

Study of nonlinear stochastic Cauchy problems in $(\mathcal{C}, \mathcal{E}, \mathcal{P})$ -algebras

Victor Dévoué¹

Abstract. We use the framework of the $(\mathcal{C}, \mathcal{E}, \mathcal{P})$ -algebras of J.-A. Marti to study some nonlinear stochastic Cauchy problems for a simple equation, namely the transport equation in basic form, with stochastic generalized processes. Until now such studies were made in Colombeau-type algebras.

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1. Introduction

To study some nonlinear stochastic Cauchy problems we reformulate them in the framework of the $(\mathcal{C}, \mathcal{E}, \mathcal{P})$ -algebras of J.-A. Marti [6, 7, 8], [2]. These algebras allow us to treat singular processes in stochastic analysis following the example of Colombeau algebras. In this article we use the notations and concepts of our previous paper, [5].

The plan of this article is as follows. This section is followed by Section 2, which introduces the definitions and properties for stochastic analysis, $(\mathcal{C}, \mathcal{E}, \mathcal{P})$ -algebras and algebras of generalized stochastic processes. We refer the reader to [5] in which we make similar studies.

In Section 3, we examine the following Cauchy problems associated to a simple equation, namely the transport equation in basic form, formally written:

$$(P) : \frac{\partial U}{\partial t} = F(U) + W, \quad U|_{\gamma} = f,$$

and

$$(P') : \frac{\partial U}{\partial t} = F(U)W, \quad U|_{\gamma} = f,$$

with a smooth function F on the right-hand side. F can be non Lipschitz (in U) but F and all its derivatives have polynomial growth. γ is a monotonic curve with the equation $x = l(t)$, γ is not a characteristic curve, f is a generalized stochastic process on \mathbb{R} , W a generalized process on \mathbb{R}^2 . That is, f and W are weakly measurable maps of some probability space (Ω, Σ, μ) with values in the distribution space $\mathcal{D}'(\mathbb{R})$, respectively $\mathcal{D}'(\mathbb{R}^2)$.

For ω fixed, ω in Ω , using regularizations and cutoff techniques, we define a well formulated problem $(P(\omega)_{gen})$ (resp. $(P'(\omega)_{gen})$) associated to problem

¹Laboratoire MEMIAD, Université des Antilles, Campus de Shoelcher, BP 7209, 97275 Schoelcher Cedex, Martinique, e-mail: devoue-vi@orange.fr

(P) (resp. (P')) in a convenient algebra. We must use two parameters. The first parameter is used to regularize the data and the second one to replace the problem by a family of Lipschitz problems. We prove that problem (P) (resp. (P')) has a unique solution in some algebras of generalized stochastic processes.

Section 4, is devoted to a nonlinear stochastic Cauchy problem with the white noise as initial data

$$(P_1) : \frac{\partial U}{\partial t} = F(U), U|_\gamma = W,$$

where γ is the curve of equation $x = l(t)$, γ is not a characteristic curve, W is the white noise on \mathbb{R} . The function F is smooth, it can be non Lipschitz but F and all derivatives have polynomial growth. We study problem (P_1) as the previous ones and we examine the limiting behavior of the generalized solution.

2. Algebra of generalized stochastic processes

2.1. The presheaves of $(\mathcal{C}, \mathcal{E}, \mathcal{P})$ -algebras

We refer the reader to the references [3], [4], [5], for the definition and the properties of $(\mathcal{C}, \mathcal{E}, \mathcal{P})$ -algebras, the notion of overgenerated rings, the relationship with distribution theory and the association process, the notion of algebra $\mathcal{A}(\Omega)$ stable under the family $(F_\eta)_\lambda$ [2], the definition of the generalized operator \mathcal{F} associated to the family $(F_\eta)_\lambda$, the definition of the generalized second order restriction mapping \mathcal{R}_g associated to the function g .

We use the same notations and the same notions as the references. All these elements of the theory of the $(\mathcal{C}, \mathcal{E}, \mathcal{P})$ -algebras are now well-known.

2.2. Algebras $\mathcal{A}_p(O)$, $p \in \mathbb{N}^*$ and $\mathcal{A}(O)$

Take $\mathcal{E} = C^\infty$, $X = \mathbb{R}^d$ for $d = 1, 2$, $E = \mathcal{D}'$ and Λ a set of indices, $\lambda \in \Lambda$. Take $p \in \mathbb{N}^*$. For any open set O in \mathbb{R}^d $\mathcal{E}(O) = C^\infty(O)$, is endowed with the $\mathcal{P}_p(O)$ topology defined by the family of the seminorms

$$N_{K,l}^p(u_\lambda) = \sup_{|\alpha| \leq l} N_{K,\alpha}^p(u_\lambda), \text{ with}$$

$$N_{K,\alpha}^p(u_\lambda) = \|D^\alpha u_\lambda(x)\|_{L^p(K)}, K \Subset O,$$

and $D^\alpha = \frac{\partial^{\alpha_1 + \dots + \alpha_d}}{\partial z_1^{\alpha_1} \dots \partial z_d^{\alpha_d}}$ for $z = (z_1, \dots, z_d) \in O$, $l \in \mathbb{N}$, $\alpha = (\alpha_1, \dots, \alpha_d) \in \mathbb{N}^d$, $K \Subset O$ means that K is a compact subset of O . Let A be a subring of the ring \mathbb{R}^Λ . We consider a solid ideal I_A of A . Put

$$\mathcal{X}_p(O) = \{(u_\lambda)_\lambda \in [C^\infty(O)]^\Lambda : \forall K \Subset O, \forall l \in \mathbb{N}, \left(N_{K,l}^p(u_\lambda)\right)_\lambda \in |A|\},$$

$$\mathcal{N}_p(O) = \{(u_\lambda)_\lambda \in [C^\infty(O)]^\Lambda : \forall K \Subset O, \forall l \in \mathbb{N}, \left(N_{K,l}^p(u_\lambda)\right)_\lambda \in |I_A|\},$$

$$\mathcal{A}_p(O) = \mathcal{X}_p(O)/\mathcal{N}_p(O).$$

The generalized derivation $D^\alpha : u(= [u_\varepsilon]) \mapsto D^\alpha u = [D^\alpha u_\varepsilon]$ provides $\mathcal{A}_p(O)$ with a differential algebraic structure.

