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SOFT BCI-IMPLICATIVE IDEALS OF SOFT BCI-ALGEBRAS

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Abstract. The notion of soft BCI-implicative ideals and BCI-implicative idealistic soft BCI-algebras is introduced and their basic properties are discussed. Relations between soft ideals and soft BCI-implicative ideals of soft BCI-algebras are provided. Also idealistic soft BCI-algebras and BCI-implicative idealistic soft BCI-algebras are being related. The intersection, union, "AND" operation and "OR" operation of soft BCI-implicative ideals and BCI-implicative idealistic soft BCI-algebras are established. The characterizations of (fuzzy) BCI-implicative ideals in BCI-algebras are given by using the concept of soft sets. Relations between fuzzy BCI-implicative ideals and BCI-implicative idealistic soft BCI-algebras are discussed.

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1. Introduction

The real world is inherently uncertain, imprecise and vague. Because of various uncertainties, classical methods are not successful for solving complicated problems in economics, engineering and environment. The theories such as the probability theory, the (intuitionistic) fuzzy sets theory, the vague set theory, the theory of interval mathematics and the rough set theory, which are used for handling uncertainties have their own difficulties. One of the reasons for these difficulties is due to the inadequacy of the parametrization tool of the theory, which was pointed out by Molodtsov [15]. To overcome these difficulties, Molodtsov introduced the concept of soft sets as a new mathematical tool for dealing with uncertainties. Soft set is a parameterized general mathematical tool which deals with a collection of approximate description of objects. In the soft set theory, the initial description of the object has an approximate nature and there is no need to introduce the notion of exact solution. The absence of any restrictions on the approximate description in soft set theory makes this theory very convenient and easily applicable in practice. Applications of soft set theory in different disciplines and real life problems are now catching momentum some of which are being discussed here.

Majumdar and Samanta [12, 13] gave the idea of soft mappings and discussed the images and inverse images of crisp sets and soft sets under soft

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mappings. An application of soft mappings in medical diagnosis was also discussed. They also studied similarity measures of fuzzy soft sets. Xu et al. [20] introduced the notion of a vague set as an extension to the soft set and the concept of the intersection of two soft sets given in [11] was redefined. Park et al. [17] studied the equivalence soft set relations and obtained soft analogues of many results concerning ordinary equivalence relations and partitions. Zou and Xiao [21] presented data analysis approaches of soft sets under incomplete information, in view of the particularity of the value domains of mapping functions in soft sets. Atagün and Sezgin [1] introduced soft sub-rings and soft ideals of a ring. Soft subfields of a field and soft submodule of a left R-module were also introduced. Moreover they also investigated properties related to soft substructures of rings, fields and modules. Neong [16] made an attempt to solve a decision problem using imprecise soft sets by considering a hypothetical case study. Sezgin and Atagün [19] introduced the concepts of normalistic soft group and normalistic soft group homomorphism and discussed some structures that are preserved under normalistic soft group homomorphisms. We refer the readers to [2, 18] for further information regarding development of soft set theory.

Jun [4] applied the concept of soft sets by Molodtsov to the theory of BCK/BCI-algebras. He introduced the notion of soft BCK/BCI-algebras and soft subalgebras. Jun et al. [6] introduced the notion of soft p-ideals and p-idealistic soft BCI-algebras and provided the relations between fuzzy p-ideals and p-idealistic soft BCI-algebras. In [5] Jun et al. further introduced the notions of fuzzy soft BCK/BCI-algebras, (closed) fuzzy soft ideals and fuzzy soft p-ideals and discussed the related properties. In this paper, we introduce the notion of soft BCI-implicative ideals and BCI-implicative idealistic soft BCI-algebras. Using soft sets, we give characterizations of (fuzzy) BCI-implicative ideals in BCI-algebras. We provide relations between fuzzy BCI-implicative ideals and BCI-implicative idealistic soft BCI-algebras.

2. Definitions

BCK/BCI-algebras are important classes of logical algebras introduced by Y. Imai and K. Iséki [3] and were extensively investigated by several researchers.

An algebra $(X, *, 0)$ of type $(2, 0)$ is called a BCI-algebra if it satisfies the following conditions:

(I) $((x * y) * (x * z)) * (z * y) = 0$

(II) $(x * (x * y)) * y = 0$

(III) $x * x = 0$

(IV) $x * y = 0$ and $y * x = 0$ imply $x = y$

for all $x, y, z \in X$. In a BCI-algebra X, we can define a partial ordering " $\leq$ " by putting $x \leq y$ if and only if $x * y = 0$.

If a BCI-algebra X satisfies the identity:

(V) $0 * x = 0$,

for all $x \in X$, then X is called a BCK-algebra.

In any BCI-algebra the following hold:

(VI) $(x * y) * z = (x * z) * y$

(VII) $x * 0 = x$

(VIII) $x \leq y$ implies $x * z \leq y * z$ and $z * y \leq z * x$

(IX) $0 * (x * y) = (0 * x) * (0 * y)$

(X) $x * (x * (x * y)) = (x * y)$

(XI) $(x * z) * (y * z) \leq x * y$

for all $x, y, z \in X$.

A non-empty subset S of a BCI-algebras X is called a subalgebra of X if $x * y \in S$ for all $x, y \in S$. A non-empty subset $\mathcal{I}$ of a BCI-algebra X is called an ideal of X if for any $x \in X$

($\mathcal{I}1$) $0 \in \mathcal{I}$

($\mathcal{I}1$) $x * y \in \mathcal{I}$ and $y \in \mathcal{I}$ implies $x \in \mathcal{I}$

Any ideal $\mathcal{I}$ of a BCI-algebra X satisfies the following implication:

$$x \leq y \text{ and } y \in \mathcal{I} \Rightarrow x \in \mathcal{I},\ \forall\, x \in X.$$

A non-empty subset $\mathcal{I}$ of a BCI-algebra X is called an BCI-implicative ideal (see Liu et al. [10]) of X if it satisfies ($\mathcal{I}1$) and

($\mathcal{I}3$) $(((x*y)*y)*(0*y))*z \in \mathcal{I}$ and $z \in \mathcal{I} \Rightarrow x*((y*(y*x))*(0*(0*(x*y)))) \in \mathcal{I}$ for all $x, y \in X$.

We know that every BCI-implicative ideal of a BCI-algebra X is also an ideal of X.

We refer the readers to [9, 14] for further study about ideals in BCK/BCI-algebras.

In [15] the soft set is defined in the following way: Let U be an initial universe set and E be a set of parameters. Let $\mathfrak{P}(U)$ denotes the power set of U and $A \subset E$.

