Novi Sad J. Math.
Vol. 44, No. 2, 2014, 137-147

HYPER BE–ALGEBRAS

Akefe Radfar[1], Akbar Rezaei[2] and Arsham Borumand Saeid[3]

Abstract. In this paper, we introduce the notion of hyper BE–algebra and investigate some properties. Also, some types of hyper filters in hyper BE–algebras are studied and the relationship between them are stated. We try to show that these notions are independent by some examples. Furthermore, it shows that under special condition hyper BE–algebras are equivalent to dual hyper K–algebras.

AMS Mathematics Subject Classification (2010): 06F35, 03G25.

Key words and phrases: (hyper)BE–algebra, hyper K–algebra, hyper (weak)filter.

1. Introduction

H. S. Kim and Y. H. Kim introduced the notion of a BE–algebra as a generalization of a dual BCK–algebra [5]. Using the notion of upper sets, they gave an equivalent condition of upper sets in BE–algebras and discussed some properties of them. A. Rezaei et al. in [7, 8] study commutative ideals in BE–algebras and give some properties. They showed that a commutative implicative BE–algebra is equivalent to the commutative self distributive BE–algebra. Also, they proved that every Hilbert algebra is a self distributive BE–algebra and commutative self distributive BE–algebra is a Hilbert algebra and showed that one can not remove the conditions of commutativity and self distributivity. In [1], S. S. Ahn et al. introduced the notions of terminal sections of a BE–algebras and gave some characterization of commutative BE–algebras in terms of lattice order relations and terminal sections. Recently, R. A. Borzooei et al. introduced the notion of pseudo BE–algebra which is a generalization of BE–algebra. They defined the basic concepts of pseudo subalgebras and pseudo filters and prove that, under some conditions, pseudo subalgebra can be a pseudo filter [3].

The hyper algebraic structure theory was introduced in 1934 [6], by F. Marty at the 8th congress of Scandinavian Mathematicians. Hyperstructures have many applications to several sectors of both pure and applied sciences. In [4], Y. B. Jun et al. applied the hyperstructures to BCK–algebras and introduced the notion of a hyper BCK–algebra which is a generalization of

[1]Dept. of Math., Payame Noor University, p. o. box, 19395-3697, Tehran, Iran,
e-mail: ateferadfar@yahoo.com
[2]Dept. of Math., Payame Noor University, p. o. box, 19395-3697, Tehran, Iran,
e-mail: rezaei@pnu.ac.ir
[3]Dept. of Math., Shahid Bahonar University of Kerman, Kerman, Iran,
e-mail: arsham@uk.ac.ir

BCK–algebra and investigated some related properties. R. A. Borzooei et al. defined the notion of a hyper K–algebra, bounded hyper K–algebra and consider the zero condition in hyper K–algebras. They show that every hyper K–algebra with the zero condition can be extended to a bounded hyper K–algebra [2, 11].

The goal of this paper is to generalize the notion of BE–algebras by considering the notion of hyperoperation, define some types of hyper filters in this structure and describe the relationship between them.

Definition 1.1. *[5] An algebra* $(X;*,1)$ *of type* $(2,0)$ *is called a BE–algebra if the following axioms hold:*

$(BE1)$ $x * x = 1$,

$(BE2)$ $x * 1 = 1$,

$(BE3)$ $1 * x = x$,

$(BE4)$ $x * (y * z) = y * (x * z)$, *for all* $x, y, z \in X$.

We introduce the relation " $\leq$ " *on* X *by* $x \leq y$ *if and only if* $x * y = 1$.

Proposition 1.2. *[5] Let* X *be a BE–algebra. Then*

(i) $x * (y * x) = 1$,

(ii) $y * ((y * x) * x) = 1$, *for all* $x, y \in X$.

Definition 1.3. *[10] An algebra* $(X;*,1)$ *of type* $(2,0)$ *is called a dual BCK–algebra if*

(BE1) $x * x = 1$ *for all* $x \in X$;

(BE2) $x * 1 = 1$ *for all* $x \in X$;

(dBCK1) $x * y = y * x = 1 \Longrightarrow x = y$;

(dBCK2) $(x * y) * ((y * z) * (x * z)) = 1$;

(dBCK3) $x * ((x * y) * y) = 1$.