For $p = +\infty$, $\mathcal{E}(O) = C^\infty(O)$ is endowed with the $\mathcal{P}(O)$ topology defined by the family of the seminorms

$$P_{K,l}(u_\lambda) = \sup_{|\alpha| \leq l} P_{K,\alpha}(u_\lambda), \text{ with}$$

$$P_{K,\alpha}(u_\lambda) = \|D^\alpha u_\lambda(x)\|_{L^\infty(K)}, \quad K \Subset O$$

Put

$$\mathcal{X}(O) = \{(u_\lambda)_\lambda \in [C^\infty(O)]^\Lambda : \forall K \Subset O, \forall l \in \mathbb{N}, (P_{K,l}(u_\lambda))_\lambda \in |A|\},$$

$$\mathcal{N}(O) = \{(u_\lambda)_\lambda \in [C^\infty(O)]^\Lambda : \forall K \Subset O, \forall l \in \mathbb{N}, (P_{K,l}(u_\lambda))_\lambda \in |I_A|\}.$$

The generalized derivation $D^\alpha : u(= [u_\varepsilon]) \mapsto D^\alpha u = [D^\alpha u_\varepsilon]$ provides $\mathcal{A}(O) = \mathcal{X}(O)/\mathcal{N}(O)$ with a differential algebraic structure.

Remark 2.1. The $N_{K,l}^2$ norms are bounded by the $P_{K,l}$ norms. We have $\mathcal{A}(O) \subset \mathcal{A}_2(O)$.

Remark 2.2. $\mathcal{A}_p(O)$ have properties similar to those of $\mathcal{A}(O)$.

2.3. Stochastic analysis

We refer the reader to [11], [12], [10] and [9], for construction of white noise and the relation between the white noise and Wiener process on \mathbb{R}^d .

Let (Ω, Σ, μ) be a probability space. A generalized stochastic process on \mathbb{R}^d is a weakly measurable map

$$X : \Omega \rightarrow \mathcal{D}'(\mathbb{R}^d)$$

For any fixed test function $\varphi \in \mathcal{D}(\mathbb{R}^d)$, the map $\Omega \rightarrow \mathbb{R}; \omega \mapsto \langle X(\omega), \varphi \rangle$ is a random variable. The space of generalized stochastic processes is denoted by $\mathcal{D}'_\Omega(\mathbb{R}^d)$.

2.4. Algebras of generalized stochastic processes

Let O be an open set in \mathbb{R}^d , (Ω, Σ, μ) a probability space.

Definition 2.3. A mapping $U : \Omega \rightarrow \mathcal{A}(O)$ such that there is a representing function

$$u = R_U : \Lambda \times O \times \Omega \rightarrow \mathbb{R}$$

with the properties:

- (i) for fixed $\lambda \in \Lambda$, the map $(x, \omega) \mapsto u(\lambda, x, \omega)$ is jointly measurable on $O \times \Omega$;
- (ii) almost surely in $\omega \in \Omega$, the map $\lambda \mapsto u(\lambda, \cdot, \omega)$ belongs to $\mathcal{X}(O)$ and it is a representative of $U(\omega)$, i.e. almost surely in $\omega \in \Omega$, $(U(\omega)_\lambda)_\lambda = (u(\lambda, \cdot, \omega))_\lambda \in \mathcal{X}(O)$, is called a $\mathcal{A}(O)$ -generalized stochastic processes on the probability space (Ω, Σ, μ) .

The algebra of generalized stochastic processes is denoted by $\mathcal{A}^\Omega(O)$.

Definition 2.4. A map $U : \Omega \rightarrow \mathcal{A}_2(O)$ such that there is a representing function

$$u = R_U : \Lambda \times O \times \Omega \rightarrow \mathbb{R}$$

with the properties:

(i) for fixed $\lambda \in \Lambda$, the map $(x, \omega) \mapsto u(\lambda, x, \omega)$ is jointly measurable on $O \times \Omega$;
(ii) almost surely in $\omega \in \Omega$, the map $\lambda \mapsto u(\lambda, \cdot, \omega)$ belongs to $\mathcal{X}_2(O)$ and it is a representative of $U(\omega)$, i.e. almost surely in $\omega \in \Omega$, $(U(\omega)_\lambda)_\lambda = (u(\lambda, \cdot, \omega))_\lambda \in \mathcal{X}_2(O)$, is called a $\mathcal{A}_2(0)$ -generalized stochastic processes on the probability space (Ω, Σ, μ) .

The algebra of generalized stochastic processes is denoted by $\mathcal{A}_2^\Omega(O)$.

Remark 2.5. Let φ of the form $\varphi(x, y) = \chi(x)\chi(y)$ with $\chi \in \mathcal{D}(\mathbb{R})$ with the property

$$\int \chi(s)ds = 1; \int s^p \chi(s)ds = 0, 1 \leq p \leq 2.$$

Let $V \in \mathcal{D}'_\Omega(\mathbb{R}^d)$ be a generalized stochastic process. If $\lambda \in \Lambda$, then $V(\omega) * \varphi_\lambda$ is measurable with respect to $\omega \in \Omega$ and smooth with respect to $x \in \mathbb{R}^d$, hence jointly measurable. So, $(V(\omega) * \varphi_\lambda)_\lambda$ belongs to $\mathcal{X}(\mathbb{R}^d)$. Then

$$R_V(\lambda, x, \omega) = (V(\omega) * \varphi_\lambda)(x) = V(\omega)_\lambda(x)$$

qualifies as a representative for a random generalized function. Thus we have an imbedding $\tau : \mathcal{D}'_\Omega(\mathbb{R}^d) \hookrightarrow \mathcal{A}^\Omega(\mathbb{R}^d)$.