Definition 2.1. (Molodtsov [15]) A pair $(\mathcal{F},\ A)$ is called a soft set over U, where $\mathcal{F}$ is a mapping given by

$$\mathcal{F} : A \to \mathfrak{P}(U)$$

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $a \in A$, $\mathcal{F}(a)$ may be considered as the set of a-approximate elements of the soft set $(\mathcal{F},\ A)$.

Definition 2.2. (Maji etal. [11]) Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ be two soft sets over a common universe U. The intersection of $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ is defined to be the soft set $(\mathcal{H},\ C)$ satisfying the following conditions:

(i) $C = A \cap B$

(ii) $\mathcal{H}(x) = \mathcal{F}(x)$ or $\mathcal{G}(x)$ for all $x \in C$, (as both are same sets)

In this case, we write $(\mathcal{F},\ A) \tilde{\cap} (\mathcal{G},\ B) = (\mathcal{H},\ C)$.

Definition 2.3. (Maji et al. [11]) Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ be two soft sets over a common universe U. The union of $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ is defined to be the soft set $(\mathcal{H},\ C)$ satisfying the following conditions:

(i) $C = A \cup B$

(ii) for all $x \in C$,

$$\mathcal{H}(x) = \begin{cases} \mathcal{F}(x) & if\ x \in A \setminus B \\ \mathcal{G}(x) & if\ x \in B \setminus A \\ \mathcal{F}(x) \cup \mathcal{G}(x) & if\ x \in A \cap B \end{cases}$$

In this case, we write $(\mathcal{F},\ A) \tilde{\cup} (\mathcal{G},\ B) = (\mathcal{H},\ C)$.

Definition 2.4. (Maji et al. [11]) Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ be two soft sets over a common universe U. Then "$(\mathcal{F},\ A)\ AND\ (\mathcal{G},\ B)$" denoted by $(\mathcal{F},\ A) \tilde{\wedge} (\mathcal{G},\ B)$ is defined as $(\mathcal{F},\ A) \tilde{\wedge} (\mathcal{G},\ B) = (\mathcal{H},\ A \times B)$, where $\mathcal{H}(x,y) = \mathcal{F}(x)\ \cap\ \mathcal{G}(y)$ for all $(x,y) \in A \times B$.

Definition 2.5. (Maji et al. [11]) Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ be two soft sets over a common universe U. Then "$(\mathcal{F},\ A)\ OR\ (\mathcal{G},\ B)$" denoted by $(\mathcal{F},\ A) \tilde{\vee} (\mathcal{G},\ B)$ is defined as $(\mathcal{F},\ A) \tilde{\vee} (\mathcal{G},\ B) = (\mathcal{H},\ A \times B)$, where $\mathcal{H}(x,y) = \mathcal{F}(x)\ \cup\ \mathcal{G}(y)$ for all $(x,y) \in A \times B$.

Definition 2.6. (Maji et al. [11]) For two soft sets $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ over a common universe U, we say that $(\mathcal{F},\ A)$ is a soft subset of $(\mathcal{G},\ B)$, denoted by $(\mathcal{F},\ A) \tilde{\subset} (\mathcal{G},\ B)$, if it satisfies:

(i) $A \subset B$

(ii) For every $a \in A$, $\mathcal{F}(a)$ and $\mathcal{G}(a)$ are identical approximations.

3. Main results

In what follows let X and A be a BCI-algebra and a nonempty set, respectively and R will refer to an arbitrary binary relation between an element of A and an element of X, that is, R is a subset of $A \times X$ without otherwise specified. A set valued function $\mathcal{F} : A \to \mathfrak{P}(X)$ can be defined as $\mathcal{F}(x) = \{y \in X \mid xRy\}$ for all $x \in A$. The pair $(\mathcal{F},\ A)$ is then a soft set over X.

Definition 3.1. (Jun and Park [7]) Let S be a subalgebra of X. A subset $\mathcal{I}$ of X is called an ideal of X related to S (briefly, S-ideal of X), denoted by $\mathcal{I} \lhd S$, if it satisfies:

(i) $0 \in \mathcal{I}$

(ii) $x * y \in \mathcal{I}$ and $y \in \mathcal{I} \Rightarrow x \in \mathcal{I}$ for all $x \in S$.

Definition 3.2. Let S be a subalgebra of X. A subset $\mathcal{I}$ of X is called a BCI-implicative ideal of X related to S (briefly, $S-(BCI-I)-$ideal of X), denoted by $\mathcal{I} \lhd_{bci-i} S$, if it satisfies:

(i) $0 \in \mathcal{I}$

(ii) $(((x*y)*y)*(0*y))*z \in \mathcal{I}$ and $z \in \mathcal{I} \Rightarrow x*((y*(y*x))*(0*(0*(x*y)))) \in \mathcal{I}$, for all $x, y \in S$.

Example 3.3. Let $X = \{0, a, b, c\}$ be the BCI-algebra with the following Cayley table:

$*$	0	a	b	c
0	0	0	c	b
a	a	0	c	b
b	b	b	0	c
c	c	c	b	0

Then $S = \{0, a\}$ is a subalgebra of X and $\mathcal{I} = \{0, a, b\}$ is an $S-(BCI-I)-$ideal of X.

Note that every $S-(BCI-I)-$ideal of X is an S-ideal of X.

Definition 3.4. (Jun [4]) Let $(\mathcal{F}, A)$ be a soft set over X. Then $(\mathcal{F}, A)$ is called a soft BCI-algebra over X if $\mathcal{F}(x)$ is a subalgebra of X for all $x \in A$.

Definition 3.5. (Jun and Park [7]) Let $(\mathcal{F}, A)$ be a soft BCI-algebra over X. A soft set $(\mathcal{G}, \mathcal{I})$ over X is called a soft ideal of $(\mathcal{F}, A)$, denoted $(\mathcal{G}, \mathcal{I}) \mathrel{\tilde{\lhd}} (\mathcal{F}, A)$, if it satisfies:

(i) $\mathcal{I} \subset A$

(ii) $\mathcal{G}(x) \lhd \mathcal{F}(x)$ for all $x \in \mathcal{I}$

Definition 3.6. Let $(\mathcal{F}, A)$ be a soft BCI-algebra over X. A soft set $(\mathcal{G}, \mathcal{I})$ over X is called a soft BCI-implicative ideal of $(\mathcal{F}, A)$, denoted by $(\mathcal{G}, \mathcal{I}) \mathrel{\tilde{\lhd}_{bci-i}} (\mathcal{F}, A)$, if it satisfies:

(i) $\mathcal{I} \subset A$

(ii) $\mathcal{G}(x) \lhd_{bci-i} \mathcal{F}(x)$ for all $x \in \mathcal{I}$.

Let us illustrate this definition using the following example.