Lemma 1.4. *[10] Let* $(X;*,1)$ *be a dual BCK–algebra. Then*

(i) $x * (y * z) = y * (x * z)$,

(ii) $1 * x = x$, *for all* $x, y, z \in X$.

Proposition 1.5. *[10] Any dual BCK–algebra is a BE–algebra.*

Example 1.6. *[9] Let* $X = \{1, 2, \ldots\}$ *and the operation* $*$ *be defined as follows:*

$$x * y = \begin{cases} 1 & \text{if } y \leq x \\ y & \text{otherwise} \end{cases}$$

Then $(X;*,1)$ *is a BE–algebra, but it is not a dual BCK–algebra.*

Definition 1.7. *[2] Let H be a nonempty set and $\circ : H \times \ H \to P^*(H)$ be a hyperoperation. Then $(H; \circ, 0)$ is called a hyper K–algebra, if it satisfies the following axioms:*

(HK_1) $(x \circ z) \circ (y \circ z) < x \circ y$,

(HK_2) $(x \circ y) \circ z = (x \circ z) \circ y$,

(HK_3) $x < x$,

(HK_4) $x < y$ *and* $y < x$ *imply that* $x = y$,

(HK_5) $0 < x$, *for all* $x, y, z \in H$.

For every $A, B \subseteq H$, where $x < y$ is defined by $0 \in x \circ y$, $A < B$ is defined by: there exist $a \in A$ and $b \in B$ such that $a < b$. Note that if $A, B \subseteq H$, then by $A \circ B$ we mean the subset $\bigcup_{a \in A, b \in B} a \circ b$ of H.

Theorem 1.8. *[2] Let H be a hyper K–algebra. Then*

(i) $x \in x \circ 0$,

(ii) $x < 0$ *implies* $x = 0$, *for all* $x \in H$.

2. On hyper BE–algebras

Definition 2.1. *Let H be a nonempty set and $\circ : H \times \ H \to P^*(H)$ be a hyperoperation. Then $(H; \circ, 1)$ is called a hyper BE–algebra, if it satisfies the following axioms:*

(HBE_1) $x < 1$ *and* $x < x$,

(HBE_2) $x \circ (y \circ z) = y \circ (x \circ z)$,

(HBE_3) $x \in 1 \circ x$,

(HBE_4) $1 < x$ *implies* $x = 1$, *for all* $x, y, z \in H$.

$(H; \circ, 1)$ is called a dual hyper K–algebra if satisfies (HBE_1), (HBE_2) and the following axioms:

(DHK_1) $x \circ y < (y \circ z) \circ (x \circ z)$,

(DHK_4) $x < y$ *and* $y < x$ *imply that* $x = y$, *for all* $x, y, z \in H$.

Where the relation "$<$" is defined by $x < y \Leftrightarrow 1 \in x \circ y$. For any two nonempty subsets A and B of H, we define $A < B$ if and only if there exist $a \in A$ and $b \in B$ such that $a < b$ and $A \circ B = \bigcup_{a \in A, b \in B} a \circ b$.

In the following examples show that axioms for hyper BE–algebras and dual hyper K–algebras are independent.

Example 2.2. *(i). Let $H = \{1, a, b\}$. Define the hyperoperations "$\circ_1$" and "$\circ_2$" as follows:*

$\circ_1$	1	a	b
1	$\{1\}$	$\{a, b\}$	$\{b\}$
a	$\{1\}$	$\{1, a\}$	$\{1, b\}$
b	$\{1\}$	$\{1, a, b\}$	$\{1\}$

$\circ_2$	1	a	b
1	$\{1\}$	$\{a, b\}$	$\{b\}$
a	$\{1\}$	$\{1, a, b\}$	$\{b\}$
b	$\{1, b\}$	$\{1, a, b\}$	$\{1, a, b\}$

Then $(H; \circ_1, 1)$ is a hyper BE–algebra and $(H; \circ_2, 1)$ is a dual hyper K–algebra.

