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Screen slant lightlike submanifolds of golden semi-Riemannian manifolds

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Abstract. The aim of our paper is to introduce the notion of screen slant lightlike submanifolds of golden semi-Riemannian manifolds. We give some non-trivial examples of screen slant lightlike submanifolds and provide a characterization theorem of such submanifolds. Further, we obtain necessary and sufficient conditions for integrability of the distributions and investigate the geometry of the leaves of the foliation determined by the distributions. We also obtain a necessary and sufficient condition for the induced connection to be a metric connection.

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1. Introduction

A submanifold of a semi-Riemannian manifold is called a lightlike submanifold if the induced metric on it is degenerate, i.e. there exists a non zero $X \in \Gamma(TM)$ such that $g(X, Z) = 0$, for all $Z \in \Gamma(TM)$. The first studies on the geometry of lightlike submanifolds are given by Duggal and Bejancu [4]. The notion of screen lightlike submanifold in a semi-Riemannian manifold was introduced by K. L. Duggal and A. Bejancu in [4]. The geometry of slant and screen slant lightlike submanifolds has been studied in [6] and screen slant lightlike submanifold is a natural generalization of invariant and screen real lightlike submanifolds. Many authors have studied lightlike submanifolds in various spaces ([6], [17], [16]). In [15], authors established some equivalent conditions for integrability and totally geodesic foliation of distributions. In [18], authors introduced the notion of screen pseudo-slant lightlike submanifolds of indefinite Kaehler manifolds and gave characterization theorem with some non-trivial examples of such submanifolds.

The Golden ratio ψ is the real positive root of the equation $x^2 - x - 1 = 0$ (thus $\psi = \frac{1+\sqrt{5}}{2} \approx 1.618...$). Inspired by the Golden ratio, Crasmareanu and Hretcanu defined a golden structure $\tilde{P}$ which is a tensor field satisfying $\tilde{P^2} - \tilde{P} - I = 0$ on $\overline{M}$ [3]. The golden structure was inspired by the Golden ratio, which was described by Kepler (1571-1630). A Riemannian manifold

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$\overline{M}$ with a golden structure $\tilde{P}$ is called a golden Riemannian manifold and was studied in ([10], [3]). In [10], authors studied invariant submanifolds of a golden Riemannian manifold. In [19], authors studied the geometry of invariant lightlike submanifolds of golden semi-Riemannian manifolds and investigated the geometry of such submanifolds.

In [14], Poyraz and Yasar studied lightlike hypersurface of a golden semi-Riemannian manifold and they proved that there is no radical anti-invariant lightlike hypersurface of a golden semi-Riemannian manifold. They also studied screen semi-invariant and screen conformal screen semi-invariant lightlike hypersurfaces of a golden semi-Riemannian manifold. Transversal and screen transversal lightlike submanifolds of metallic semi-Riemannian manifolds were studied in ([7], [9]). In [13], authors proved that there is no radical anti-invariant lightlike submanifold of a golden semi-Riemannian manifold. In [8], the author studied the geometry of screen transversal, radical screen transversal and screen transversal anti-invariant lightlike submanifolds of golden semi-Riemannian manifolds and investigated the geometry of the distributions.

In [1], B. E. Acet introduced the notion of screen pseudo-slant lightlike submanifolds of a golden semi-Riemannian manifold and studied the geometry of such submanifolds. In [12], N. Onen Poyraz studied golden GCR-lightlike submanifolds of golden semi-Riemannian manifolds and found equivalent conditions for integrability and totally geodesic foliation of distributions. The purpose of this paper is to study screen slant lightlike submanifold of golden semi-Riemannian manifolds. The paper is arranged as follows. In Section 2, some definitions and basic results about lightlike submanifolds and golden semi-Riemannian manifold are given. In Section 3, we study screen slant lightlike submanifolds of a golden semi-Riemannian manifold, giving examples and investigate the integrability of the distributions. In Section 4, we obtain necessary and sufficient conditions of totally geodesic foliation on screen slant lightlike submanifolds of golden semi-Riemannian manifolds.

2. Preliminaries

Let $\overline{M}$ be a C^{∞}-differentiable manifold. If a (1, 1) type tensor field $\tilde{P}$ on $\overline{M}$ satisfies the following equation

$$\tilde{P}^2 = \tilde{P} + I, \tag{2.1}$$

then $\tilde{P}$ is called a golden structure on $\overline{M}$, where I is the identity transformation. Let $(\overline{M}, \overline{g})$ be a semi-Riemannian manifold and $\tilde{P}$ be a golden structure on $\overline{M}$. If $\tilde{P}$ satisfies the following equation

$$\overline{g}(\tilde{P}U, W) = \overline{g}(U, \tilde{P}W), \tag{2.2}$$

then $(\overline{M}, \overline{g}, \tilde{P})$ is called a golden semi-Riemannian manifold [11]. Also, if $\tilde{P}$ is integrable then we have [3]

$$\overline{\nabla}_U \tilde{P}W = \tilde{P}\overline{\nabla}_U W. \tag{2.3}$$

Now, from (2.2), we get

$$\overline{g}(\tilde{P}U, \tilde{P}W) = \overline{g}(\tilde{P}U, W) + \overline{g}(U, W), \tag{2.4}$$

for all $U, W \in \Gamma(T\overline{M})$.
Let $(\overline{M}, \overline{g})$ be a real $(m+n)$-dimensional semi-Riemannian manifold of constant index q, such that $m, n \geq 1$, $1 \leq q \leq m+n-1$ and (M, g) be an m-dimensional submanifold of $\overline{M}$, where g is the induced metric of $\overline{g}$ on M. If g is degenerate on the tangent bundle TM of M, then M is called a lightlike submanifold [4] of $\overline{M}$. Let $S(TM)$ be a screen distribution which is a semi-Riemannian complementary distribution of $RadTM$ in TM, that is

$$TM = RadTM \oplus_{orth} S(TM). \tag{2.5}$$

Consider a screen transversal vector bundle $S(TM^{\perp})$, which is a semi-Rieman nian complementary vector bundle of $RadTM$ in $TM^{\perp}$. Then, we have