Example 3.7. Consider the BCI-algebra $X = \{0, a, b, c\}$ which is given in Example 3.3. Let $(\mathcal{F}, A)$ be a soft set over X, where $A = X$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ is a set-valued function defined by:

$$\mathcal{F}(x) = \{0\} \cup \{y \in X \mid y * (y * x) \in \{0, a\}\}$$

for all $x \in A$. Then $\mathcal{F}(0) = \mathcal{F}(a) = X$, $\mathcal{F}(b) = \mathcal{F}(c) = \{0\}$, which are subalgebras of X. Hence $(\mathcal{F}, A)$ is a soft BCI-algebra over X. Let $\mathcal{I} = \{0, a, b\} \subset A$ and $\mathcal{G} : \mathcal{I} \to \mathfrak{P}(X)$ be a set-valued function defined by:

$$\mathcal{G}(x) = \begin{cases} Z(\{0, a\}) & if\ x = b \\ \{0\} & if\ x \in \{0, a\} \end{cases}$$

where $Z(\{0, a\}) = \{x \in X \mid 0 * (0 * x) \in \{0, a\}\}$. Then $\mathcal{G}(0) = \{0\} \lhd_{bci-i} X = \mathcal{F}(0)$, $\mathcal{G}(a) = \{0\} \lhd_{bci-i} X = \mathcal{F}(a)$, $\mathcal{G}(b) = \{0, a\} \lhd_{bci-i} \{0\} = \mathcal{F}(b)$. Hence $(\mathcal{G}, \mathcal{I})$ is a soft BCI-implicative ideal of $(\mathcal{F}, A)$.

Note that every soft BCI-implicative ideal is a soft ideal but the converse is not true as seen in the following example.

Example 3.8. Let $X = \{0, a, b, c, d\}$ be the BCK-algebra and hence a BCI-algebra, with the following Cayley table:

$*$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	0	0	0
b	b	b	0	0	0
c	c	c	c	0	0
d	d	d	d	c	0

Let $(\mathcal{F}, A)$ be a soft set over X, where $A = X$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ is a set-valued function defined by:

$$\mathcal{F}(x) = \{y \in X \mid y * (y * x) \in \{0, a\}\}$$

for all $x \in A$. Then $\mathcal{F}(0) = \mathcal{F}(a) = X$, $\mathcal{F}(b) = \{0, a, c, d\}$ and $\mathcal{F}(c) = \mathcal{F}(d) = \{0, a\}$, which are subalgebras of X. Hence $(\mathcal{F}, A)$ is a soft BCI-algebra over X.

Let $(\mathcal{G}, \mathcal{I})$ be a soft set over X, where $\mathcal{I} = \{a, b\} \subset A$ and $\mathcal{G} : \mathcal{I} \to \mathfrak{P}(X)$ be a set-valued function defined by:

$$\mathcal{G}(x) = \{y \in X \mid y * x = 0\}$$

for all $x \in \mathcal{I}$. Then $\mathcal{G}(a) = \{0, a\} \lhd X = \mathcal{F}(a)$, $\mathcal{G}(b) = \{0, a, b\} \lhd \{0, a, c, d\} = \mathcal{F}(b)$. Hence $(\mathcal{G}, \mathcal{I})$ is a soft ideal of $(\mathcal{F}, A)$ but it is not a soft BCI-implicative ideal of $(\mathcal{F}, A)$ because $\mathcal{G}(a)$ is not a BCI-implicative ideal of X related to $\mathcal{F}(a)$ since $(((b * c) * c) * (0 * c)) * a = 0 \in \mathcal{G}(a)$ and $a \in \mathcal{G}(a)$ but $b * ((c * (c * b)) * (0 * (0 * (b * c)))) = b * 0 = b \notin \mathcal{G}(a)$.

Theorem 3.9. *Let* $(\mathcal{F},\ A)$ *be a soft BCI-algebra over* X. *For any soft sets* $(\mathcal{G}_1,\ \mathcal{I}_1)$ *and* $(\mathcal{G}_2,\ \mathcal{I}_2)$ *over* X *where* $\mathcal{I}_1 \cap \mathcal{I}_2 \neq \emptyset$, *we have*

$$(\mathcal{G}_1,\ \mathcal{I}_1)\ \tilde{\lhd}_{bci-i}\ (\mathcal{F},\ A),\ (\mathcal{G}_2,\ \mathcal{I}_2)\ \tilde{\lhd}_{bci-i}\ (\mathcal{F},\ A) \\ \Rightarrow (\mathcal{G}_1,\ \mathcal{I}_1)\ \tilde{\cap}\ (\mathcal{G}_2,\ \mathcal{I}_2)\ \tilde{\lhd}_{bci-i}\ (\mathcal{F},\ A)$$

Proof. Using Definition 2.2, we can write

$$(\mathcal{G}_1,\ \mathcal{I}_1)\ \tilde{\cap}\ (\mathcal{G}_2,\ \mathcal{I}_2) = (\mathcal{G},\ \mathcal{I})$$

where $\mathcal{I} = \mathcal{I}_1 \cap \mathcal{I}_2$ and $\mathcal{G}(e) = \mathcal{G}_1(e)$ or $\mathcal{G}_2(e)$ for all $e \in \mathcal{I}$. Obviously, $\mathcal{I} \subset A$ and $\mathcal{G} : \mathcal{I} \to \mathfrak{P}(X)$ is a mapping. Hence $(\mathcal{G},\ \mathcal{I})$ is a soft set over X. Since $(\mathcal{G}_1,\ \mathcal{I}_1)\ \tilde{\lhd}_{bci-i}\ (\mathcal{F},\ A)$ and $(\mathcal{G}_2,\ \mathcal{I}_2)\ \tilde{\lhd}_{bci-i}\ (\mathcal{F},\ A)$, it follows that $\mathcal{G}(e) = \mathcal{G}_1(e) \lhd_{bci-i} \mathcal{F}(e)$ or $\mathcal{G}(e) = \mathcal{G}_2(e) \lhd_{bci-i} \mathcal{F}(e)$ for all $e \in \mathcal{I}$. Hence

$$(\mathcal{G}_1,\ \mathcal{I}_1)\ \tilde{\cap}\ (\mathcal{G}_2,\ \mathcal{I}_2) = (\mathcal{G},\ \mathcal{I})\ \tilde{\lhd}_{bci-i}\ (\mathcal{F},\ A)$$