3. Some nonlinear stochastic problems

3.1. A nonlinear stochastic problem with additive generalized stochastic process

Consider the Cauchy problem formally written:

$$(1) \quad (P) : \frac{\partial U}{\partial t} = F(U) + W, \quad U|_\gamma = f,$$

where γ is a monotonic curve of equation $x = l(t)$, γ is not a characteristic curve, $f \in \mathcal{A}^\Omega(\mathbb{R})$, $W \in \mathcal{A}^\Omega(\mathbb{R}^2)$ is a $\mathcal{A}(\mathbb{R}^2)$ -generalized stochastic process on a probability space (Ω, Σ, μ) . F is smooth, it can be non Lipschitz but F and all derivatives have polynomial growth and $F(0) = 0$. We look for a solution $(U : \Omega \rightarrow \mathcal{A}_2(\mathbb{R}^2)) \in \mathcal{A}_2^\Omega(\mathbb{R}^2)$. (For example, $F(U) = -U - U^2$ or $F(U) = -U^2$.)

Thus U is a solution to problem (P) if and only if for each $\omega \in \Omega$, $U(\omega)$ is solution to the formally written problem

$$(P(\omega)) \left\{ \begin{array}{l} \frac{\partial U(\omega)}{\partial t} = F(U(\omega)) + W(\omega), \\ U(\omega)|_\gamma = f(\omega). \end{array} \right.$$

3.2. A nonlinear stochastic problem with multiplicative generalized stochastic process

Consider the Cauchy problem formally written:

$$(2) \quad (P') : \frac{\partial U}{\partial t} = F(U)W, \quad U|_{\gamma} = f,$$

where γ is a monotonic curve of equation $x = l(t)$, γ is not a characteristic curve, $f \in \mathcal{A}^{\Omega}(\mathbb{R})$, $W \in \mathcal{A}^{\Omega}(\mathbb{R}^2)$ is a $\mathcal{A}(\mathbb{R}^2)$ -generalized stochastic process on a probability space (Ω, Σ, μ) . F is smooth, it can be non Lipschitz, but F and all derivatives have polynomial growth and $F(0) = 0$. We look for a solution $(U : \Omega \rightarrow \mathcal{A}_2(\mathbb{R}^2)) \in \mathcal{A}_2^{\Omega}(\mathbb{R}^2)$. (For example, $F(U) = -U^2$).

Thus U is a solution to problem (P) if and only if for each $\omega \in \Omega$, $U(\omega)$ is solution to the formally written problem

$$P'(\omega) \left\{ \begin{array}{l} \frac{\partial U(\omega)}{\partial t} = F(U(\omega))W(\omega), \\ U(\omega)|_{\gamma} = f(\omega). \end{array} \right.$$

3.3. Cut off procedure

Take $(r_{\eta})_{\eta}$ be in $\mathbb{R}_*^{(0,1]}$ such that $r_{\eta} > 0$ and $\lim_{\eta \rightarrow 0} r_{\eta} = +\infty$. Set $E_{\eta} = [-r_{\eta}, r_{\eta}]$.

Set a family of smooth one-variable functions $(h_{\eta})_{\eta}$ such that

$$(3) \quad \sup_{z \in I_{\eta}} |h_{\eta}(z)| = 1, \quad h_{\eta}(z) = \begin{cases} 0, & \text{if } |z| \geq r_{\eta} \\ 1, & \text{if } -r_{\eta} + 1 \leq z \leq r_{\eta} - 1 \end{cases}$$

Suppose that $\frac{\partial^n h_{\eta}}{\partial z^n}$ is bounded on E_{η} for any integer n , $n > 0$. Set

$$\sup_{z \in E_{\eta}} \left| \frac{\partial^n h_{\eta}}{\partial z^n}(z) \right| = M_n.$$

Let $\phi_{\eta}(z) = zh_{\eta}(z)$. We approximate the function F by the family of functions $(F_{\eta})_{\eta}$ defined by

$$F_{\eta}(z) = F(\phi_{\eta}(z)) = F(zh_{\eta}(z)).$$

Suppose that $F(0) = 0$. F is smooth, it can be non Lipschitz but F and all derivatives have polynomial growth. More precisely, we assume the existence of $p \in \mathbb{N}$ such that

$$\forall l \in \mathbb{N}, \exists c_l > 0, \sup_{z \in \mathbb{R}} |D^l F(z)| \leq c_l(1 + |z|)^p.$$

Then

$$\forall l \in \mathbb{N}, \exists \mu_l > 0, \sup_{z \in \mathbb{R}; |\alpha| \leq l} |D^{\alpha} F_{\eta}(z)| = \sup_{|z| \leq r_{\eta}; |\alpha| \leq l} |D^{\alpha} F(\phi_{\eta}(z))| \leq \mu_l(1 + r_{\eta})^p.$$

Thus, according to [3], [4], $\mathcal{A}(\mathbb{R})$ is stable under the family $(F_{\eta})_{\eta}$.

3.4. Construction of $\mathcal{A}(\mathbb{R}^2)$

Take $U(\omega) = [U(\omega)_{\varepsilon, \eta}]$ and

$$W(\omega)_{\varepsilon, \eta}(t, x) = (\phi_\eta(W(\omega) * \varphi_\varepsilon))(t, x),$$

φ of the form $\varphi(t, x) = \chi(t)\chi(x)$, $\chi \in \mathcal{D}(\mathbb{R})$ having the property

$$\int \chi(s)ds = 1; \int s^p \chi(s)ds = 0, 1 \leq p \leq 2$$

and $(\chi_\varepsilon)_\varepsilon$ being a family of mollifiers such that $\chi_\varepsilon(x) = \frac{1}{\varepsilon} \chi(\frac{x}{\varepsilon})$, thus $\varphi_\varepsilon(t, x) = \chi_\varepsilon(t)\chi_\varepsilon(x)$. Take $f(\omega)_\varepsilon = f(\omega) * \chi_\varepsilon$.