Theorem 2.1 ([4]). *Let $(M, g, S(TM), S(TM^{\perp}))$ be an r-lightlike submanifold of a semi-Riemannian manifold $(\overline{M}, \overline{g})$. Suppose U is a coordinate neighbourhood of M and $\{\xi_i\}$, $i \in \{1, 2, ..., r\}$ is a basis of $\Gamma(Rad(TM|_U))$. Then, there exist a complementary vector subbundle $ltr(TM)$ of $Rad(TM)$ in $S(TM^{\perp})^{\perp}$ and a basis $\{N_i\}$, $i \in \{1, 2, .., r\}$ of $\Gamma(ltr(TM|_U))$ such that $\overline{g}(N_i, \xi_j) = \delta_{ij}$ and $\overline{g}(N_i, N_j) = 0$, for any $i, j \in \{1, 2, ..., r\}$.*

Let $tr(TM)$ be complementary (but not orthogonal) vector bundles to TM in $T\overline{M}|_M$. Then from the above theorem, we obtain

$$tr(TM) = ltr(TM) \oplus_{orth} S(TM^{\perp}), \tag{2.6}$$

$$T\overline{M}|_M = TM \oplus tr(TM), \tag{2.7}$$

$$T\overline{M}|_M = S(TM) \oplus_{orth} [RadTM \oplus ltr(TM)] \oplus_{orth} S(TM^{\perp}). \tag{2.8}$$

The Gauss and Weingarten formulae are given as

$$\overline{\nabla}_X Y = \nabla_X Y + h(X, Y), \tag{2.9}$$

$$\overline{\nabla}_X V = -A_V X + \nabla^t_X V, \tag{2.10}$$

for all $X, Y \in \Gamma(TM)$ and $V \in \Gamma(tr(TM))$, where $\{\nabla_X Y, A_V X\}$ belong to $\Gamma(TM)$ and $\{h(X, Y), \nabla^t_X V\}$ belong to $\Gamma(tr(TM))$. ∇ and ∇^t are linear connections on M and on the vector bundle $tr(TM)$, respectively. From (2.9) and (2.10), for any $X, Y \in \Gamma(TM)$, $N \in \Gamma(ltr(TM))$ and $W \in \Gamma(S(TM^{\perp}))$, we have

$$\overline{\nabla}_X Y = \nabla_X Y + h^l(X, Y) + h^s(X, Y), \tag{2.11}$$

(2.12) $$\overline{\nabla}_X N = -A_N X + \nabla^l_X N + D^s(X, N),$$

(2.13) $$\overline{\nabla}_X W = -A_W X + \nabla^s_X W + D^l(X, W),$$

where

$$h^l(X,Y) = L(h(X,Y)), h^s(X,Y) = S(h(X,Y)),$$
$$D^l(X,W) = L(\nabla^t_X W), D^s(X,N) = S(\nabla^t_X N).$$

L and S are the projection morphisms of $tr(TM)$ on $ltr(TM)$ and $S(TM^\perp)$, respectively. ∇^l and ∇^s are linear connections on $ltr(TM)$ and $S(TM^\perp)$ called the lightlike connection and screen transversal connection on M, respectively. Also by using (2.9), (2.11)-(2.13) and metric connection $\overline{\nabla}$, we obtain

(2.14) $$\overline{g}(h^s(X,Y), W) + \overline{g}(Y, D^l(X,W)) = g(A_W X, Y),$$

(2.15) $$\overline{g}(D^s(X,N), W) = \overline{g}(N, A_W X).$$

Now, denote the projection of TM on $S(TM)$ by $\tilde{S}$. Then from the decomposition of the tangent bundle of a lightlike submanifold, for any $X, Y \in \Gamma(TM)$ and $\xi \in \Gamma(RadTM)$, we have

(2.16) $$\nabla_X \tilde{S}Y = \nabla^*_X \tilde{S}Y + h^*(X, \tilde{S}Y),$$

(2.17) $$\nabla_X \xi = -A^*_\xi X + \nabla^{*t}_X \xi.$$

By using the above equations, we obtain

(2.18) $$\overline{g}(h^l(X, \tilde{S}Y), \xi) = g(A^*_\xi X, \tilde{S}Y).$$

It is important to note that in general ∇ is not a metric connection on M. Since $\overline{\nabla}$ is a metric connection, by using (2.11), we get

(2.19) $$(\nabla_X g)(Y,Z) = \overline{g}(h^l(X,Y), Z) + \overline{g}(h^l(X,Z), Y),$$

for all $X, Y, Z \in \Gamma(T\overline{M})$.

Definition 2.2 ([5])**.** A lightlike submanifold (M, g) of a semi-Riemannian manifold $(\overline{M}, \overline{g})$ is said to be totally umbilical in $\overline{M}$ if there is a smooth transversal vector field $H \in \Gamma(tr(TM))$ on M, called the transversal curvature vector field of M, such that for any $X, Y \in \Gamma(TM)$,

(2.20) $$h(X,Y) = g(X,Y)H.$$

In case $H = 0$, M is called totally geodesic. Now, using (2.11) and (2.20), we conclude that M is totally umbilical if and only if there exist smooth vector fields $H^l \in \Gamma(ltr(TM))$ and $H^s \in \Gamma(S(TM^\perp))$ such that

(2.21) $$h^l(X,Y) = g(X,Y)H^l, h^s(X,Y) = g(X,Y)H^s \textit{and } D^l(X,W) = 0,$$

for any $X, Y \in \Gamma(TM)$ and $W \in \Gamma(S(TM^\perp))$.