This completes the proof. □

Corollary 3.10. *Let* $(\mathcal{F},\ A)$ *be a soft BCI-algebra over* X. *For any soft sets* $(\mathcal{G},\ \mathcal{I})$ *and* $(\mathcal{H},\ \mathcal{I})$ *over* X, *we have*

$$(\mathcal{G},\ \mathcal{I})\ \tilde{\lhd}_{bci-i}\ (\mathcal{F},\ A),\ (\mathcal{H},\ \mathcal{I})\ \tilde{\lhd}_{bci-i}\ (\mathcal{F},\ A) \Rightarrow (\mathcal{G},\ \mathcal{I})\ \tilde{\cap}\ (\mathcal{H},\ \mathcal{I})\ \tilde{\lhd}_{bci-i}\ (\mathcal{F},\ A)$$

Proof. Straightforward. □

Theorem 3.11. *Let* $(\mathcal{F},\ A)$ *be a soft BCI-algebra over* X. *For any soft sets* $(\mathcal{G},\ \mathcal{I})$ *and* $(\mathcal{H},\ \mathcal{J})$ *over* X *in which* $\mathcal{I}$ *and* $\mathcal{J}$ *are disjoint, we have*

$$(\mathcal{G},\ \mathcal{I})\ \tilde{\lhd}_{bci-i}\ (\mathcal{F},\ A),\ (\mathcal{H},\ \mathcal{J})\ \tilde{\lhd}_{bci-i}\ (\mathcal{F},\ A) \Rightarrow (\mathcal{G},\ \mathcal{I})\ \tilde{\cup}\ (\mathcal{H},\ \mathcal{J})\ \tilde{\lhd}_{bci-i}\ (\mathcal{F},\ A)$$

Proof. Assume that $(\mathcal{G},\ \mathcal{I})\ \tilde{\lhd}_{bci-i}\ (\mathcal{F},\ A)$ and $(\mathcal{H},\ \mathcal{J})\ \tilde{\lhd}_{bci-i}\ (\mathcal{F},\ A)$. By means of Definition 2.3, we can write $(\mathcal{G},\ \mathcal{I})\ \tilde{\cup}\ (\mathcal{H},\ \mathcal{J}) = (\mathcal{R},\ \mathcal{U})$, where $\mathcal{U} = \mathcal{I} \cup \mathcal{J}$ and for every $e \in \mathcal{U}$,

$$\mathcal{R}(x) = \begin{cases} \mathcal{G}(e) & if\ e \in \mathcal{I} \setminus \mathcal{J} \\ \mathcal{H}(e) & if\ e \in \mathcal{J} \setminus \mathcal{I} \\ \mathcal{G}(e) \cup \mathcal{H}(e) & if\ e \in \mathcal{I} \cap \mathcal{J} \end{cases}$$

Since $\mathcal{I} \cap \mathcal{J} = \emptyset$, either $e \in \mathcal{I} \setminus \mathcal{J}$ or $e \in \mathcal{J} \setminus \mathcal{I}$ for all $e \in \mathcal{U}$. If $e \in \mathcal{I} \setminus \mathcal{J}$, then $\mathcal{R}(e) = \mathcal{G}(e) \lhd_{bci-i} \mathcal{F}(e)$ since $(\mathcal{G},\ \mathcal{I})\ \tilde{\lhd}_{bci-i}\ (\mathcal{F},\ A)$. If $e \in \mathcal{J} \setminus \mathcal{I}$, then $\mathcal{R}(e) = \mathcal{H}(e) \lhd_{bci-i} \mathcal{F}(e)$ since $(\mathcal{H},\ \mathcal{J})\ \tilde{\lhd}_{bci-i}\ (\mathcal{F},\ A)$. Thus $\mathcal{R}(e) \lhd_{bci-i} \mathcal{F}(e)$ for all $e \in \mathcal{U}$ and so

$$(\mathcal{G},\ \mathcal{I})\ \tilde{\cup}\ (\mathcal{H},\ \mathcal{J}) = (\mathcal{R},\ \mathcal{U})\ \tilde{\lhd}_{bci-i}\ (\mathcal{F},\ A)$$

□

It $\mathcal{I}$ and $\mathcal{J}$ are not disjoint in Theorem 3.11, then Theorem 3.11 is not true in general as seen in the following example.

Example 3.12. Let $X = \{0, a, b, c, d\}$ be the BCK-algebra and hence a BCI-algebra, with the following Cayley table:

$*$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	0	0	0
b	b	b	0	b	0
c	c	c	c	0	0
d	d	d	c	b	0

Let $(\mathcal{F},\ A)$ be a soft set over X, where $A = X$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ is a set-valued function defined by:

$$\mathcal{F}(x) = \{y \in X \mid y * (y * x) \in \{0, b\}\}$$

for all $x \in A$. Then $\mathcal{F}(0) = X$, $\mathcal{F}(a) = \mathcal{F}(b) = \{0, b, c, d\}$ and $\mathcal{F}(c) = \mathcal{F}(d) = \{0, b\}$, which are subalgebras of X. Hence $(\mathcal{F},\ A)$ is a soft BCI-algebra over X.

Let $(\mathcal{G},\ \mathcal{I})$ be a soft set over X, where $\mathcal{I} = \{b, c, d\} \subset A$ and $\mathcal{G} : \mathcal{I} \to \mathfrak{P}(X)$ be a set-valued function defined by:

$$\mathcal{G}(x) = \{y \in X \mid y * x = 0\}$$

for all $x \in \mathcal{I}$. Then $\mathcal{G}(b) = \{0, a, b\} \lhd_{bci-i} \{0, b, c, d\} = \mathcal{F}(b)$, $\mathcal{G}(c) = \{0, a, c\} \lhd_{bci-i} \{0, b\} = \mathcal{F}(c)$, $\mathcal{G}(d) = X \lhd_{bci-i} \{0, b\} = \mathcal{F}(d)$. Hence $(\mathcal{G},\ \mathcal{I})$ is a soft BCI-implicative ideal of $(\mathcal{F},\ A)$.

Now consider $\mathcal{J} = \{b\}$ which is not disjoint with $\mathcal{I}$ and let $\mathcal{H} : \mathcal{J} \to \mathfrak{P}(X)$ be a set valued function by:

$$\mathcal{H}(x) = \{y \in X \mid y * (y * x) = 0\}\}$$

for all $x \in \mathcal{J}$. Then $\mathcal{H}(b) = \{0, c\} \lhd_{bci-i} \{0, b, c, d\} = \mathcal{F}(b)$. Hence $(\mathcal{H},\ \mathcal{J})$ is a soft BCI-implicative ideal of $(\mathcal{F},\ A)$. But if $(\mathcal{R},\ \mathcal{U}) = (\mathcal{G},\ \mathcal{I})\ \tilde{\cup}\ (\mathcal{H},\ \mathcal{J})$, then $\mathcal{R}(b) = \mathcal{G}(b) \cup \mathcal{H}(b) = \{0, a, b, c\}$, which is not a BCI-implicative ideal of X related to $\mathcal{F}(b)$ since $(((d * 0) * 0) * (0 * 0)) * b = d * b = c \in \mathcal{R}(b)$ and $b \in \mathcal{R}(b)$ but $d * ((0 * (0 * d)) * (0 * (0 * (d * 0)))) = d * 0 = d \notin \mathcal{R}(b)$. Hence $(\mathcal{R},\ \mathcal{U}) = (\mathcal{G},\ \mathcal{I})\ \tilde{\cup}\ (\mathcal{H},\ \mathcal{J})$ is not a soft BCI-implicative ideal of $(\mathcal{F},\ A)$.