(ii). Define the hyper operation "$\circ$" on $\mathbb{R}$ as follows:

$$x \circ y = \begin{cases} \{y\} & \text{if } x = 1 \\ \mathbb{R} & \text{otherwise} \end{cases}$$

Then $(\mathbb{R}; \circ, 1)$ is a hyper BE–algebra.

(iii). Let $H = \{1, a, b\}$. Define the hyperoperations "$\circ_3$" and "$\circ_4$" as follows:

$\circ_3$	1	a	b
1	$\{1\}$	$\{a\}$	$\{b\}$
a	$\{b\}$	$\{1\}$	$\{a\}$
b	$\{a, b\}$	$\{1, b\}$	$\{1, a\}$

$\circ_4$	1	a	b
1	$\{1\}$	$\{a\}$	$\{a, b\}$
a	$\{1, a\}$	$\{a, b\}$	$\{1, b\}$
b	$\{1\}$	$\{a\}$	$\{a, b\}$

Then $(H; \circ_3, 1)$ and $(H; \circ_4, 1)$ satisfy (HBE_2), (HBE_3) and (HBE_4). Since $a \not< 1$ and $a \not< a$, it follows that they do not satisfy (HBE_1).

(iv). Let $H = \{1, a, b\}$. Define the hyperoperations "$\circ_5$", "$\circ_6$" and "$\circ_7$" as follows:

$\circ_5$	1	a	b
1	$\{1\}$	$\{a\}$	$\{b\}$
a	$\{1, b\}$	$\{1\}$	$\{1, a, b\}$
b	$\{1\}$	$\{1, b\}$	$\{1, b\}$

$\circ_6$	1	a	b
1	$\{1\}$	$\{b\}$	$\{b\}$
a	$\{1\}$	$\{1\}$	$\{1\}$
b	$\{1\}$	$\{1, b\}$	$\{1, b\}$

$\circ_7$	1	a	b
1	$\{1\}$	$\{1, a\}$	$\{b\}$
a	$\{1\}$	$\{1\}$	$\{b\}$
b	$\{1\}$	$\{a\}$	$\{1, b\}$

Then $(H; \circ_5, 1)$ satisfies (HBE_1), (HBE_3) and (HBE_4). Since

$$a \circ_5 (b \circ_5 b) = \{1, a, b\} \neq \{1, b\} = b \circ_5 (a \circ_5 b),$$

we can see that $(H; \circ_5, 1)$ does not satisfy (HBE_2). Also, $(H; \circ_6, 1)$ satisfies (HBE_1), (HBE_2) and (HBE_4). Since $a \notin 1 \circ a$, $(H; \circ_6, 1)$ does not satisfy (HBE_3). Furthermore, $(H; \circ_7, 1)$ satisfies (HBE_1), (HBE_2) and (HBE_3). Since $1 < a$, $(H; \circ_7, 1)$ does not satisfy (HBE_4).

(v). Let $H = \{1, a, b, c\}$ and define $\circ_8$ as follows:

$\circ_8$	1	a	b	c
1	$\{1\}$	$\{a\}$	$\{b\}$	$\{c\}$
a	$\{1\}$	$\{1\}$	$\{a\}$	$\{b, c\}$
b	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$
c	$\{1\}$	$\{1\}$	$\{a\}$	$\{1, b, c\}$

Then $(H; \circ_8, 1)$ satisfies (HBE_1), (HBE_2) and (DHK_4). Since

$$a \circ_8 b \not< (b \circ_8 c) \circ_8 (a \circ_8 c),$$

$(H; \circ_8, 1)$ does not satisfy (DHK_1).