3. Screen Slant Lightlike Submanifolds

In this section, we study screen slant lightlike submanifolds of golden semi-Riemannian manifolds. First we give the following lemma which will be useful to define slant notion on the screen distribution.

Lemma 3.1. *Let M be a $2q$-lightlike submanifold of a golden semi-Riemannian manifold $\overline{M}$ with constant index $2q$ such that $2q < dim(M)$. Then the screen distribution $S(TM)$ of lightlike submanifold M is Riemannian.*

Proof. The proof of the above lemma is similar to Lemma 6.8.1 of [6]. So we skip it. □

Definition 3.2. Let M be a $2q$- lightlike submanifolds of golden semi-Rieman nian manifolds $\overline{M}$ of index $2q < dim(M)$. Then we say that M is a screen slant lightlike submanifold of $\overline{M}$ if the following conditions are satisfied:
(i) $RadTM$ is invariant with respect to $\tilde{P}$, i.e. $\tilde{P}(RadTM) = RadTM$,
(ii) the distribution $S(TM)$ is slant with angle $\theta(\neq 0)$, i.e. for each $x \in M$ and each non-zero vector $X \in \Gamma S(TM)$, the angle θ between $\tilde{P}X$ and the vector subspace $S(TM)$ is a non-zero constant, which is independent of the choice of $x \in M$ and $X \in \Gamma S(TM)$.
This constant angle θ is called the slant angle of distribution $S(TM)$. A screen slant lightlike submanifold is said to be proper if it is neither invariant $(\theta = 0)$ nor screen real $(\theta = \frac{\pi}{2})$. On the other hand, the screen transversal bundle $S(TM^{\perp})$ has the following decomposition

$$S(TM^{\perp}) = F(S(TM)) \oplus_{orth} \nu, \tag{3.1}$$

where $F(S(TM))$ is the transversal part of $\tilde{P}(S(TM))$ and distribution ν is non-degenerate complementary orthogonal to $F(S(TM))$.

Let M be a screen slant-lightlike submanifold of a golden semi-Riemannian manifold $(\overline{M}, \overline{g}, \tilde{P})$. Thus for any $X \in \Gamma(TM)$, we derive

$$\tilde{P}X = PX + FX, \tag{3.2}$$

where PX and FX are tangential and transversal parts of $\tilde{P}X$, respectively. Also for any $V \in \Gamma(tr(TM))$, we write

$$\tilde{P}V = BV + CV, \tag{3.3}$$

where BV and CV are tangential and transversal parts of $\tilde{P}V$, respectively. Now, we denote the projections on $RadTM$ and $S(TM)$ in TM by P_1 and P_2, respectively. Similarly, we denote the projections of $tr(TM)$ on $ltr(TM)$, $F(S(TM))$ and ν by Q_1 Q_2 and Q_3, respectively. Then for any $X \in \Gamma(TM)$, we get

$$X = P_1X + P_2X. \tag{3.4}$$

Now applying $\tilde{P}$ to (3.4), we have

$$\tilde{P}X = \tilde{P}P_1X + \tilde{P}P_2X, \tag{3.5}$$

which gives

$$\tilde{P}X = \tilde{P}P_1X + PP_2X + FP_2X, \tag{3.6}$$

where PP_2X (resp. FP_2X) denotes the tangential (resp. transversal) component of $\tilde{P}P_2X$. Thus we get $\tilde{P}P_1X \in \Gamma(RadTM)$, $PP_2X \in \Gamma(S(TM))$ and $FP_2X \in \Gamma(F(S(TM))$. Also, for any $W \in \Gamma(tr(TM))$, we have

$$W = Q_1W + Q_2W + Q_3W. \tag{3.7}$$

Applying $\tilde{P}$ to (3.7), we obtain

$$\tilde{P}W = \tilde{P}Q_1W + \tilde{P}Q_2W + \tilde{P}Q_3W, \tag{3.8}$$

which gives

$$\tilde{P}W = \tilde{P}Q_1W + BQ_2W + CQ_2W + \tilde{P}Q_3W, \tag{3.9}$$

where BQ_2W (resp.CQ_2W) denotes the tangential (resp. transversal) component of $\tilde{P}Q_2W$. Thus we get $\tilde{P}Q_1W \in \Gamma(ltr(TM))$, $BQ_2W \in \Gamma(S(TM))$, $CQ_2W \in \Gamma(F(S(TM))$ and $\tilde{P}Q_3W \in \Gamma(\nu)$.

Proposition 3.3. *Let M be a screen slant lightlike submanifold of a golden semi-Riemannian manifold $(\overline{M}, \overline{g}, \tilde{P})$. Then the distribution ν is invariant with respect to $\tilde{P}$.*

Proof. Using (2.2), we obtain $\overline{g}(\tilde{P}V, \xi) = \overline{g}(V, \tilde{P}\xi) = 0$, which show that $\tilde{P}V$ does not belong to $ltr(TM)$. Also, $\overline{g}(\tilde{P}V, N) = \overline{g}(V, \tilde{P}N) = 0$, which show that $\tilde{P}V$ does not belong to $Rad(TM)$. Now, from (2.4), we have $\overline{g}(\tilde{P}V, \tilde{P}N) = \overline{g}(\tilde{P}V, N) + \overline{g}(V, N) = 0$, $\overline{g}(\tilde{P}V, \tilde{P}\xi) = \overline{g}(\tilde{P}V, \xi) + \overline{g}(V, \xi) = 0$ and $\overline{g}(\tilde{P}V, Z) = \overline{g}(V, \tilde{P}Z) = 0$, which show that $\tilde{P}V$ does not belong to $S(TM)$. Also, from (2.4), we get $\overline{g}(\tilde{P}V, \tilde{P}Z) = \overline{g}(\tilde{P}V, Z) + \overline{g}(V, Z) = 0$, $\overline{g}(\tilde{P}V, W) = \overline{g}(V, \tilde{P}W) = \overline{g}(V, \tilde{P}^2Z) = 0$ and $\overline{g}(\tilde{P}V, \tilde{P}W) = \overline{g}(\tilde{P}V, W) + \overline{g}(V, W) = 0$, which show that $\tilde{P}V$ does not belong $F(S(TM))$, for any $V \in \Gamma(\nu)$, $\xi \in \Gamma Rad(TM)$, $N \in \Gamma ltr(TM)$, $Z \in \Gamma S(TM)$ and $W \in \Gamma F(S(TM))$. Therefore, the distribution ν is invariant with respect to $\tilde{P}$. □