We make the following assumptions to generate a convenient $(\mathcal{C}, \mathcal{E}, \mathcal{P})$ -algebra adapted to our problem.

$$(H_1) : \exists p > 0, \forall n \in \mathbb{N}, \exists a_n > 0, \sup_{z \in \mathbb{R}; |\alpha| \leq n} |D^\alpha F_\eta(z)| \leq a_n(1 + r_\eta)^p.$$

$$(H_2) : \forall K \in \mathbb{R}^2, \forall n \in \mathbb{N}, \exists \rho_n > 0, P_{K, n}(W(\omega)_{\varepsilon, \eta}) \leq \rho_n(1 + r_\eta)^p.$$

$$(H_3) \left\{ \begin{array}{l} \mathcal{C} = A/I_A \text{ is overgenerated by the following elements of } \mathbb{R}_*^{(0,1]} \\ (\varepsilon)_{\varepsilon, \eta}, (\eta)_{\varepsilon, \eta}, (r_\eta)_{\varepsilon, \eta}, (\exp(1 + r_\eta))_{\varepsilon, \eta}. \end{array} \right.$$

$$(H_4) \left\{ \begin{array}{l} \mathcal{A}(\mathbb{R}^2) = \mathcal{X}(\mathbb{R}^2)/\mathcal{N}(\mathbb{R}^2) \text{ is built on } \mathcal{C} \text{ with} \\ (\mathcal{E}, \mathcal{P}) = (C^\infty(\mathbb{R}^2), (P_{K, l})_{K \in \mathbb{R}^2, l \in \mathbb{N}}). \end{array} \right.$$

$$\begin{aligned} (H_5) \mathcal{A}_2(\mathbb{R}^2) &= \mathcal{X}_2(\mathbb{R}^2)/\mathcal{N}_2(\mathbb{R}^2) \text{ is built on } \mathcal{C} \\ \text{with } (\mathcal{E}, \mathcal{P}') &= (C^\infty(\mathbb{R}^2), (N_{K, l}^2)_{K \in \mathbb{R}^2, l \in \mathbb{N}}). \end{aligned}$$

3.5. Generalized differential problems associated to the formal ones

We give a meaning to the problems formally written as (P) and (P') .

F_η is defined above. Let \mathcal{F} be the generalized operator associated to F via the family $(h_\eta)_\eta$. \mathcal{R}_l is the generalized second-size mapping associated with l [5].

3.5.1. Generalized differential problem associated to (P)

For ω fixed, the problem associated to $(P(\omega))$ can be written as the well-formulated problem

$$(P(\omega)_{gen}) \left\{ \begin{array}{l} \frac{\partial U(\omega)}{\partial t} = \mathcal{F}(U(\omega)) + [W(\omega)_{\varepsilon, \eta}], \\ \mathcal{R}_l(U(\omega)) = [f(\omega)_\varepsilon], \end{array} \right.$$

then

$$\begin{cases} \frac{\partial U(\omega)}{\partial t} = [F_\eta(U(\omega))] + [W(\omega)_{\varepsilon,\eta}], \\ U(\omega)|_\gamma = [f(\omega) * \chi_\varepsilon]. \end{cases}$$

In terms of representatives, and thanks to the stability and restriction hypothesis, if we find $U(\omega)_{\varepsilon,\eta} \in C^\infty(\mathbb{R}^2)$ verifying

$$(P(\omega)_{(\varepsilon,\eta)}) \begin{cases} \frac{\partial U(\omega)_{\varepsilon,\eta}}{\partial t}(t, x) = F_\eta(U(\omega)_{\varepsilon,\eta}(t, x)) + W(\omega)_{\varepsilon,\eta}(t, x), \\ U(\omega)_{\varepsilon,\eta}(t, l(t)) = f(\omega)_\varepsilon(t) = (f(\omega) * \chi_\varepsilon)(t), \end{cases}$$

and if we prove that $(U(\omega)_{\varepsilon,\eta})_{\varepsilon,\eta} \in \mathcal{X}_2(\mathbb{R}^2)$, thus $U(\omega) = [U(\omega)_{\varepsilon,\eta}]$ is a solution of $P(\omega)_{gen}$.

Let $V(\omega) = [V(\omega)_{\varepsilon,\eta}]$ be another solution to $P(\omega)_{gen}$.

If $(V(\omega)_{\varepsilon,\eta} - U(\omega)_{\varepsilon,\eta})_{\varepsilon,\eta} \in \mathcal{N}(\mathbb{R}^2)$ the solution to $P_{gen}(\omega)$ is unique.

Remark 3.1. Dependence on some regularizing family. The problem $P(\omega)_{gen}$ itself, so a solution of it, a priori depends on the family of cutoff functions and, in the case of irregular data, on the family of mollifiers [4].

Remark 3.2. $F(U)$ is such that

$$F(U) : \Omega \rightarrow \mathcal{A}(\mathbb{R}^2), \omega \mapsto [F_\eta(U(\omega)_{\varepsilon,\eta})]$$

and

$$F_\eta(U(\omega)_{\varepsilon,\eta}) : \mathbb{R}^2 \rightarrow \mathbb{R}, (t, x) \mapsto F_\eta(U(\omega)_{\varepsilon,\eta}(t, x)).$$

Moreover

$$R_U = u : \Lambda \times \mathbb{R}^2 \times \Omega \rightarrow \mathbb{R}; (\lambda, (t, x), \omega) \mapsto U(\omega)_{\varepsilon,\eta}(t, x) = u(\lambda, t, x, \omega),$$

with $\lambda = (\varepsilon, \eta)$.