4. BCI-implicative idealistic soft BCI-algebras

Definition 4.1. (Jun and Park [7]) Let $(\mathcal{F},\ A)$ be soft set over X. Then $(\mathcal{F},\ A)$ is called an idealistic soft BCI-algebra over X if $\mathcal{F}(x)$ is an ideal of X for all $x \in A$.

Definition 4.2. Let $(\mathcal{F},\ A)$ be soft set over X. Then $(\mathcal{F},\ A)$ is called a BCI-implicative idealistic soft BCI-algebra over X if $\mathcal{F}(x)$ is a BCI-implicative ideal of X for all $x \in A$.

Example 4.3. Consider the BCI-algebra $X = \{0, a, b, c\}$ which is given in Example 3.3. Let $(\mathcal{F},\ A)$ be a soft set over X, where $A = X$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ is a set-valued function defined by:

$$\mathcal{F}(x) = \begin{cases} Z(\{0, a\}) & if\ x \in \{b, c\} \\ X & if\ x \in \{0, a\} \end{cases}$$

where $Z(\{0, a\}) = \{x \in X \mid 0 * (0 * x) \in \{0, a\}\}$. Then $(\mathcal{F},\ A)$ is a BCI-implicative idealistic soft BCI-algebra over X.

For any element x of a BCI-algebra X, we define the order of x, denoted by $o(x)$, as

$$o(x) = \min\{n \in N \mid 0 * x^n = 0\}$$

where $0 * x^n = (...((0 * x) * x)...) * x$, in which x appears n times.

Example 4.4. Let $X = \{0, a, b, c, d, e, f, g\}$ be a BCI-algebra defined by the following Cayley table:

$*$	0	a	b	c	d	e	f	g
0	0	0	0	0	d	d	d	d
a	a	0	0	0	e	d	d	d
b	b	b	0	0	f	f	d	d
c	c	b	a	0	g	f	e	d
d	d	d	d	d	0	0	0	0
e	e	d	d	d	a	0	0	0
f	f	f	d	d	b	b	0	0
g	g	f	e	d	c	b	a	0

Let $(\mathcal{F},\ A)$ be a soft set over X, where $A = \{a, b, c\} \subset X$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ is a set-valued function defined by:

$$\mathcal{F}(x) = \{y \in X \mid o(x) = o(y)\}$$

for all $x \in A$. Then $\mathcal{F}(a) = \mathcal{F}(b) = \mathcal{F}(c) = \{0, a, b, c\}$ is a BCI-implicative ideal of X. Hence $(\mathcal{F},\ A)$ is a BCI-implicative idealistic soft BCI-algebra over X. But if we take $B = \{a, b, f, g\} \subset X$ and define a set-valued function $\mathcal{G} : B \to \mathfrak{P}(X)$ by:

$$\mathcal{G}(x) = \{0\} \cup \{y \in X \mid o(x) = o(y)\}$$

for all $x \in B$, then $(\mathcal{G},\ B)$ is not a BCI-implicative idealistic soft BCI-algebra over X, since $\mathcal{G}(f) = \{0, d, e, f, g\}$ is not a BCI-implicative ideal of X because $(((g * d) * d) * (0 * d)) * d = c * d = g \in \mathcal{G}(f)$ and $d \in \mathcal{G}(f)$, but $g * ((d * (d * g)) * (0 * (0 * (g * d)))) = g * d = c \notin \mathcal{G}(f)$.

Example 4.5. Consider the BCI-algebra X $= \{0, a, b, c\}$ with the following Cayley table:

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Let $(\mathcal{F},\ A)$ be a soft set over X, where $A = X$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ is a set-valued function defined by:

$$\mathcal{F}(x) = \{y \in X \mid y = x^n,\ n \in N\}$$

for all $x \in A$. Then $\mathcal{F}(0) = \{0\},\ \mathcal{F}(a) = \{0, a\},\ \mathcal{F}(b) = \{0, b\},\ \mathcal{F}(c) = \{0, c\}$, which are BCI-implicative ideals of X. Hence $(\mathcal{F},\ A)$ is a BCI-implicative idealistic soft BCI-algebra over X.

Obviously, every BCI-implicative idealistic soft BCI-algebra over X is an idealistic soft BCI-algebra over X, but the converse is not true in general as seen in the following example.

Example 4.6. Consider a BCI-algebra $X := Y \times Z$, where $(Y, *, 0)$ is a BCI-algebra and $(Z, -, 0)$ is the adjoint BCI-algebra of the additive group $(Z, +, 0)$ of integers. Let $\mathcal{F} : X \to \mathfrak{P}(X)$ be the set-valued function defined as follows:

$$\mathcal{F}(y, n) = \begin{cases} Y \times N_\circ & if\ n \in N_\circ \\ \{(0, 0)\} & otherwise \end{cases}$$

for all $(y, n) \in X$, where $N_\circ$ is the set of all non-negative integers. Then $(\mathcal{F},\ X)$ is an idealistic soft BCI-algebra over X but it is not a BCI-implicative idealistic soft BCI-algebra over X since $\{(0, 0)\}$ may not be a BCI-implicative ideal of X.

Proposition 4.7. *Let $(\mathcal{F},\ A)$ and $(\mathcal{F},\ B)$ be soft sets over X where $B \subseteq A \subseteq X$. If $(\mathcal{F},\ A)$ is a BCI-implicative idealistic soft BCI-algebra over X, then so is $(\mathcal{F},\ B)$.*

Proof. Straightforward. □

The converse of Proposition 4.7 is not true in general as seen in the following example.

Example 4.8. Consider the BCI-implicative idealistic soft BCI-algebra over X which is described in Example 4.4. If we take $B = \{a, b, c, d\} \supseteq A$, then $(\mathcal{F},\ B)$ is not a BCI-implicative idealistic soft BCI-algebra over X since $\mathcal{F}(d) = \{d, e, f, g\}$ is not a BCI-implicative ideal of X.