Lemma 3.4. *Let (M, g) be a screen slant lightlike submanifold of a golden semi-Riemannian manifold $(\overline{M}, \overline{g}, \tilde{P})$. Then we have*

$$(\nabla_X P)Y = A_{FY}X + Bh(X, Y), \tag{3.10}$$

$$(\nabla^t_X F)Y = Ch(X, Y) - h(X, PY), \tag{3.11}$$

$$P^2X = PX + X - BFX, \tag{3.12}$$

$$FX = FPX + CFX, \tag{3.13}$$

$$g(PX, Y) - g(X, PY) = g(X, FY) - g(FX, Y), \tag{3.14}$$

$$\begin{aligned} g(PX, PY) =& g(PX, Y) + g(X, Y) + g(FX, Y) - g(PX, FY) \\ & - g(FX, PY) - g(FX, FY), \end{aligned} \tag{3.15}$$

where $(\nabla_X P)Y = \nabla_X PY - P\nabla_X Y$ *and* $(\nabla^t_X F)Y = \nabla^t_X FY - F\nabla_X Y$*, for all* $X, Y \in \Gamma(TM)$*.*

Proof. Using (2.3), (2.9), (2.10), (3.2) and (3.3), on comparing tangential and transversal parts of the resulting equation, we obtain (3.10) and (3.11). Applying $\tilde{P}$ to (3.2), using (2.1) and (3.3), taking tangential and transversal parts of the resulting equation, we get (3.12) and (3.13). Finally, using (2.2), (2.4) and (3.2), we obtain (3.14) and (3.15). □

Proposition 3.5. *Let* (M, g) *be a screen slant lightlike submanifold of a golden semi-Riemannian manifold* $(\overline{M}, \overline{g}, \tilde{P})$*. Then* P *is a golden structure on* M *if and only if* $FX = 0$*.*

Proof. Let P be a golden structure on M then, from (3.12), $FX = 0$. Conversely, let $FX = 0$ then our result follows from (3.12). □

Example 3.6. Let $(\mathbb{R}^{10}_2, \overline{g}, \tilde{P})$ be a golden semi-Riemannian manifold, where the metric $\overline{g}$ is of signature $(-, -, +, +, +, +, +, +, +, +)$ with respect to the canonical basis $\{\partial x^1, \partial x^2, \partial x^3, \partial x^4, \partial x^5, \partial x^6, \partial x^7, \partial x^8, \partial x^9, \partial x^{10}\}$ and let $(x^1, x^2, x^3, x^4, x^5, x^6, x^7, x^8, x^9, x^{10})$ be the standard coordinate system of $\mathbb{R}^{10}_2$. Take $\tilde{P}(\partial x^1, ..., \partial x^{10}) = (\psi\partial x^1, \psi\partial x^2, \psi\partial x^3, \psi\partial x^4, \psi\partial x^5, \psi\partial x^6, (1-\psi)\partial x^7, (1-\psi)\partial x^8, (1-\psi)\partial x^9, (1-\psi)\partial x^{10})$, where $\psi = \frac{1+\sqrt{5}}{2}$ *and* $(1-\psi) = \frac{1-\sqrt{5}}{2}$ are the roots of the equation $x^2 - x - 1 = 0$. Thus, $\tilde{P^2} = \tilde{P} + I$ and $\tilde{P}$ is a golden structure on $\mathbb{R}^{10}_2$. Suppose M is a submanifold of $\mathbb{R}^{10}_2$ given by $x^1 = u^1$, $x^2 = u^2$, $x^3 = \psi u^3$, $x^4 = \psi u^4$, $x^5 = u^1 \cos\alpha - u^2 \sin\alpha$, $x^6 = u^1 \sin\alpha + u^2 \cos\alpha$, $x^7 = (1-\psi)u^3$, $x^8 = (1-\psi)u^4$, $x^9 = x^{10} = 0$. The local frame of TM is given by $\{Z_1, Z_2, Z_3, Z_4\}$, where $Z_1 = \partial x^1 + \cos\alpha\partial x^5 + \sin\alpha\partial x^6$, $Z_2 = \partial x^2 - \sin\alpha\partial x^5 + \cos\alpha\partial x^6$, $Z_3 = \psi\partial x^3 + (1-\psi)\partial x^7$, $Z_4 = \psi\partial x^4 + (1-\psi)\partial x^8$. Hence $RadTM = span\{Z_1, Z_2\}$ and $S(TM) = span\{Z_3, Z_4\}$.
Now $ltr(TM)$ is spanned by $N_1 = \frac{1}{2}(-\partial x^1 + \cos\alpha\partial x^5 + \sin\alpha\partial x^6)$, $N_2 = \frac{1}{2}(-\partial x^2 - \sin\alpha\partial x^5 + \cos\alpha\partial x^6)$ and $S(TM^\perp)$ is spanned by $W_1 = (1-\psi)\partial x^3 - \psi\partial x^7$, $W_2 = (1-\psi)\partial x^4 - \psi\partial x^8$, $W_3 = \partial x^9 + \partial x^{10}$, $W_4 = \partial x^9 - \partial x^{10}$.
It follows that $\tilde{P}Z_1 = \psi Z_1$ and $\tilde{P}Z_2 = \psi Z_2$, which implies that $RadTM$ is invariant, i.e. $\tilde{P}RadTM = RadTM$ and $\tilde{P}N_1 = \frac{1}{2}\psi N_1$, $\tilde{P}N_2 = \frac{1}{2}\psi N_2$, which shows $\tilde{P}ltr(TM) = ltr(TM)$ and the Distribution $\nu = span\{W_3, W_4\}$, $\tilde{P}W_3 = (1-\psi)W_3$ and $\tilde{P}W_4 = (1-\psi)W_4$, which implies that ν is invariant, i.e. $\tilde{P}\nu = \nu$ and $S(TM) = span\{Z_3, Z_4\}$ is a slant distribution with slant angle $\theta = \arccos(\frac{4}{\sqrt{21}})$. Hence M is a screen slant 2-lightlike submanifold of $\mathbb{R}^{10}_2$.