(vi). Let $H = \{1, a, b\}$. Define the hyperoperations " $\circ_9$ " to " $\circ_{12}$ " as follows:

$\circ_9$	1	a	b
1	$\{1\}$	$\{a\}$	$\{b\}$
a	$\{1\}$	$\{1\}$	$\{1, a\}$
b	$\{1\}$	$\{1\}$	$\{1, a\}$

$\circ_{10}$	1	a	b
1	$\{1\}$	$\{a\}$	$\{b\}$
a	$\{b\}$	$\{1, a, b\}$	$\{1, a, b\}$
b	$\{b\}$	$\{a, b\}$	$\{1, a, b\}$

$\circ_{11}$	1	a	b
1	$\{1, a\}$	$\{a\}$	$\{a, b\}$
a	$\{1, b\}$	$\{a, b\}$	$\{1, a, b\}$
b	$\{1, b\}$	$\{a\}$	$\{a, b\}$

$\circ_{12}$	1	a	b
1	$\{1\}$	$\{b\}$	$\{a, b\}$
a	$\{1\}$	$\{1, a\}$	$\{b\}$
b	$\{1\}$	$\{1\}$	$\{1, a\}$

Then $(H; \circ_9, 1)$ satisfies (HBE_1), (HBE_2) and (DHK_1). Since $a < b$ and $b < a$, $(H; \circ_9, 1)$ does not satisfy (DHK_4). Also, $(H; \circ_{10}, 1)$ and $(H; \circ_{11}, 1)$ satisfies (HBE_2), (DHK_1) and (DHK_4). But in $(H; \circ_{10}, 1)$, $a \not< 1$ and in $(H; \circ_{11}, 1)$ $a \not< a$ and so they do not satisfy (HBE_1).
Furthermore, $(H; \circ_{12}, 1)$ satisfies (HBE_1), (DHK_1) and (DHK_4). But since

$$1 \circ_{12} (a \circ_{12} b) = 1 \circ_{12} \{b\} = \{a, b\} \neq \{1, a, b\} = a \circ_{12} \{a, b\} = a \circ_{12} (1 \circ_{12} b),$$

$(H; \circ_{12}, 1)$ does not satisfy (HBE_2).

Theorem 2.3. *Let H be a hyper BE–algebra. Then*

(i) $A \circ (B \circ C) = B \circ (A \circ C)$,

(ii) $A < A$,

(iii) $1 < A$ *implies* $1 \in A$,

(iv) $x < y \circ x$,

(v) $x < y \circ z$ *implies* $y < x \circ z$,

(vi) $x < (x \circ y) \circ y$,

(*vii*) $z \in x \circ y$ *implies* $x < z \circ y$,

(*viii*) $y \in 1 \circ x$ *implies* $y < x$, *for all* $x, y, z \in H$ *and* $A, B, C \subseteq H$.

Proof. We prove just (*iii*) and (*viii*).

(*iii*) Let $1 < A$. It means that there is $a \in A$ such that $1 < a$. By using (HBE_4), $a = 1$, and so $1 \in A$.

(*viii*) Let $y \in 1 \circ x$. Then by (HBE_2), $1 \in y \circ (1 \circ x) = 1 \circ (y \circ x)$. Thus there is $a \in y \circ x$ such that $1 \in 1 \circ a$. Hence $1 < a$. By (HBE_4), $a = 1$ and $1 \in y \circ x$. Therefore, $y < x$. □

Proposition 2.4. *Let* $(X; *, 1)$ *be a (dual BCK–algebra) BE–algebra. If we define* $x \circ y = \{x * y\}$, *for all* $x, y \in X$, *then* $(X; \circ, 1)$ *is a (dual hyper K–algebra) hyper BE–algebra.*

Theorem 2.5. *Let* $(H; \diamond, 0)$ *be a hyper K–algebra. Then* $(H; \circ, 1)$ *is a dual hyper K–algebra, whenever* $1 := 0$ *and* $x \circ y := y \diamond x$, *for all* $x, y \in H$.