Theorem 4.9. *Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ be two BCI-implicative idealistic soft BCI-algebras over X. If $A \cap B \neq \emptyset$, then the intersection $(\mathcal{F},\ A)\ \tilde{\cap}\ (\mathcal{G},\ B)$ is a BCI-implicative idealistic soft BCI-algebra over X.*

Proof. Using Definition 2.2, we can write

$$(\mathcal{F},\ A)\ \tilde{\cap}\ (\mathcal{G},\ B) = (\mathcal{H},\ C)$$

where $C = A \cap B$ and $\mathcal{H}(e) = \mathcal{F}(e)$ or $\mathcal{G}(e)$ for all $e \in C$. Note that $\mathcal{H} : C \to \mathfrak{P}(X)$ is a mapping, therefore $(\mathcal{H},\ C)$ is a soft set over X. Since $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ are BCI-implicative idealistic soft BCI-algebras over X, it follows that $\mathcal{H}(e) = \mathcal{F}(e)$ is a BCI-implicative ideal of X or $\mathcal{H}(e) = \mathcal{G}(e)$ is a BCI-implicative ideal of X for all $e \in C$. Hence $(\mathcal{H},\ C) = (\mathcal{F},\ A)\ \tilde{\cap}\ (\mathcal{G},\ B)$ is a BCI-implicative idealistic soft BCI-algebra over X. □

Corollary 4.10. *Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ A)$ be two BCI-implicative idealistic soft BCI-algebras over X. Then their intersection $(\mathcal{F},\ A)\ \tilde{\cap}\ (\mathcal{G},\ A)$ is a BCI-implicative idealistic soft BCI-algebra over X.*

Proof. Straightforward. □

Theorem 4.11. *Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ be two BCI-implicative idealistic soft BCI-algebras over X. If A and B are disjoint, then the union $(\mathcal{F},\ A)\ \tilde{\cup}\ (\mathcal{G},\ B)$ is a BCI-implicative idealistic soft BCI-algebra over X.*

Proof. By means of Definition 2.3, we can write $(\mathcal{F},\ A)\ \tilde{\cup}\ (\mathcal{G},\ B) = (\mathcal{H},\ C)$, where $C = A \cup B$ and for every $e \in C$,

$$\mathcal{H}(x) = \begin{cases} \mathcal{F}(e) & if\ e \in A \setminus B \\ \mathcal{G}(e) & if\ e \in A \setminus B \\ \mathcal{F}(e) \cup \mathcal{G}(e) & if\ e \in A \cap B \end{cases}$$

Since $A \cap B = \emptyset$, either $e \in A \setminus B$ or $e \in B \setminus A$ for all $e \in C$. If $e \in A \setminus B$, then $\mathcal{H}(e) = \mathcal{F}(e)$ is a BCI-implicative ideal of X since $(\mathcal{F},\ A)$ is a BCI-implicative idealistic soft BCI-algebra over X. If $e \in B \setminus A$, then $\mathcal{H}(e) = \mathcal{G}(e)$ is a BCI-implicative ideal of X since $(\mathcal{G},\ B)$ is a BCI-implicative idealistic soft BCI-algebra over X. Hence $(\mathcal{H},\ C) = (\mathcal{F},\ A)\ \tilde{\cup}\ (\mathcal{G},\ B)$ is a BCI-implicative idealistic soft BCI-algebra over X. □

Theorem 4.12. *Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ be two BCI-implicative idealistic soft BCI-algebras over X, then $(\mathcal{F},\ A)\ \tilde{\wedge}\ (\mathcal{G},\ B)$ is a BCI-implicative idealistic soft BCI-algebra over X.*

Proof. By means of Definition 2.4, we know that

$$(\mathcal{F},\ A)\ \tilde{\wedge}\ (\mathcal{G},\ B) = (\mathcal{H},\ A \times B),$$

where $H(x, y) = \mathcal{F}(x)\ \cap\ \mathcal{G}(y)$ for all $(x, y) \in A \times B$. Since $\mathcal{F}(x)$ and $\mathcal{G}(y)$ are BCI-implicative ideals of X, the intersection $\mathcal{F}(x)\ \cap\ \mathcal{G}(y)$ is also a BCI-implicative ideal of X. Hence $H(x, y)$ is a BCI-implicative ideal of X for all $(x, y) \in A \times B$.

Hence $(\mathcal{F},\ A)\ \tilde{\wedge}\ (\mathcal{G},\ B) = (\mathcal{H},\ A \times B)$ is a BCI-implicative idealistic soft BCI-algebra over X. □

Definition 4.13. A BCI-implicative idealistic soft BCI-algebra $(\mathcal{F},\ A)$ over X is said to be trivial (resp., whole) if $\mathcal{F}(x) = 0$ (resp., $\mathcal{F}(x) = X$) for all $x \in A$.

Example 4.14. Let X be a BCI-algebra which is given in Example 4.5 and let $\mathcal{F} : X \to \mathfrak{P}(X)$ be a set-valued function defined by

$$\mathcal{F}(x) = \{0\} \cup \{y \in X \mid o(x) = o(y)\}$$

for all $x \in X$. Then $\mathcal{F}(0) = \{0\}$ and $\mathcal{F}(a) = \mathcal{F}(b) = \mathcal{F}(c) = X$, which are BCI-implicative ideals of X. Hence $(\mathcal{F},\ \{0\})$ is a trivial BCI-implicative idealistic soft BCI-algebra over X and $(\mathcal{F},\ X \setminus \{0\})$ is a whole BCI-implicative idealistic soft BCI-algebra over X.

The proofs of the following three lemmas are straightforward, so they are omitted.

Lemma 4.15. *Let $f : X \to Y$ be an onto homomorphism of BCI-algebras. If I is an ideal of X, then $f(I)$ is an ideal of Y.*

Lemma 4.16. *Let $f : X \to Y$ be an isomorphism of BCI-algebras. If I is a BCI-implicative ideal of X, then $f(I)$ is a BCI-implicative ideal of Y.*

Let $f : X \to Y$ be a mapping of BCI-algebras. For a soft set $(\mathcal{F},\ A)$ over X, $(f(\mathcal{F}),\ A)$ is soft set over Y, where $f(\mathcal{F}) : A \to \mathfrak{P}(Y)$ is defined by $f(\mathcal{F})(x) = f(\mathcal{F}(x))$ for all $x \in A$.

Lemma 4.17. *Let $f : X \to Y$ be an isomorphism of BCI-algebras. If $(\mathcal{F},\ A)$ is a BCI-implicative idealistic soft BCI-algebra over X, then $(f(\mathcal{F}),\ A)$ is a BCI-implicative idealistic soft BCI-algebra over Y.*

Theorem 4.18. *Let $f : X \to Y$ be an isomorphism of BCI-algebras and let $(\mathcal{F},\ A)$ be a BCI-implicative idealistic soft BCI-algebra over X.*

(1) If $\mathcal{F}(x) = ker(f)$ for all $x \in A$, then $(f(\mathcal{F}),\ A)$ is a trivial BCI-implicative idealistic soft BCI-algebra over Y.

