ A(W)[S(Y,Z)-\frac{\tau}{n}g(Y,Z)]+(n-1)B(W)g(Y,Z).\end{aligned}$$

Replacing $Z$ by $\xi$ in above relation then using (2.1) and (4.6), we get

$$(D_WS)(Y,\xi)=\frac{d\tau(W)}{n}\eta(Y). \tag{4.8}$$

We know that

$$(D_WS)(Y,\xi)=D_WS(Y,\xi)-S(D_WY,\xi)-S(Y,D_W\xi). \tag{4.9}$$

Using (2.4) and (2.5) in the above relation, it follows that

$$(D_WS)(Y,\xi)=-(n-1)g(\phi Y,W)-S(Y,\phi W). \tag{4.10}$$

Comparing equations (4.8) and (4.10), we get

$$-(n-1)g(\phi Y,W)-S(Y,\phi W)=\frac{d\tau(W)}{n}\eta(Y). \tag{4.11}$$

Replacing $Y$ by $\phi Y$ in (4.11) and using (2.1), we get

$$S(Y,W)=2(1-n)g(Y,W)+(n-1)\eta(Y)\eta(W). \tag{4.12}$$

Thus, we can state the following:

**Theorem 4.2.** *A generalized concircular $\phi$ - recurrent P-Sasakian manifold an $\eta$ - Einstein manifold.*

Now taking inner product of (4.3) and using (2.1), we get

$$\begin{aligned}A(W)\eta(C(X,Y)Z) &+ B(W)[g(Y,Z)\eta(X)\\ &- g(X,Z)\eta(Y)]=0,\end{aligned} \tag{4.13}$$

from which it follows that

$$\begin{aligned} A(W)\eta(K(X,Y)Z) &= \{A(W)\frac{\tau}{n(n-1)} - B(W)\}[g(Y,Z)\eta(X) \\ &- g(X,Z)\eta(Y)]. \end{aligned} \tag{4.14}$$

Taking the cyclic rotation of $W, X, Y$ in (4.14), we get

$$\begin{aligned} & A(W)\eta(K(X,Y)Z) + A(X)\eta(K(Y,W,)Z) + A(Y)\eta(K(W,X)Z) \\ = & \{A(W)\frac{\tau}{n(n-1)} - B(W)\}[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)] \\ + & \{A(X)\frac{\tau}{n(n-1)} - B(X)\}[g(W,Z)\eta(Y) - g(Y,Z)\eta(W)] \\ + & \{A(W)\frac{\tau}{n(n-1)} - B(W)\}[g(X,Z)\eta(W) - g(W,Z)\eta(X)]. \end{aligned}$$

Using (2.8) in above equation, we get

$$\begin{aligned} & A(W)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)] \\ + & A(X))[g(W,Z)\eta(Y) - g(Y,Z)\eta(W)] \\ + & A(Y)[g(X,Z)\eta(W) - g(W,Z)\eta(X)] \\ = & \{A(W)\frac{\tau}{n(n-1)} - B(W)\}[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)] \\ + & \{A(X)\frac{\tau}{n(n-1)} - B(X)\}[g(W,Z)\eta(Y) - g(Y,Z)\eta(W)] \\ + & \{A(Y)\frac{\tau}{n(n-1)} - B(Y)\}[g(X,Z)\eta(W) - g(W,Z)\eta(X)]. \end{aligned} \tag{4.15}$$

Putting $Y = Z = e_i$ in (4.15) and taking summation over $i$, $1 \le i \le n$, we get

$$\begin{aligned} \{\frac{\tau}{n-1} &+ 2 - n\}[A(W)\eta(X) - A(X)\eta(W)] \\ &= (n-2)[B(W)\eta(X) - B(X)\eta(W)] \end{aligned}$$

which implies that

$$\begin{aligned} (a) \;\; & A(W)\eta(X) = A(X)\eta(W), \\ (b) \;\; & B(W)\eta(X) = B(X)\eta(W). \end{aligned} \tag{4.16}$$

Replacing $X$ by $\xi$ in above, we get

$$\begin{aligned} (a) \;\; & A(W) = \eta(\rho_1)\eta(W), \\ (b) \;\; & B(W) = \eta(\rho_2)\eta(W). \end{aligned} \tag{4.17}$$

From (4.16) and (4.17), we have the following:

**Theorem 4.3.** *In a generalized concircularly $\phi-$ recurrent P-Sasakian manifold $M^n$, $(n > 2)$ the characteristic vector fields $\rho_1, \rho_2$ associated to the 1-forms $A, B$ respectively are co-directional and the 1-forms $A, B$ are given by (4.17).*