Proof. Let $(H; \diamond, 0)$ be a hyper K–algebra. Then

$$\begin{aligned}(x \circ y) \circ ((y \circ z) \circ (x \circ z)) &= (y \diamond x) \circ ((z \diamond y) \circ (z \diamond x)) \\ &= (y \diamond x) \circ ((z \diamond x) \diamond (z \diamond y)) \\ &= ((z \diamond x) \diamond (z \diamond y)) \diamond (y \diamond x) \\ &= ((z \diamond x) \diamond (y \diamond x)) \diamond (z \diamond y).\end{aligned}$$

By (HK_1), $0 \in ((z \diamond x) \diamond (y \diamond x)) \diamond (z \diamond y)$ and so $1 \in (x \circ y) \circ ((y \circ z) \circ (x \circ z))$. Thus H satisfies (DHK_1). Also by (HK_2),

$$x \circ (y \circ z) = x \circ (z \diamond y) = (z \diamond y) \diamond x = (z \diamond x) \diamond y = (x \circ z) \diamond y = y \circ (x \circ z).$$

Thus H satisfies (HBE_2). By (HK_3) and (HK_5), $0 \in x \diamond x$ and $0 \in 0 \diamond x$, which means that $1 \in x \circ x$ and $1 \in x \circ 1$. Hence, H satisfies (HBE_1). By (HK_4) and definition of "$\circ$", we can easily conclude that H satisfies (DHK_4), and so $(H; \circ, 1)$ is a dual hyper K–algebra. □

Proposition 2.6. *Every dual hyper K–algebra is a hyper BE–algebra.*

Proof. Let H be a dual hyper K–algebra. By definition of dual hyper K–algebra and hyper BE–algebra, it is sufficient to prove that H satisfies (HBE_3) and (HBE_4).

By Theorem 2.5, if we define $x \diamond y := y \circ x$ and $0 := 1$, then $(H; \diamond, 0)$ is a hyper K–algebra. By Theorem 1.8 (*i*), $x \in x \diamond 0$ and so $x \in 1 \circ x$, which means that $(H; \circ, 1)$ satisfies (HBE_3). Also, by Theorem 1.8(*ii*), if $0 \in x \diamond 0$, then $x = 0$. It means that $1 \in 1 \circ x$ implies $x = 1$. Thus $1 \leq x$ implies $x = 1$. Therefore, H satisfies (HBE_4) and H is a hyper BE–algebra. □

Note. In a similar way, we can define a dual hyper BCK–algebra. Since every hyper BCK–algebra is a hyper K–algebra, consequently, every dual hyper BCK–algebra is a dual hyper K–algebra. By Proposition 2.6, every dual hyper BCK–algebra is a hyper BE–algebra. We can see that the converse of Proposition 2.6 is not correct in general. In Example 2.2(*iv*), $(H; \circ_8, 1)$ is a hyper BE–algebra, but it is not a dual hyper K–algebra.

3. Some types of hyper BE–algebras

Definition 3.1. *A hyper BE–algebra is said*

(*i*) *row hyper BE–algebra (briefly, R–hyper BE–algebra), if* $1 \circ x = \{x\}$, *for all* $x \in H$,

(*ii*) *column hyper BE–algebra (briefly, C–hyper BE–algebra), if* $x \circ 1 = \{1\}$, *for all* $x \in H$,

(*iii*) *diagonal hyper BE–algebra (briefly, D–hyper BE–algebra), if* $x \circ x = \{1\}$, *for all* $x \in H$,

(*iv*) *thin hyper BE–algebra (briefly, T–hyper BE–algebra), if it is a RC–hyper BE–algebra,*

(*v*) *very thin hyper BE–algebra (briefly, V–hyper BE–algebra), if it is a RCD–hyper BE–algebra,*

Example 3.2. (*i*). *Every BE–algebra is a RCD–hyper BE–algebra. In Example 2.2* (*i*), $(H; \circ_1, 1)$ *is a C–hyper BE–algebra.*

(*ii*). *Let* $H = \{1, a, b\}$. *Define hyper operations* $\circ_{13}$ *as follows:*

$\circ_{13}$	1	a	b
1	$\{1\}$	$\{a\}$	$\{b\}$
a	$\{1, b\}$	$\{1, a, b\}$	$\{1, a\}$
b	$\{1, a, b\}$	$\{a\}$	$\{1, a, b\}$

Then H is a R–hyper BE–algebra.