Example 2. Let $(\mathbb{R}_2^{10}, \overline{g}, \tilde{P})$ be a golden semi-Riemannian manifold, where the metric $\overline{g}$ is of signature $(-,-,+,+,+,+,+,+,+,+)$ with respect to the canonical basis $\{\partial x^1, \partial x^2, \partial x^3, \partial x^4, \partial x^5, \partial x^6, \partial x^7, \partial x^8, \partial x^9, x^{10},\}$ and let $(x^1, x^2, x^3, x^4, x^5, x^6, x^7, x^8, x^9, x^{10})$ be the standard coordinate system of $\mathbb{R}_2^{10}$.
Taking $\tilde{P}(\partial x^1, ..., \partial x^{10}) = (\psi \partial x^1, (1-\psi)\partial x^2, (1-\psi)\partial x^3, \psi \partial x^4, (1-\psi)\partial x^5, (1-\psi)\partial x^6, \psi \partial x^7, \psi \partial x^8, \psi \partial x^9, \psi \partial x^{10})$, where $\psi = \frac{1+\sqrt{5}}{2}$ *and* $(1-\psi) = \frac{1-\sqrt{5}}{2}$ are the roots of the equation $x^2 - x - 1 = 0$. Thus $\tilde{P^2} = \tilde{P} + I$ and $\tilde{P}$ is a golden structure on $\mathbb{R}_2^{10}$. Suppose M is a submanifold of $\mathbb{R}_2^{10}$ given by $x^1 = x^4 = u^1$, $x^2 = x^3 = u^2$, $x^5 = u^3$, $x^6 = u^4$, $x^7 = \sin u^3$, $x^8 = -\sin u^4$, $x^9 = \cos u^4$, $x^{10} = \cos u^3$. The local frame of TM is given by $\{Z_1, Z_2, Z_3, Z_4\}$, where $Z_1 = \partial x^1 + \partial x^4$, $Z_2 = \partial x^2 + \partial x^3$, $Z_3 = \partial x^5 + \cos u^3 \partial x^7 - \sin u^3 \partial x^{10}$, $Z_4 = \partial x^6 - \cos u^4 \partial x^8 - \sin u^4 \partial x^9$.
Hence $RadTM = span\,\{Z_1, Z_2\}$ and $S(TM) = span\,\{Z_3, Z_4\}$.
Now $ltr(TM)$ is spanned by $N_1 = -\frac{1}{2}(\partial x^1 - \partial x^4)$, $N_2 = -\frac{1}{2}(\partial x^2 - \partial x^3)$ and $S(TM^\perp)$ is spanned by $W_1 = -\partial x^5 + \cos u^3 \partial x^7 - \sin u^3 \partial x^{10}$, $W_2 = -\partial x^6 - \cos u^4 \partial x^8 - \sin u^4 \partial x^9$, $W_3 = -\sin u^3 \partial x^7 - \cos u^3 \partial x^{10}$, $W_4 = \sin u^4 \partial x^8 - \cos u^4 \partial x^9$.
It follows that $\tilde{P}Z_1 = \psi Z_1$ and $\tilde{P}Z_2 = (1-\psi)Z_2$, which implies that $RadTM$ is invariant, i.e. $\tilde{P}RadTM = RadTM$ and $\tilde{P}N_1 = -\frac{1}{2}\psi N_1$, $\tilde{P}N_2 = -\frac{1}{2}(1-\psi)N_2$, which shows $\tilde{P}ltr(TM) = ltr(TM)$ and the distribution $\nu = span\{W_3, W_4\}$, $\tilde{P}W_3 = \psi W_3$ and $\tilde{P}W_4 = \psi W_4$, which implies $\tilde{P}\nu = \nu$, i.e. ν is invariant and $S(TM) = span\{Z_3, Z_4\}$ is a slant distribution with slant angle $\theta = \arccos(1/\sqrt{6})$. Hence M is a screen slant 2-lightlike submanifold of $\mathbb{R}_2^{10}$.

Theorem 3.7. *Let M be a $2q$-lightlike submanifold of a golden semi-Rieman nian manifold $\overline{M}$. Then M is a screen slant lightlike submanifold of $\overline{M}$ if and only if*

(i) $ltr(TM)$ is invariant with respect to $\tilde{P}$,

(ii) there exists a constant $\lambda \in [0,1)$ such that $P^2X = \lambda(PX + X)$,

for any $X \in \Gamma(S(TM))$. Moreover, in this case $\lambda = \cos^2\theta$ and θ is the slant angle of $S(TM)$.