(2) If $(\mathcal{F},\ A)$ is whole, then $(f(\mathcal{F}),\ A)$ is a whole BCI-implicative idealistic soft BCI-algebra over Y.

Proof. (1) Assume that $\mathcal{F}(x) = ker(f)$ for all $x \in A$. Then $f(\mathcal{F})(x) = f(\mathcal{F}(x)) = \{0_Y\}$ for all $x \in A$. Hence $(\mathcal{F},\ A)$ is a trivial BCI-implicative idealistic soft BCI-algebra over Y by Lemma 4.17 and Definition 4.13.

(2) Suppose that $(\mathcal{F},\ A)$ is whole. Then $\mathcal{F}(x) = X$ for all $x \in A$ and so $f(\mathcal{F})(x) = f(\mathcal{F}(x)) = f(X) = Y$ for all $x \in A$. It follows from Lemma 4.17 and Definition 4.13 that $(f(\mathcal{F}),\ A)$ is a whole BCI-implicative idealistic soft BCI-algebra over Y. $\square$

Definition 4.19. (Liu and Meng [9]) A fuzzy set μ in X is called a fuzzy BCI-implicative ideal of X, if for all $x, y, z \in X$,

(i) $\mu(0) \geq \mu(x)$

(ii) $\mu(x*((y*(y*x))*(0*(0*(x*y))))) \geq min\{\mu((((x*y)*y)*(0*y))*z),\ \mu(z)\}$

The transfer principle for fuzzy sets described in [8] suggest the following theorem.

Lemma 4.20. *(Liu and Meng [9]) A fuzzy set μ in X is a fuzzy BCI-implicative ideal of X if and only if for any $t \in [0, 1]$, the level subset $U(\mu; t) := \{x \in X \mid \mu(x) \geq t\}$ is either empty or a BCI-implicative ideal of X.*

Theorem 4.21. *For every fuzzy BCI-implicative ideal μ of X, there exists a BCI-implicative idealistic soft BCI-algebra $(\mathcal{F},\ A)$ over X.*

Proof. Let μ be a fuzzy BCI-implicative ideal of X. Then $U(\mu; t) := \{x \in X \mid \mu(x) \geq t\}$ is an BCI-implicative ideal of X for all $t \in Im(\mu)$. If we take $A = Im(\mu)$ and consider the set valued function $\mathcal{F} : A \to \mathfrak{P}(X)$ given by $\mathcal{F}(t) = U(\mu; t)$ for all $t \in A$, then $(\mathcal{F},\ A)$ is a BCI-implicative idealistic soft BCI-algebra over X. $\square$

Conversely, the following theorem is straightforward.

Theorem 4.22. *For any fuzzy set μ in X, if a BCI-implicative idealistic soft BCI-algebra $(\mathcal{F},\ A)$ over X is given by $A = Im(\mu)$ and $\mathcal{F}(t) = U(\mu; t)$ for all $t \in A$, then μ is a fuzzy BCI-implicative ideal of X.*

Let μ be a fuzzy set in X and let $(\mathcal{F},\ A)$ be a soft set over X in which $A = Im(\mu)$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ is a set-valued function defined by

$$\mathcal{F}(t) = \{x \in X \mid \mu(x) + t > 1\} \tag{1}$$

for all $t \in A$. Then there exists $t \in A$ such that $\mathcal{F}(t)$ is not a BCI-implicative ideal of X as seen in the following example.

Example 4.23. For any BCI-algebra X, define a fuzzy set μ in X by $\mu(0) = t_\circ < 0.5$ and $\mu(x) = 1 - t_\circ$ for all $x \neq 0$. Let $A = Im(\mu)$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ be a set-valued function defined by (1). Then $\mathcal{F}(1 - t_\circ) = X \setminus \{0\}$, which is not a BCI-implicative ideal of X.

Theorem 4.24. *Let μ be a fuzzy set in X and let $(\mathcal{F},\ A)$ be a soft set over X in which $A = [0, 1]$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ is given by (1). Then the following assertions are equivalent:*

(1) μ is a fuzzy BCI-implicative ideal of X.

(2) for every $t \in A$ with $\mathcal{F}(t) \neq \emptyset$, $\mathcal{F}(t)$ is an BCI-implicative ideal of X.

Proof. Assume that μ is a fuzzy BCI-implicative ideal of X. Let $t \in A$ be such that $\mathcal{F}(t) \neq \emptyset$. Then for any $x \in \mathcal{F}(t)$, we have $\mu(0) + t \geq \mu(x) + t > 1$, that is, $0 \in \mathcal{F}(t)$. Let $(((x * y) * y) * (0 * y)) * z \in \mathcal{F}(t)$ and $z \in \mathcal{F}(t)$ for any $t \in A$ and $x, y, z \in X$. Then $\mu((((x * y) * y) * (0 * y)) * z) + t > 1$ and $\mu(z) + t > 1$. Since μ is a fuzzy BCI-implicative ideal of X, it follows that

$$\mu(x * ((y * (y * x)) * (0 * (0 * (x * y))))) + t \ \geq$$

$$\min\{\mu((((x * y) * y) * (0 * y)) * z),\ \mu(z)\} + t =$$

$$\min\{\mu((((x * y) * y) * (0 * y)) * z) + t,\ \mu(z) + t\} > 1$$

so that $x * ((y * (y * x)) * (0 * (0 * (x * y)))) \in \mathcal{F}(t)$. Hence $\mathcal{F}(t)$ is a BCI-implicative ideal of X for all $t \in A$ such that $\mathcal{F}(t) \neq \emptyset$.