3.5.2. Generalized differential problem associated to (P')

For ω fixed, the problem associated to $(P'(\omega))$ can be written as the well-formulated problem

$$(P'_{gen}(\omega)) \begin{cases} \frac{\partial U(\omega)}{\partial t} = \mathcal{F}(U(\omega)) [W(\omega)_{\varepsilon,\eta}], \\ \mathcal{R}_l(U(\omega)) = [f(\omega)_\varepsilon], \end{cases}$$

then

$$\begin{cases} \frac{\partial U(\omega)}{\partial t} = [F_\eta(U(\omega))] [W(\omega)_{\varepsilon,\eta}], \\ U(\omega)|_\gamma = [f(\omega) * \chi_\varepsilon]. \end{cases}$$

In terms of representatives, and thanks to the stability and restriction hypothesis, if we find $U(\omega)_{\varepsilon,\eta} \in C^\infty(\mathbb{R}^2)$ verifying

$$(P'(\omega)_{(\varepsilon,\eta)}) \begin{cases} \frac{\partial U(\omega)_{\varepsilon,\eta}}{\partial t}(t, x) = F_\eta(U(\omega)_{\varepsilon,\eta}(t, x)) W(\omega)_{\varepsilon,\eta}(t, x), \\ U(\omega)_{\varepsilon,\eta}(t, l(t)) = f(\omega)_\varepsilon(t) = (f(\omega) * \chi_\varepsilon)(t), \end{cases}$$

and if we prove that $(U(\omega)_{\varepsilon,\eta})_{\varepsilon,\eta} \in \mathcal{X}_2(\mathbb{R}^2)$, thus $U(\omega) = [U(\omega)_{\varepsilon,\eta}]$ is a solution of $(P'(\omega)_{gen})$.

3.6. Generalized problems

3.6.1. Solution to the parametrized regular problems

Fix ω , consider the regularized problems $(P(\omega)_{(\varepsilon,\eta)})$ and $(P'(\omega)_{(\varepsilon,\eta)})$.

Under assumptions (H_1) , (H_2) and the assumptions

$$(H_{\varepsilon,\eta}) \begin{cases} \text{a)} & l \in C^\infty(\mathbb{R}), l' > 0, l(\mathbb{R}) = \mathbb{R}, \\ \text{b)} & F_\eta \in C^\infty(\mathbb{R}, \mathbb{R}), \\ \text{c)} & f(\omega)_\varepsilon \in C^\infty(\mathbb{R}), \end{cases}$$

one can prove that $(P(\omega)_{(\varepsilon,\eta)})$ admits a unique smooth solution $U(\omega)_{\varepsilon,\eta}$ such that

$$U(\omega)_{\varepsilon,\eta}(t, x) = f(\omega)_\varepsilon(l^{-1}(x)) + \int_{l^{-1}(x)}^t (F_\eta(U(\omega)_{\varepsilon,\eta}(\zeta, x)) + W(\omega)_{\varepsilon,\eta}(\zeta, x)) d\zeta$$

and $(P'(\omega)_{(\varepsilon,\eta)})$ admits a unique smooth solution $U(\omega)_{\varepsilon,\eta}$ such that

$$U(\omega)_{\varepsilon,\eta}(t, x) = f(\omega)_\varepsilon(l^{-1}(x)) + \int_{l^{-1}(x)}^t (F_\eta(U(\omega)_{\varepsilon,\eta}(\zeta, x))W(\omega)_{\varepsilon,\eta}(\zeta, x)) d\zeta,$$

Theorem 3.3. *Under assumptions $(H_{\varepsilon,\eta})$, (H_1) and (H_2) , problem $(P(\omega)_{(\varepsilon,\eta)})$ (resp. $(P'(\omega)_{(\varepsilon,\eta)})$) has a unique solution, $U(\omega)_{\varepsilon,\eta}$, in $C^\infty(\mathbb{R}^2)$.*

See [1].

3.6.2. Solution to the problems

Theorem 3.4. *Suppose that $U(\omega)_{\varepsilon,\eta}$ is the solution to problem $(P(\omega)_{(\varepsilon,\eta)})$ (resp. $(P'(\omega)_{(\varepsilon,\eta)})$) then problem $(P(\omega)_{gen})$ (resp. $(P'(\omega)_{gen})$) has a unique solution $U(\omega) = [U(\omega)_{\varepsilon,\eta}]$ in $\mathcal{A}(\mathbb{R}^2)$.*

$U(\omega)$ is the solution to $(P(\omega)_{(\varepsilon,\eta)})$ (resp. $(P'(\omega)_{(\varepsilon,\eta)})$) if $(U(\omega)_{\varepsilon,\eta})_{\varepsilon,\eta} \in \mathcal{X}(\mathbb{R}^2)$, that is

$$\forall K \in \mathbb{R}^2, \forall l \in \mathbb{N}, (P_{K,l}(U(\omega)_{\varepsilon,\eta}))_{\varepsilon,\eta} \in A.$$

The proof follows the same steps as the existence results which can be found in [3], replacing $u_{\varepsilon,\eta}$ by $U(\omega)_{\varepsilon,\eta}$ and $F_\eta(x, y, u_{\varepsilon,\eta}(x, y))$ by $F_\eta(U(\omega)_{\varepsilon,\eta}(x, y)) + W(\omega)_{\varepsilon,\eta}(x, y)$ (resp. $F_\eta(U(\omega)_{\varepsilon,\eta}(x, y))W(\omega)_{\varepsilon,\eta}(x, y)$). An induction process on the order of the successive derivatives shows that $(U(\omega)_{\varepsilon,\eta})_{\varepsilon,\eta}$ belongs to $\mathcal{X}(\mathbb{R}^2)$. For the uniqueness, the Gronwall lemma is an essential tool.

Theorem 3.5. *Suppose that $U(\omega)_{\varepsilon,\eta}$ is the solution to problem $(P(\omega)_{(\varepsilon,\eta)})$ (resp. $(P'(\omega)_{(\varepsilon,\eta)})$) then problem $(P(\omega)_{gen})$ (resp. $(P'(\omega)_{gen})$) has a unique solution $U(\omega) = [U(\omega)_{\varepsilon,\eta}]$ in $\mathcal{A}_2(\mathbb{R}^2)$.*

Proof. $U(\omega)$ is the solution to $(P(\omega))_{(\varepsilon, \eta)}$ if $(U(\omega)_{\varepsilon, \eta})_{\varepsilon, \eta} \in \mathcal{X}_2(\mathbb{R}^2)$. We must prove that

$$\forall K \in \mathbb{R}^2, \forall l \in \mathbb{N}, (N_{K, l}^2(U(\omega)_{\varepsilon, \eta}))_{\varepsilon, \eta} \in A.$$