Proof. Let M be a screen slant lightlike submanifold of a golden semi-Rieman nian manifold $\overline{M}$. Then the distribution $RadTM$ is invariant with respect to $\tilde{P}$. Now for any $N \in \Gamma(ltr(TM))$ and $X \in \Gamma(S(TM))$, using (2.2) and (3.6), we obtain $\overline{g}(\tilde{P}N, X) = \overline{g}(N, \tilde{P}X) = \overline{g}(N, PP_2X + FP_2X) = 0$. Thus $\tilde{P}N$ does not belong to $\Gamma(S(TM))$. For any $N \in \Gamma(ltr(TM))$ and $W \in \Gamma(S(TM^\perp))$, from (2.2) and (3.9), we have $\overline{g}(\tilde{P}N, W) = \overline{g}(N, \tilde{P}W) = \overline{g}(N, BQ_2W + CQ_2W + \tilde{P}Q_3W) = 0$. Hence, we conclude that $\tilde{P}N$ does not belong to $\Gamma(S(TM^\perp))$. Now suppose that $\tilde{P}N \in \Gamma(RadTM)$. Then $\tilde{P}(\tilde{P}N) = \tilde{P}^2N = \tilde{P}N + N \in \Gamma(Rad(TM) + ltr(TM))$, which contradicts that RadTM is invariant. Thus $ltr(TM)$ is invariant with respect to $\tilde{P}$.
Now for any $X \in \Gamma(S(TM))$ we have $|PX| = |\tilde{P}X| \cos\theta$, which implies

$$\cos\theta = \frac{|PX|}{|\tilde{P}X|}. \tag{3.16}$$

In view of (3.16), we get $\cos^2\theta = \frac{|PX|^2}{|\tilde{P}X|^2} = \frac{g(PX,PX)}{g(\tilde{P}X,\tilde{P}X)} = \frac{g(X,P^2X)}{g(X,\tilde{P^2}X)}$, which gives

$$g(X, P^2X) = \cos^2\theta g(X, \tilde{P^2}X). \tag{3.17}$$

Since M is a screen slant lightlike submanifold, $\cos^2\theta = \lambda(constant) \in [0, 1)$ and therefore from (3.17), we get

$$g(X, P^2X) = \lambda g(X, \tilde{P^2}X) = g(X, \lambda\tilde{P^2}X) = g(X, \lambda(\tilde{P}X + X)),$$

which implies

$$g(X, P^2X - \lambda(PX + X)) = 0. \tag{3.18}$$

Since

$$P^2X - \lambda(PX + X) \in \Gamma(S(TM))$$

and the induced metric $g = g|_{S(TM)\times S(TM)}$ is non-degenerate (positive definite) from (3.18), we have

$$(P^2X - \lambda(PX + X)) = 0,$$

which implies

$$P^2X = \lambda(PX + X), \tag{3.19}$$

for all $X \in \Gamma(S(TM))$. This proves (ii)
Conversely, suppose that conditions (i) and (ii) are satisfied. We can show that $RadTM$ is invariant in similar way to the proof that $ltr(TM)$ is invariant. Now

$$\begin{aligned} \cos\theta &= \frac{g(\tilde{P}X, PX)}{|\tilde{P}X||PX|} = \frac{g(X, \tilde{P}PX)}{|\tilde{P}X||PX|} = \frac{g(X, P^2X)}{|\tilde{P}X||PX|} = \frac{g(X, \lambda(PX + X))}{|\tilde{P}X||PX|} \\ &= \frac{g(X, \lambda(\tilde{P}X + X))}{|\tilde{P}X||PX|} = \lambda\frac{g(X, \tilde{P^2}X)}{|\tilde{P}X||PX|} = \lambda\frac{g(\tilde{P}X, \tilde{P}X)}{|\tilde{P}X||PX|}. \end{aligned}$$

From the above equation, we get

$$\cos\theta = \lambda\frac{|\tilde{P}X|}{|PX|}. \tag{3.20}$$

Therefore (3.16) and (3.20), give $\cos^2\theta = \lambda(constant)$. Hence M is a screen slant lightlike submanifold. □

Corollary 3.8. *Let M be a screen slant lightlike submanifold of a golden semi-Riemannian manifold $\overline{M}$ with slant angle θ, then for any $X, Y \in \Gamma(S(TM))$, we have*
(i) $g(PX, PY) = \cos^2\theta\ (g(X, Y) + g(X, PY))$,
(ii) $g(FX, FY) = \sin^2\theta\ (g(X, Y) + g(PX, Y))$.

Proof. From (2.2), (3.2) and (3.19), we obtain
$g(PX, PY) = g(X, \lambda(PY + Y)) = \cos^2\theta(g(X, Y) + g(X, PY))$.
Moreover, from (2.2), (3.2) and part (i) of Corollary (3.8), we get

$$g(FX, FY) = g(X, Y) + g(PX, Y) - g(PX, PY) = \sin^2\theta(g(X, Y) + g(PX, Y)).$$

Hence, the proof is complete. □

Theorem 3.9. *Let M be a screen slant lightlike submanifold of a golden semi-Riemannian manifold $\overline{M}$. Then the following statements are equivalent:*
(i) $Rad(TM)$ is integrable,
(ii) $\overline{g}(\nabla_X \tilde{P}Y - \nabla_Y \tilde{P}X, PZ) + \overline{g}(h^s(X, \tilde{P}Y) - h^s(Y, \tilde{P}X), FZ) = \overline{g}(\nabla_X \tilde{P}Y - \nabla_Y \tilde{P}X, Z)$,
(iii) $\overline{g}(h^l(Y, PZ) - h^l(Y, Z) + D^l(Y, FZ), \tilde{P}X) = \overline{g}(h^l(X, PZ) - h^l(X, Z) + D^l(X, FZ), \tilde{P}Y)$,
for all $X, Y \in \Gamma(RadTM)$ and $Z \in \Gamma(S(TM))$.