(*iii*). *Let* $H = \{1, a\}$. *Define the hyper operations* $\circ_{14}$ *to* $\circ_{16}$ *as follows:*

$\circ_{14}$	1	a
1	$\{1\}$	$\{a\}$
a	$\{1\}$	$\{1, a\}$

$\circ_{15}$	1	a
1	$\{1\}$	$\{a\}$
a	$\{1, a\}$	$\{1\}$

$\circ_{16}$	1	a
1	$\{1\}$	$\{a\}$
a	$\{1, a\}$	$\{1, a\}$

Then $(H; \circ_{14}, 1)$ *is a T–hyper BE–algebra,* $(H; \circ_{15}, 1)$ *is a RD–hyper BE-algebra and* $(H; \circ_{16}, 1)$ *is a R–hyper BE–algebra.*

(*iv*). *Let* $H = \{1, a, b\}$. *Define the hyperoperations* " $\circ_{17}$ " *to* " $\circ_{20}$ " *as follows:*

$\circ_{17}$	1	a	b
1	$\{1\}$	$\{a, b\}$	$\{b\}$
a	$\{1, b\}$	$\{1\}$	$\{1\}$
b	$\{1, b\}$	$\{1\}$	$\{1\}$

$\circ_{18}$	1	a	b
1	$\{1\}$	$\{a\}$	$\{b\}$
a	$\{1\}$	$\{1, a, b\}$	$\{b\}$
b	$\{1\}$	$\{a, b\}$	$\{1, b\}$

$\circ_{19}$	1	a	b
1	$\{1\}$	$\{a\}$	$\{b\}$
a	$\{1\}$	$\{1\}$	$\{b\}$
b	$\{1\}$	$\{1,a\}$	$\{1\}$

$\circ_{20}$	1	a	b
1	$\{1\}$	$\{a,b\}$	$\{b\}$
a	$\{1\}$	$\{1\}$	$\{1\}$
b	$\{1\}$	$\{1,b\}$	$\{1\}$

Then $(H;\circ_{17},1)$ is a D–hyper BE–algebra, $(H;\circ_{18},1)$ is a T–hyper BE–algebra, $(H;\circ_{19},1)$ is a V–hyper BE–algebra and $(H;\circ_{20},1)$ is a CD–hyper BE–algebra.

Theorem 3.3. *Let H be a D–hyper BE–algebra. Then*

(*i*) $a\in 1\circ x$ *implies* $x\circ a=\{1\}$,

(*ii*) $y\circ(x\circ y)=x\circ 1$,

(*iii*) $1\circ(x\circ 1)=x\circ 1$, *for all* $a,x,y\in H$.

Proof. (i). By (HBE_2) and Definition 3.1, $\{1\}=1\circ 1=1\circ(x\circ x)=x\circ(1\circ x)$. It follows that, for all $a\in 1\circ x$, $x\circ a=\{1\}$.

(ii). $y\circ(x\circ y)=x\circ(y\circ y)=x\circ 1$.

(iii). $1\circ(x\circ 1)=x\circ(1\circ 1)=x\circ 1$. □

Theorem 3.4. *Let H be a CD–hyper BE–algebra. Then*

(*i*) $x\circ(y\circ x)=\{1\}$,

(*ii*) $z\in x\circ y$ *implies* $y\circ z=\{1\}$, *for all* $x,y,z\in H$.

Proof. (i). By (HBE_2) and Definition 3.1, $x\circ(y\circ x)=y\circ(x\circ x)=y\circ\{1\}=\{1\}$.