However

$$\|D^\alpha(U(\omega)_{\varepsilon, \eta})\|_{L^2(K)} \leq (\mu(K))^{1/2} \|D^\alpha(U(\omega)_{\varepsilon, \eta})\|_{L^\infty(K)}$$

and, as $(U(\omega)_{\varepsilon, \eta})_{\varepsilon, \eta} \in \mathcal{X}(\mathbb{R}^2)$, we have $(\|D^\alpha(U(\omega)_{\varepsilon, \eta})\|_\infty)_{\varepsilon, \eta} \in A$. Then

$$(\|D^\alpha(U(\omega)_{\varepsilon, \eta})\|_{L^2(K)})_{\varepsilon, \eta} = (N_{K, l}^2(U(\omega)_{\varepsilon, \eta}))_{\varepsilon, \eta} \in A.$$

So $U(\omega) \in \mathcal{A}_2(\mathbb{R}^2)$ and it is the solution to problem $(P(\omega))_{gen}$ in $\mathcal{A}_2(\mathbb{R}^2)$. Set

$$U : \Omega \rightarrow \mathcal{A}_2(\mathbb{R}^2), \omega \mapsto U(\omega).$$

Thus $U \in \mathcal{A}_2^\Omega(\mathbb{R}^2)$. □

Theorem 3.6. *The mapping U is the solution to problem (P) (resp. (P')) and it is almost surely unique in $\mathcal{A}_2^\Omega(\mathbb{R}^2)$.*

Proof. Since $U(\omega)$ is the unique solution to problem $(P(\omega))_{gen}$ in $\mathcal{A}_2(\mathbb{R}^2)$ then almost surely in $\omega \in \Omega$, the map $\lambda \mapsto R_U(\lambda, (\cdot, \cdot), \omega) = U(\omega)_\lambda$, ($\lambda = (\varepsilon, \eta)$), belongs to $\mathcal{X}_2(\mathbb{R}^2)$ and it is a representative of $U(\omega)$ (i.e. $U(\omega) = [U(\omega)_\lambda]$). For fixed $\lambda = (\varepsilon, \eta) \in \Lambda$, the map

$$((x, y), \omega) \mapsto R_U(\lambda, (x, y), \omega) = U(\omega)_\lambda(x, y) = u_\lambda((x, y), \omega)$$

is jointly measurable on $\mathbb{R}^2 \times \Omega$. Then U is the solution to problem (P) almost surely unique in $\mathcal{A}_2^\Omega(\mathbb{R}^2)$. □

3.7. A special case

Consider the Cauchy problem formally written as

$$(2) \quad (S) : \frac{\partial U}{\partial t} = W, U|_\gamma = f,$$

where γ is a monotonic curve of equation $x = l(t)$, γ is not a characteristic curve, $f \in \mathcal{A}^\Omega(\mathbb{R})$, $W \in \mathcal{A}^\Omega(\mathbb{R}^2)$ is a $\mathcal{A}(\mathbb{R}^2)$ -generalized stochastic process on a probability space (Ω, Σ, μ) .

This problem coincides with problem (P) for $F = 0$ and with problem (P') for $F = 1$. Problem (S) admits a solution U . $U(\omega) = [U(\omega)_{\varepsilon, \eta}]$ in $\mathcal{A}_2(\mathbb{R}^2)$ is defined, with the previous notations, by

$$U(\omega)_{\varepsilon, \eta}(t, x) = f(\omega)_\varepsilon(l^{-1}(x)) + \int_{l^{-1}(x)}^t W(\omega)_{\varepsilon, \eta}(\zeta, x) d\zeta.$$

4. A nonlinear stochastic Cauchy problem with the white noise as data

Consider the Cauchy problems formally written:

$$(4) \quad (P_1) : \frac{\partial U}{\partial t} = F(U), U|_\gamma = W,$$

and

$$(5) \quad (P_2) : \frac{\partial V}{\partial t} = 0, V|_\gamma = W,$$

where γ is a monotonic curve of equation $x = l(t)$, γ is not a characteristic curve, $W \in \mathcal{A}^\Omega(\mathbb{R})$ is the white noise on \mathbb{R} . F is smooth, it can be non Lipschitz but F and all its derivatives have polynomial growth. We look for a solution $(U : \Omega \rightarrow \mathcal{A}(\mathbb{R}^2)) \in \mathcal{A}^\Omega(\mathbb{R}^2)$ and $(V : \Omega \rightarrow \mathcal{A}(\mathbb{R}^2)) \in \mathcal{A}^\Omega(\mathbb{R}^2)$.

U is a solution to problem (P_1) if and only if for every $\omega \in \Omega$, $U(\omega)$ is a solution to the formal problem

$$(P_1(\omega)) : \frac{\partial U(\omega)}{\partial t} = F(U(\omega)), U(\omega)|_\gamma = W(\omega).$$

V is a solution to problem (P_2) if and only if, for any $\omega \in \Omega$, $V(\omega)$ is a solution to the formally problem

$$(P_2(\omega)) : \frac{\partial V(\omega)}{\partial t} = 0, V(\omega)|_\gamma = W(\omega).$$

We make the same hypotheses and we take the same spaces $\mathcal{A}(\mathbb{R}^2)$ and $\mathcal{A}_2(\mathbb{R}^2)$ built for problems (P) and (P') .

4.1. A generalized differential problem associated to the formal one

For ω fixed, the problem associated to $(P_1(\omega))$ can be written as the well-formulated problem

$$(P_{1gen}(\omega)) \left\{ \begin{array}{l} \frac{\partial U(\omega)}{\partial t} = \mathcal{F}(U(\omega)), \\ \mathcal{R}_l(U(\omega)) = [W(\omega)_\varepsilon], \end{array} \right.$$

then

$$\left\{ \begin{array}{l} \frac{\partial U(\omega)}{\partial t} = [F_\eta(U(\omega))], \\ U(\omega)|_\gamma = [W(\omega) * \chi_\varepsilon]. \end{array} \right.$$

The problem associated to $(P_2(\omega))$ can be written as the well-formulated problem

$$(P_{2gen}(\omega)) : \frac{\partial V(\omega)}{\partial t} = 0, \mathcal{R}_l(V(\omega)) = [W(\omega)_\varepsilon],$$

so

$$\frac{\partial V(\omega)}{\partial t} = 0, V(\omega)|_\gamma = [W(\omega) * \chi_\varepsilon].$$