## 5. On 3-dimensional locally generalized concircularly $\phi-$ recurrent p-Sasakian manifold

It is known that in a 3-dimensional P-Sasakian manifold the curvature tensor has the following form [6]

$$\begin{aligned} K(X,Y)Z &= \frac{(\tau+4)}{2}\{g(Y,Z)X - g(X,Z)Y\} \\ &- \frac{(\tau+6)}{2}[g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi \\ &+ \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y]. \end{aligned} \tag{5.1}$$

Differentiating (5.1) covariantly with respect to $W$, we obtain

$$\begin{aligned} (D_W K)(X,Y)Z &= \frac{d\tau(W)}{2}\{g(Y,Z)X - g(X,Z)Y\} \\ &- \frac{d(\tau)(W)}{2}[g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi \\ &+ \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y] \\ &- \frac{(\tau+6)}{2}[g(Y,Z)(D_W\eta)(X)\xi + g(Y,Z)\eta(X)(D_W\xi) \\ &- g(X,Z)(D_W\eta)(Y)\xi - g(X,Z)\eta(Y)(D_W\xi) \\ &+ (D_W\eta)(Y)\eta(Z)X + (D_W\eta)(Z)\eta(Y)X \\ &- (D_W\eta)(X)\eta(Z)Y - (D_W\eta)(Z)\eta(X)Y]. \end{aligned} \tag{5.2}$$

Taking $X, Y, Z, W$ orthogonal to $\xi$ and using (2.4) and (2.5), we get

$$\begin{aligned} (D_W K)(X,Y)Z &= \frac{d\tau(W)}{2}\{g(Y,Z)X - g(X,Z)Y\} \\ &- \frac{(\tau+6)}{2}[g(Y,Z)g(\phi X,W) \\ &- g(X,Z)g(\phi Y,W)]\xi. \end{aligned} \tag{5.3}$$

From above equation it follows that

$$\phi^2(D_W K)(X,Y)Z = \frac{d\tau(W)}{2}[g(Y,Z)\phi^2 X - g(X,Z)\phi^2 Y]. \tag{5.4}$$

Now, using (2.1) and $X, Y, Z, W$ orthogonal to $\xi$ in (5.4), we obtain

$$\phi^2(D_W K)(X,Y)Z = \frac{d\tau(W)}{2}[g(Y,Z)X - g(X,Z)Y]. \tag{5.5}$$

Taking covariant differentiation of (4.1) with respect to $W$ (for n=3), we get

$$(D_W C)(X,Y)Z = (D_W K)(X,Y)Z - \frac{d\tau(W)}{6}[g(Y,Z)X - g(X,Z)Y],$$

from which it follows that

$$\begin{aligned} \phi^2(D_W C)(X,Y)Z &= \phi^2(D_W K)(X,Y)Z \\ &- \frac{d\tau(W)}{6}\{g(Y,Z)\phi^2 X - g(X,Z)\phi^2 Y\}. \end{aligned} \tag{5.6}$$

Using (4.2), (5.5) and (2.1) in (5.6), we get

$$\begin{aligned} A(W)C(X,Y)Z &+ B(W)[g(Y,Z)X - g(X,Z)Y] \\ &= \frac{d\tau(W)}{2}\{g(Y,Z)X - g(X,Z)Y\} \\ &- \frac{d\tau(W)}{6}\{g(Y,Z)X - g(X,Z)Y \\ &+ g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\}. \end{aligned} \tag{5.7}$$

Taking $X, Y, Z, W$ orthogonal to $\xi$, we get

$$C(X,Y)Z = \{\frac{d\tau(W)}{3\,A(W)} - \frac{B(W)}{A(W)}\}[g(Y,Z)X - g(X,Z)Y], \tag{5.8}$$

from which it follows that

$$R(X,Y)Z = \{\frac{\tau}{6} + \frac{d\tau(W)}{3\,A(W)} - \frac{B(W)}{A(W)}\}[g(Y,Z)X - g(X,Z)Y]. \tag{5.9}$$

Putting $W = e_i$ in (5.9), where $e_i,\ i = 1, 2, 3$ is an orthonormal basis of the tangent space at any point of the manifold and taking summation over $i,\ 1 \leq i \leq 3$, we get

$$\begin{aligned} R(X,Y)Z &= \{\frac{\tau}{6} + \frac{d\tau(e_i)}{3\,A(e_i)} - \frac{B(e_i)}{A(e_i)}\}[g(Y,Z)X - g(X,Z)Y] \\ &= \lambda[g(Y,Z)X - g(X,Z)Y], \end{aligned} \tag{5.10}$$

where $\lambda = \{\frac{\tau}{6} + \frac{d\tau(e_i)}{3\,A(e_i)} - \frac{B(e_i)}{A(e_i)}\}$ is a scalar. Then by Schur's theorem $\lambda$ will be a constant on the manifold. Therefore $M^3$ is a space of constant curvature $\lambda$. This leads to the following theorem:

**Theorem 5.1.** *A 3-dimensional locally generalized concircularly $\phi-$ recurrent P-Sasakian manifold is of constant curvature.*

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