Conversely, suppose that (2) is valid. If there exists $x_\circ \in X$ such that $\mu(0) < \mu(x_\circ)$, then there exists $t_\circ \in A$ such that $\mu(0) + t_\circ \leq 1 < \mu(x_\circ) + t_\circ$. It follows that $x_\circ \in \mathcal{F}(t_\circ)$ and $0 \notin \mathcal{F}(t_\circ)$, which is a contradiction. Hence $\mu(0) \geq \mu(x)$ for all $x \in X$. Now assume that

$$\mu(x_\circ * ((y_\circ * (y_\circ * x_\circ)) * (0 * (0 * (x_\circ * y_\circ))))) <$$

$$\min\{\mu((((x_\circ * y_\circ) * y_\circ) * (0 * y_\circ)) * z_\circ),\ \mu(z_\circ)\}$$

for some $x_\circ, y_\circ, z_\circ \in X$. Then there exists some $s_\circ \in A$ such that

$$\mu(x_\circ * ((y_\circ * (y_\circ * x_\circ)) * (0 * (0 * (x_\circ * y_\circ))))) + s_\circ \leq 1 <$$

$$\min\{\mu((((x_\circ * y_\circ) * y_\circ) * (0 * y_\circ)) * z_\circ),\ \mu(z_\circ)\} + s_\circ$$

$$\Rightarrow \mu(x_\circ * ((y_\circ * (y_\circ * x_\circ)) * (0 * (0 * (x_\circ * y_\circ))))) + s_\circ \leq 1 <$$

$$\min\{\mu((((x_\circ * y_\circ) * y_\circ) * (0 * y_\circ)) * z_\circ) + s_\circ,\ \mu(z_\circ) + s_\circ\}$$

which implies that $(((x_\circ * y_\circ) * y_\circ) * (0 * y_\circ)) * z_\circ \in \mathcal{F}(s_\circ)$ and $z_\circ \in \mathcal{F}(s_\circ)$ but $x_\circ * ((y_\circ * (y_\circ * x_\circ)) * (0 * (0 * (x_\circ * y_\circ)))) \notin \mathcal{F}(s_\circ)$. This is a contradiction. Therefore

$$\mu(x * ((y * (y * x)) * (0 * (0 * (x * y))))) \geq \min\{\mu((((x * y) * y) * (0 * y)) * z),\ \mu(z)\}$$

for all $x, y, z \in X$ and thus μ is fuzzy BCI-implicative ideal of X. □

Corollary 4.25. *Let μ be a fuzzy set in X such that $\mu(x) > 0.5$ for all $x \in X$ and let $(\mathcal{F},\ A)$ be a soft set over X in which*

$$A := \{t \in Im(\mu) \mid t > 0.5\}$$

and $\mathcal{F} : A \to \mathfrak{P}(X)$ is given by (1). If μ is a fuzzy BCI-implicative ideal of X, then $(\mathcal{F},\ A)$ is a BCI-implicative idealistic soft BCI-algebra over X.

Proof. Straightforward. □

Theorem 4.26. *Let μ be a fuzzy set in X and let $(\mathcal{F},\ A)$ be a soft set over X in which $A = (0.5, 1]$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ is defined by $\mathcal{F}(t) = U(\mu; t)$ for all $t \in A$.*

Then $\mathcal{F}(t)$ is a BCI-implicative ideal of X for all $t \in A$ with $\mathcal{F}(t) \neq \emptyset$ if and only if the following assertions are valid:

(1) $max\{\mu(0),\ 0.5\} \geq \mu(x)$ for all $x \in X$.

(2) $max\{\mu(x * ((y * (y * x)) * (0 * (0 * (x * y))))),\ 0.5\} \geq \min\{\mu((((x * y) * y) * (0 * y)) * z),\ \mu(z)\}$ for all $x, y, z \in X$.

Proof. Assume that $\mathcal{F}(t)$ is a BCI-implicative ideal of X for all $t \in A$ with $\mathcal{F}(t) \neq \emptyset$. If there exists $x_\circ \in X$ such that $max\{\mu(0),\ 0.5\} < \mu(x_\circ)$, then there exists $t_\circ \in A$ such that $max\{\mu(0),\ 0.5\} < t_\circ \leq \mu(x_\circ)$. It follows that $\mu(0) < t_\circ$, so that $x_\circ \in \mathcal{F}(t_\circ)$ and $0 \notin \mathcal{F}(t_\circ)$. This is a contradiction. Therefore (1) is valid. Suppose that there exist $a, b, c \in X$ such that

$$max\{\mu(a * ((b * (b * a)) * (0 * (0 * (a * b))))),\ 0.5\} <$$

$$min\{\mu((((a * b) * b) * (0 * b)) * c),\ \mu(c)\}$$

Then there exists $s_\circ \in A$ such that

$$\max\{\mu(a * ((b * (b * a)) * (0 * (0 * (a * b))))),\ 0.5\} < s_\circ \leq$$

$$min\{\mu((((a * b) * b) * (0 * b)) * c),\ \mu(c)\}$$

which implies $(((a * b) * b) * (0 * b)) * c \in \mathcal{F}(s_\circ)$ and $c \in \mathcal{F}(s_\circ)$, but $a * ((b * (b * a)) * (0 * (0 * (a * b)))) \notin \mathcal{F}(s_\circ)$. This is a contradiction. Hence (2) is valid.

Conversely, suppose that (1) and (2) are valid. Let $t \in A$ with $\mathcal{F}(t) \neq \emptyset$. Then for any $x \in \mathcal{F}(t)$, we have

$$max\{\mu(0),\ 0.5\} \geq \mu(x) \geq t > 0.5$$

which implies $\mu(0) \geq t$ and thus $0 \in \mathcal{F}(t)$. Let $(((x * y) * y) * (0 * y)) * z \in \mathcal{F}(t)$ and $z \in \mathcal{F}(t)$, for any $x, y, z \in X$. Then $\mu((((x * y) * y) * (0 * y)) * z) \geq t$ and $\mu(z) \geq t$. It follows from the second condition that

$$\max\{\mu(x * ((y * (y * x)) * (0 * (0 * (x * y))))),\ 0.5\} \geq$$

$$\min\{\mu((((x * y) * y) * (0 * y)) * z),\ \mu(z)\} \geq t > 0.5$$

$$\Rightarrow \mu(x * ((y * (y * x)) * (0 * (0 * (x * y))))) \geq t$$

so that $x * ((y * (y * x)) * (0 * (0 * (x * y)))) \in \mathcal{F}(t)$. Therefore $\mathcal{F}(t)$ is a BCI-implicative ideal of X for all $t \in A$ with $\mathcal{F}(t) \neq \emptyset$. $\square$

5. Conclusion

The concept of soft set, which is introduced by Molodtsov [15], is a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Soft sets are deeply related to fuzzy sets and rough sets. We introduced the notion of soft BCI-implicative ideals and BCI-implicative idealistic soft BCI-algebras and discussed related properties. We established the intersection, union, "AND" operation and "OR" operation of soft BCI-implicative ideals and BCI-implicative idealistic soft BCI-algebras. From the above discussion it can be observed that fuzzy BCI-implicative ideals can be characterized using the concept of soft sets. For a soft set $(\mathcal{F},\ A)$ over X, a fuzzy set μ in X is a fuzzy BCI-implicative ideal of X if and only if for every $t \in A$ with $\mathcal{F}(t) = \{x \in X \mid \mu(x) + t > 1\} \neq \emptyset$, $\mathcal{F}(t)$ is a BCI-implicative ideal of X. Finally we have discussed the relations between fuzzy BCI-implicative ideals and BCI-implicative idealistic soft BCI-algebras.

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