Proof. Let M be a screen slant lightlike submanifold of a golden semi-Riemannian manifold $\overline{M}$. Then the distribution $RadTM$ is integrable if and only if

$$\overline{g}([X, Y], Z) = 0, \tag{3.21}$$

for all $X, Y \in \Gamma(Rad(TM))$ and $Z \in \Gamma(S(TM))$.
(i) ⇒ (ii) From (2.3), (2.4), (2.11), (3.6) and (3.21), we obtain

$$\begin{aligned} \overline{g}(\nabla_X \tilde{P}Y - \nabla_Y \tilde{P}X, PZ) + \overline{g}(h^s(X, \tilde{P}Y) - h^s(Y, \tilde{P}X), FZ) \\ = \overline{g}(\nabla_X \tilde{P}Y - \nabla_Y \tilde{P}X, Z). \end{aligned} \tag{3.22}$$

(ii) ⇒ (iii) Since $\overline{\nabla}$ is a metric connection and using (2.11), (3.6) in (3.22), we get

$$-\overline{g}(\tilde{P}Y, \overline{\nabla}_X \tilde{P}Z) + \overline{g}(\tilde{P}X, \overline{\nabla}_Y \tilde{P}Z) = -\overline{g}(\tilde{P}Y, \overline{\nabla}_X Z) + \overline{g}(\tilde{P}X, \overline{\nabla}_Y \tilde{P}Z). \tag{3.23}$$

Now, from (2.11), (2.13), (3.6) and (3.23), we obtain

$$\begin{aligned} \overline{g}(h^l(Y, PZ) - h^l(Y, Z) + D^l(Y, FZ), \tilde{P}X) = \overline{g}(h^l(X, PZ) - h^l(X, Z) \\ + D^l(X, FZ), \tilde{P}Y), \end{aligned} \tag{3.24}$$

for all $X, Y \in \Gamma(Rad(TM))$ and $Z \in \Gamma(S(TM))$.
(iii) ⇒ (i) Using (2.11), (2.13) and (3.6) in (3.24), we get (3.23). Now since $\overline{\nabla}$ is a metric connection, from (2.3), (2.4) and (3.23), we obtain (3.21). Hence the proof is completed. □

Theorem 3.10. *Let M be a screen slant lightlike submanifold of a golden semi-Riemannian manifold $\overline{M}$. Then the following statements are equivalent:*
(i) $S(TM)$ is integrable,
(ii)

$$\begin{aligned} &\overline{g}(\nabla_X PY - A_{FY}X - \nabla_Y PX + A_{FX}Y, \tilde{P}N) \\ =&\overline{g}(\nabla_X PY - A_{FY}X - \nabla_Y PX + A_{FX}Y, N), \end{aligned}$$

(iii)

$$\begin{aligned}&\overline{g}(A_NY - A_{\tilde{P}N}Y, PX) + \overline{g}(A_{\tilde{P}N}X - A_NY, PY)\\ =&\overline{g}(D^s(Y,N) - D^s(Y,\tilde{P}N), FX) + \overline{g}(D^s(X,\tilde{P}N) - D^s(X,N), FY)\end{aligned}$$

for all $X, Y \in \Gamma(S(TM))$ *and* $N \in \Gamma ltr(TM)$*.*

Proof. Let M be a screen slant lightlike submanifold of a golden semi-Rieman nian manifold $\overline{M}$. Then the distribution $S(TM)$ is integrable if and only if

$$\overline{g}([X,Y], N) = 0, \tag{3.25}$$

for all $X, Y \in \Gamma(S(TM))$ and $N \in \Gamma(ltr(TM))$.
(i) $\Rightarrow$ (ii) From (2.3), (2.4), (2.11), (2.13), (3.6) and (3.25), we obtain

$$\begin{aligned}\overline{g}(\nabla_X PY - A_{FY}X - \nabla_Y PX + A_{FX}Y, \tilde{P}N) = \overline{g}(\nabla_X PY - A_{FY}X -\\ \nabla_Y PX + A_{FX}Y, N).\end{aligned} \tag{3.26}$$

(ii) $\Rightarrow$ (iii) Since $\overline{\nabla}$ is a metric connection and using (2.11), (2.13), (3.6) in (3.26), we get

$$-\overline{g}(\tilde{P}Y, \overline{\nabla}_X \tilde{P}N) + \overline{g}(\tilde{P}X, \overline{\nabla}_Y \tilde{P}N) = -\overline{g}(\tilde{P}Y, \overline{\nabla}_X N) + \overline{g}(\tilde{P}X, \overline{\nabla}_Y N). \tag{3.27}$$

From (2.12), (3.6) and (3.27), we obtain

$$\begin{aligned}\overline{g}(A_NY - A_{\tilde{P}N}Y, PX) + \overline{g}(A_{\tilde{P}N}X - A_NY, PY) = \overline{g}(D^s(Y,N) -\\ D^s(Y,\tilde{P}N), FX) + \overline{g}(D^s(X,\tilde{P}N) - D^s(X,N), FY),\end{aligned} \tag{3.28}$$

for all $X, Y \in \Gamma(S(TM))$ and $N \in \Gamma(ltr(TM))$.
(iii) $\Rightarrow$ (i) Using (2.12) and (3.6) in (3.28), we get (3.27). Now taking into account that $\overline{\nabla}$ is a metric connection, from (2.3), (2.4), (3.27), we obtain (3.25). Hence the proof is completed. □

Theorem 3.11. *Let* M *be a screen slant lightlike submanifold of a golden semi-Riemannian manifold* $\overline{M}$*. Then the induced connection* ∇ *is a metric connection if and only if* $PP_2\nabla_X Y + BQ_2h^s(X,Y) = 0$*, for all* $X \in \Gamma(TM)$ *and* $Y \in \Gamma(RadTM)$*.*

Proof. Let M be a screen slant lightlike submanifold of a golden semi-Rieman nian manifold $\overline{M}$. Then the induced connection ∇ is a metric connection if and only if $RadTM$ is parallel distribution with respect to ∇ [4]. From (2.3), (2.11), (3.6) and (3.9), for any $X \in \Gamma(TM)$ and $Y \in \Gamma(RadTM)$, we have $\overline{\nabla}_X \tilde{P}Y = \tilde{P}P_1\nabla_X Y + PP_2\nabla_X Y + FP_2\nabla_X Y + \tilde{P}h^l(X,Y) + BQ_2h^s(X,Y) + CQ_2h^s(X,Y) + \tilde{P}Q_3h^s(X,Y)$. By comparing tangential components of both sides of the above equation, we obtain $\nabla_X \tilde{P}Y = \tilde{P}P_1\nabla_X Y + PP_2\nabla_X Y + BQ_2h^s(X,Y)$, which completes the proof. □

4. Foliations Determined by Distributions

In this section, we obtain necessary and sufficient conditions for foliations determined by the distributions on a screen slant lightlike submanifolds of a golden semi-Riemannian manifold to be totally geodesic.