In terms of representatives, and thanks to the stability and restriction hypothesis, if we find $U(\omega)_{\varepsilon, \eta} \in C^\infty(\mathbb{R}^2)$ verifying

$$(P_1(\omega)_{(\varepsilon, \eta)}) \begin{cases} \frac{\partial U(\omega)_{\varepsilon, \eta}}{\partial t}(t, x) = F_\eta(U(\omega)_{\varepsilon, \eta}(t, x)), \\ U(\omega)_{\varepsilon, \eta}(t, l(t)) = (W(\omega) * \chi_\varepsilon)(t) \end{cases}$$

and if we prove that $(U(\omega)_{\varepsilon, \eta})_{\varepsilon, \eta} \in \mathcal{X}_2(\mathbb{R}^2)$, then $U(\omega) = [U(\omega)_{\varepsilon, \eta}]$ is a solution of $(P_1(\omega)_{gen})$.

As $\frac{\partial V(\omega)}{\partial t} = 0$, we have

$$V(\omega)_{\varepsilon, \eta}(t, x) = W(\omega) * \chi_\varepsilon(l^{-1}(x)).$$

4.2. Generalized problem

4.2.1. Solution to the parametrized regular problem

For ω fixed consider the family of regularized problems $(P_1(\omega)_{(\varepsilon, \eta)})$. We must prove that $(P_1(\omega)_{(\varepsilon, \eta)})$ has a unique smooth solution under the following assumptions

$$(H_{\varepsilon, \eta}) \begin{cases} \text{a) } l \in C^\infty(\mathbb{R}), l' > 0, l(\mathbb{R}) = \mathbb{R}, \\ \text{b) } F_\eta \in C^\infty(\mathbb{R}, \mathbb{R}), \\ \text{c) } W(\omega)_\varepsilon \in C^\infty(\mathbb{R}). \end{cases}$$

$$(H_1) : \exists p > 0, \forall l \in \mathbb{N}, \exists a_l > 0, \sup_{z \in \mathbb{R}; |\alpha| \leq l} |D^\alpha F_\eta(z)| \leq a_l(1 + r_\eta)^p.$$

one can prove that $(P_1(\omega)_{(\varepsilon, \eta)})$ admits a unique smooth solution $U(\omega)_{\varepsilon, \eta}$ such that

$$U(\omega)_{\varepsilon, \eta}(t, x) = (W(\omega) * \chi_\varepsilon)(l^{-1}(x)) + \int_{l^{-1}(x)}^t F_\eta(U(\omega)_{\varepsilon, \eta}(\zeta, x)) d\zeta$$

Theorem 4.1. *Under assumptions $(H_{\varepsilon, \eta})$, (H_1) , problem $(P_1(\omega)_{(\varepsilon, \eta)})$ has a unique solution, $U(\omega)_{\varepsilon, \eta}$, in $C^\infty(\mathbb{R}^2)$.*

4.2.2. Solution to (P_1)

Theorem 4.2. *Suppose that $U(\omega)_{\varepsilon, \eta}$ is the solution to problem $(P_1(\omega)_{(\varepsilon, \eta)})$ then problem $(P_1(\omega)_{gen})$ has a unique solution $U(\omega) = [U(\omega)_{\varepsilon, \eta}]$ in $\mathcal{A}(\mathbb{R}^2)$.*

The proof follows the same steps as the existence results which can be found in [3] (replacing $u_{\varepsilon, \eta}$ by $U(\omega)_{\varepsilon, \eta}$ and $F_\eta(x, y, u_{\varepsilon, \eta}(x, y))$ by $F_\eta(U(\omega)_{\varepsilon, \eta}(x, y))$).

Theorem 4.3. *Suppose that $U(\omega)_{\varepsilon, \eta}$ is the solution to problem $(P_1(\omega)_{(\varepsilon, \eta)})$ then problem $(P_1(\omega)_{gen})$ has a unique solution $U(\omega) = [U(\omega)_{\varepsilon, \eta}]$ in $\mathcal{A}_2(\mathbb{R}^2)$.*

Proof. $U(\omega)$ is the solution to $(P_1(\omega))_{(\varepsilon, \eta)}$ if $(U(\omega)_{\varepsilon, \eta})_{\varepsilon, \eta} \in \mathcal{X}_2(\mathbb{R}^2)$. We must prove that

$$\forall K \in \mathbb{R}^2, \forall l \in \mathbb{N}, (N_{K,l}^2(U(\omega)_{\varepsilon, \eta}))_{\varepsilon, \eta} \in A.$$

However

$$\|D^\alpha(U(\omega)_{\varepsilon, \eta})\|_{L^2(K)} \leq (\mu(K))^{1/2} \|D^\alpha(U(\omega)_{\varepsilon, \eta})\|_{L^\infty(K)}$$

and, as $(U(\omega)_{\varepsilon, \eta})_{\varepsilon, \eta} \in \mathcal{X}(\mathbb{R}^2)$, we have $(\|D^\alpha(U(\omega)_{\varepsilon, \eta})\|_\infty)_{\varepsilon, \eta} \in A$. Then

$$(\|D^\alpha(U(\omega)_{\varepsilon, \eta})\|_{L^2(K)})_{\varepsilon, \eta} = (N_{K,l}^2(U(\omega)_{\varepsilon, \eta}))_{\varepsilon, \eta} \in A.$$

Thus $U(\omega) \in \mathcal{A}_2(\mathbb{R}^2)$ and it is the solution to problem $(P_1(\omega)_{gen})$ in $\mathcal{A}_2(\mathbb{R}^2)$. Set

$$U : \Omega \rightarrow \mathcal{A}_2(\mathbb{R}^2), \omega \mapsto U(\omega).$$

Thus $U \in \mathcal{A}_2^\Omega(\mathbb{R}^2)$. □

Theorem 4.4. *The mapping U is the solution to problem (P_1) and it is almost surely unique in $\mathcal{A}_2^\Omega(\mathbb{R}^2)$.*