(ii). Let $z\in x\circ y$. Then by (i), $y\circ z\subseteq y\circ(x\circ y)=\{1\}$. Therefore, $y\circ z=\{1\}$. □

4. Hyper filters in hyper BE–algebras

Definition 4.1. *Let F be a nonempty subset of hyper BE–algebra H and $1\in F$. Then F is called*

(*i*) *a weak hyper filter of H if $x\circ y\subseteq F$ and $x\in F$ imply $y\in F$, for all $x,y\in H$,*

(*ii*) *a hyper filter of H if $x\circ y\approx F$ and $x\in F$ imply $y\in F$, for all $x,y\in H$.*

Example 4.2. *In Example 2.2 (i), $(H;\circ_1,1)$ is a hyper BE–algebra and $F_1=\{1,a\}$ is a weak hyper filter of H. Also, $(H;\circ_2,1)$ is a hyper BE–algebra and $F_2=\{1,a\}$ is a hyper filter of H.*

Theorem 4.3. *Every hyper filter is a weak hyper filter (i.e., hyper filters $\subseteq$ weak hyper filters).*

Proof. Let F be a subset of a hyper BE–algebra H and $x\circ y\subseteq F$, for some $x\in F$, $y\in H$. Since $x\circ y\subseteq F$ implies $x\circ y\approx F$. Now, since F is a hyper filter, we have $x\in F$. Therefore F is a weak hyper filter. □

Example 4.4. *In Example 4.2, F_1 is not a hyper filter, because $a \circ_1 b \approx F_1$ and $a \in F_1$, but $b \notin F_1$.*

Note. We can see that the notions of weak hyper filter and hyper filter are different in a hyper BE–algebra. In Example 4.2, $(H; \circ_1, 1)$ is a C–hyper BE–algebra, F_1 is a weak hyper filter and is not a hyper filter of H.

In Definition 4.1, the only case that we did not mention was $x \circ y < F$. In the next theorem we prove that this case is trivial.

Theorem 4.5. *Let F be a subset of a hyper BE–algebra H and $1 \in F$. If $x \circ y < F$ and $x \in F$ implies $y \in F$, for all $x, y \in H$, then $F = H$.*

Proof. Let x be an arbitrary element of H. By (HBE_1), $x < 1$ and by (HBE_3), $x \in 1 \circ x$. Thus $1 \circ x < 1$. Since $1 \in F$, consequently, $1 \circ x < F$. By hypothesis, $x \in F$. Therefore, $F = H$. □

Definition 4.6. *A subset S of hyper BE–algebra H is said to be a subalgebra, if $x \circ y \subseteq S$, for all $x, y \in S$.*

Example 4.7. *In Example 2.2(i), $\{1, b\}$ is a subalgebra of $(H; \circ_1, 1)$.*

Theorem 4.8. *Let H be a hyper BE–algebra and S be a subalgebra of H. Then*

(*i*) *S is a weak hyper filter of H if and only if for all $x \in S$ and $y \in X \setminus S$, $x \circ y \nsubseteq S$,*

(*ii*) *S is a hyper filter of H if and only if for all $x \in S$ and $y \in X/S$, $x \circ y \not\approx S$.*

Proof. (*i*). Let S be a subalgebra, weak hyper filter of H, $x \in S$ and $y \in X \setminus S$. Assume the opposite, i.e. let $x \circ y \subseteq S$. Since S is a weak filter and $x \in S$, we have $y \in S$, which is a contradiction.

Conversely, let for all $x \in S$ and $y \in X \setminus S$, $x \circ y \nsubseteq S$. Let $x \circ y \subseteq S$ and $x \in S$. If $y \notin S$, then by assumption, $x \circ y \nsubseteq S$, which is a contradiction.

(*ii*). The proof is similar to (*i*). □

Theorem 4.9. *Let H be a CD–hyper BE–algebra. Then every (weak) hyper filter of H is a subalgebra of H.*

Proof. Let F be a hyper filter of H, $x, y \in F$ and $a \in x \circ y$. Then by Theorem 3.4 (*ii*), $y \circ a = \{1\}$ and so $y \circ a \approx F$. Since F is a hyper filter and $y \in F$, we can see that $a \in F$. Thus $x \circ y \subseteq F$ and F is a subalgebra of H. By a similar way, every weak hyper filter is a subalgebra of H. □

In the next example we show that Theorem 4.9, is not correct about hyper BE–algebras in general.

Example 4.10. (*i*). *In Example 3.2, $(H; \circ_{18}, 1)$ is a T–hyper BE–algebra and $\{1, a\}$ is a (weak) hyper filter. Since $a \circ a \nsubseteq \{1, a\}$, $\{1, a\}$ is not a subalgebra.*

(*ii*). *Let $H = \{1, a, b\}$. Define a hyperoperation "$\circ_{21}$" on H as follows:*

$\circ_{21}$	1	a	b
1	$\{1\}$	$\{a\}$	$\{b\}$
a	$\{1,a,b\}$	$\{1\}$	$\{a,b\}$
b	$\{1,a,b\}$	$\{1,a,b\}$	$\{1\}$

Then H is a D–hyper BE–algebra and $\{1,a\}$ is a weak hyper filter. Since

$$a \circ 1 = \{1,a,b\} \not\subseteq \{1,a\},$$

$\{1,a\}$ is not a subalgebra of H.

Note. We can see that every subalgebra of a hyper BE–algebra H is not a (weak) hyper filter in general. In Example 2.2(i), $\{1,b\}$ is a subalgebra of a C–hyper BE–algebra $(H;\circ_1,1)$, but it is not a hyper filter of H.

Theorem 4.11. *Let F be a subset of a hyper BE–algebra H and $y \in H$. If $x \leq y$ and $x \in F$, then $y \in F$.*

Proof. Let F be a hyper filter of H, $x \in F$ and $x < y$, for some $y \in H$. Then $1 \in x \circ y$. Since $1 \in F$, we have $x \circ y \approx F$. Therefore, $y \in F$. □

5. Conclusion

In the present paper, we have introduced the concept of hyper BE–algebras and investigated some of their useful properties. This work focused on some types of hyper BE–algebras. Also, we discuss on hyper filters in this structure and present some fundamental properties.

In our future work, we will get more results in hyper BE–algebras and application.

6. Acknowledgments

The authors are very indebted to the referees for valuable suggestions that improved the readability of the paper.

References

[1] Ahn, S. S., Kim, Y. H., Ko, J. M., Filters in commutative BE–algebras. Commun. Korean Math. Soc. 27 (2012) 233-242.

[2] Borzooei, R. A., Hasankhani, A., Zahedi, M. M., Jun, Y. B., On hyper K–algebras. Sci. Math. Jpn. 52 (2000), 113-121.

[3] Borzooei, R. A., Borumand Saeid, A., Rezaei, A., Radfar, A., and Ameri, R., On pseudo BE–algebras. Discussi. Math. General Algebra Applic. 33 (2013), 95-108.

[4] Jun, Y. B., Zahedi, M. M., Lin, X. L., Borzooei, R. A., On hyper BCK–algebras. Italian journal of pure and applied Mathematics, 8 (2000), 127-136.

[5] Kim, H. S., Kim, Y. H., On BE–algebras. Sci. Math. Jpn. 66 (2007), 113-116.

[6] Marty, F., Sur une generalization de la notion de group, 8th congress Math. Scandinaves, Stockholm, (1934), 45-49.

[7] Rezaei, A., Borumand Saeid, A., Commutative ideals in BE–algebras. Kyungpook Math. J. 52 (2012), 483-494.

[8] Rezaei, A., Borumand Saeid, A., Borzooei, R. A., Relation between Hilbert algebras and BE–algebras. Appl. Appl. Math. 8 (2013), 573-584.

[9] Rezaei, A., Borumand Saeid, A., On fuzzy subalgebras of BE–algebras. Afr. Math. 22 (2011), 115-127.

[10] Walendziak, A., On commutative BE–algebras. Sci. Math. Jpn. 69 (2008), 585-588.

[11] Zahedi, M. M., Borzooei, R. A., Jun, Y. B., Hasankhani, A., Some results on hyper K–algebras. Sci. Math. Jpn. 3 (2000), 53-59.

Received by the editors February 28, 2014