Proof. Since $U(\omega)$ is the unique solution to problem $(P_1(\omega)_{gen})$ in $\mathcal{A}_2(\mathbb{R}^2)$ thus almost surely in $\omega \in \Omega$, the map $\lambda \mapsto R_U(\lambda, (\cdot, \cdot), \omega) = U(\omega)_\lambda$, $(\lambda = (\varepsilon, \eta))$, belongs to $\mathcal{X}_2(\mathbb{R}^2)$ and it is a representative of $U(\omega)$ (i.e. $U(\omega) = [U(\omega)_\lambda]$). For fixed $\lambda = (\varepsilon, \eta) \in \Lambda$, the map

$$((x, y), \omega) \mapsto R_U(\lambda, (x, y), \omega) = U(\omega)_\lambda(x, y)$$

is jointly measurable on $\mathbb{R}^2 \times \Omega$. Then U is the solution to problem (P_1) almost surely unique in $\mathcal{A}_2^\Omega(\mathbb{R}^2)$. □

4.3. Limiting behavior of the solution

See [11], [12]. Take $W_\varepsilon = (W(\omega) * \chi_\varepsilon)$. We have $E(W_\varepsilon) = 0$ and $V(W_\varepsilon) = \sigma_\varepsilon^2 = \|\chi_\varepsilon\|_{L^2(\mathbb{R})}^2$. Then the variance of W_ε tends to infinity as ε tends to 0. That implies

Theorem 4.5. *There is a subsequence $\varepsilon_k \rightarrow 0$ such that μ -almost surely in $\omega \in \Omega$,*

$$\lim_{k \rightarrow 0} |R_V((\varepsilon_k, \eta), (t, x), \omega)| = \lim_{k \rightarrow 0} |V(\omega)_{\varepsilon_k, \eta}(t, x)| = \infty$$

for almost all $(x, y) \in \mathbb{R}^2$.

Proof. See [11] Corollary 1 and [12]. □

Suppose that $\lim_{|z| \rightarrow \infty} F(z) = L$. Define the function $M : \mathbb{R}^2 \rightarrow \mathbb{R}$ by $M(t, x) = tL$.

Theorem 4.6. *Under the assumptions above, every subsequence of $\varepsilon \rightarrow 0$ has a subsequence $\varepsilon_k \rightarrow 0$ such that for any compact set $K \Subset \mathbb{R}^2$*

$$\lim_{k \rightarrow 0} \|R_U((\varepsilon_k, \eta), (\cdot, \cdot), \omega) - R_V((\varepsilon_k, \eta), (\cdot, \cdot), \omega) - M\|_{L^1(K)} = 0$$

μ -almost surely.

That is

$$\lim_{k \rightarrow 0} \|U(\omega)_{\varepsilon_k, \eta} - V(\omega)_{\varepsilon_k, \eta} - M\|_{L^1(K)} = 0$$

μ -almost surely.

Proof. Take $\lambda = (\varepsilon, \eta)$. We have

$$\frac{\partial (U(\omega)_\lambda - V(\omega)_\lambda - M)}{\partial t} = \frac{\partial (U(\omega)_\lambda)}{\partial t} - \frac{\partial (V(\omega)_\lambda + M)}{\partial t}$$

and

$$\begin{aligned} & \frac{\partial (U(\omega)_\lambda)}{\partial t} - \frac{\partial (V(\omega)_\lambda + M)}{\partial t} \\ &= F(U(\omega)_\lambda) - L \\ &= (F(U(\omega)_\lambda) - F(V(\omega)_\lambda + M)) + (F(V(\omega)_\lambda + M) - L) \\ &= (U(\omega)_\lambda - V(\omega)_\lambda - M) \int_0^1 \frac{\partial F}{\partial z} (U(\omega)_\lambda + \sigma (U(\omega)_\lambda - V(\omega)_\lambda - M)) d\sigma \\ &+ (F(V(\omega)_\lambda + M) - L). \end{aligned}$$

So

$$\begin{aligned} & \left\| \frac{\partial (U(\omega)_\lambda - V(\omega)_\lambda - M)}{\partial t} \right\|_{L^1(K)} \\ & \leq \|U(\omega)_\lambda - V(\omega)_\lambda - M\|_{L^1(K)} \left\| \frac{\partial F}{\partial z} \right\|_{L^\infty(\mathbb{R})} + \|F(V(\omega)_\lambda + M) - L\|_{L^1(K)} \end{aligned}$$

By Theorem 4.5, there is a subsequence $\varepsilon_k \rightarrow 0$ such that μ -almost surely in $\omega \in \Omega$ almost everywhere $((t, x) \in \mathbb{R}^2)$, $\lim_{k \rightarrow 0} |V(\omega)_{\varepsilon_k, \eta}(t, x)| = \infty$.

As $\lim_{|z| \rightarrow \infty} F(z) = L$, we deduce that

$$\lim_{k \rightarrow 0} \|F(V(\omega)_\lambda + M) - L\|_{L^1(K)} = 0$$

almost everywhere.

Hence by Lebesgue's theorem and Gronwall's lemma the assertion follows. \square

Theorem 4.7. *Let $V \in \mathcal{D}'_\Omega(\mathbb{R}^2)$ be the distributional solution to the free equation (P_2) . Then the representative $U(\omega)_{\varepsilon, \eta}$ of the generalized solution to the nonlinear problem (P_1) converges to $V + M$ with respect to the strong topology of $\mathcal{D}'(\mathbb{R}^2)$, in probability as $\varepsilon \rightarrow 0$.*

Proof. Let q be one of the defining seminorms of the strong topology of $\mathcal{D}'(\mathbb{R}^2)$. According to Theorem 4.6, every subsequence of $\varepsilon \rightarrow 0$ has a subsequence $\varepsilon_k \rightarrow 0$ such that for any compact set $K \subseteq \mathbb{R}^2$

$$q(R_U((\varepsilon_k, \eta), (\cdot, \cdot), \omega) - R_V((\varepsilon_k, \eta), (\cdot, \cdot), \omega) - M) \rightarrow 0$$

almost surely. This is equivalent to convergence in probability. \square

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