Definition 4.1 ([2])**.** An equivalence relation on an n-dimensional semi-Riemannian manifold $(\overline{M}, \overline{g})$ in which the equivalence classes are connected, immersed submanifolds (called the leaves of the foliation) of a common dimension k, $0 < k \leq n$ is called a foliation on $\overline{M}$. If each leaf of a foliation F on a semi-Riemannian manifold $(\overline{M}, \overline{g})$ is totally geodesic submanifold of $\overline{M}$, we say that F is a totally geodesic foliation.

Theorem 4.2. *Let M be a screen slant lightlike submanifold of a golden semi-Riemannian manifold $\overline{M}$. Then the following statements are equivalent:*
(i) $Rad(TM)$ defines a totally geodesic foliation,
(ii) $\overline{g}(\nabla_X \tilde{P}Y + h^s(X, \tilde{P}Y), \tilde{P}Z) = \overline{g}(\nabla_X \tilde{P}Y, Z)$,
(iii) $\overline{g}(h^l(X, PZ) + D^l(X, FZ), \tilde{P}Y) = \overline{g}(h^l(X, PZ) + D^l(X, FZ), Y)$,
for all $X, Y \in \Gamma(Rad(TM))$ and $Z \in \Gamma(S(TM))$.

Proof. Let M be a screen slant lightlike submanifold of a golden semi-Rieman nian manifold $\overline{M}$. Then the distribution $Rad(TM)$ defines a totally geodesic foliation if and only if

$$\overline{g}(\nabla_X Y, Z) = 0, \tag{4.1}$$

for all $X, Y \in \Gamma(Rad(TM))$ and $Z \in \Gamma(S(TM))$.
(i) $\Rightarrow$ (ii) From (2.3), (2.4), (2.11) and (4.1), we obtain

$$\overline{g}(\nabla_X \tilde{P}Y + h^s(X, \tilde{P}Y), \tilde{P}Z) = \overline{g}(\nabla_X \tilde{P}Y, Z). \tag{4.2}$$

(ii) $\Rightarrow$ (iii) Since $\overline{\nabla}$ is a metric connection and using (2.11) in (4.2), we obtain

$$\overline{g}(\tilde{P}Y, \overline{\nabla}_X \tilde{P}Z) = \overline{g}(Y, \overline{\nabla}_X \tilde{P}Z), \tag{4.3}$$

for all $X, Y \in \Gamma(Rad(TM))$ and $Z \in \Gamma(S(TM))$.
From (2.11), (2.13), (3.6) and (4.3), we get

$$\overline{g}(h^l(X, PZ) + D^l(X, FZ), \tilde{P}Y) = \overline{g}(h^l(X, PZ) + D^l(X, FZ), Y). \tag{4.4}$$

(iii) $\Rightarrow$ (i) Using (2.11), (2.13) and (3.6) in (4.4), we get (4.3). Since $\overline{\nabla}$ is a metric connection then from (2.4), (4.3), we get (4.1). Thus, the proof is completed. □

Theorem 4.3. *Let M be a screen slant lightlike submanifold of a golden semi-Riemannian manifold $\overline{M}$. Then the following statements are equivalent:*
(i) $S(TM)$ defines a totally geodesic foliation,
(ii) $\overline{g}(\nabla_X PY - A_{FY}X, \tilde{P}N) = \overline{g}(\nabla_X PY - A_{FY}X, N)$,
(iii) $\overline{g}(D^s(X, \tilde{P}N), FY) + \overline{g}(A_{\tilde{P}N}X, Y) = \overline{g}(A_{\tilde{P}N}X, PY)$,
for all $X, Y \in \Gamma(S(TM))$ and $N \in \Gamma ltr(TM)$.

Proof. Let M be a screen slant lightlike submanifold of a golden semi-Rieman nian manifold $\overline{M}$. Then the distribution $S(TM)$ defines a totally geodesic foliation if and only if

$$\overline{g}(\nabla_X Y, N) = 0, \tag{4.5}$$

for all $X, Y \in \Gamma(S(TM))$ and $N \in \Gamma(ltr(TM))$.
(i) $\Rightarrow$ (ii) From (2.3), (2.4), (2.11), (2.13), (3.6) and (4.5), we obtain

$$\overline{g}(\nabla_X PY - A_{FY}X, \tilde{P}N) = \overline{g}(\nabla_X PY - A_{FY}X, N), \tag{4.6}$$

(ii) $\Rightarrow$ (iii) Since $\overline{\nabla}$ is a metric connection and using (2.11), (2.13) and (3.6) in (4.6), we obtain

$$\overline{g}(\tilde{P}Y, \overline{\nabla}_X \tilde{P}N) = \overline{g}(Y, \overline{\nabla}_X \tilde{P}N), \tag{4.7}$$

for all $X, Y \in \Gamma(S(TM))$ and $N \in \Gamma(ltr(TM))$.
From (2.12), (3.6) and (4.7), we get

$$\overline{g}(D^s(X, \tilde{P}N), FY) + \overline{g}(A_{\tilde{P}N}X, Y) = \overline{g}(A_{\tilde{P}N}X, PY). \tag{4.8}$$

(iii) $\Rightarrow$ (i) Using (2.12) and (3.6) in (4.8), we get (4.7). Since $\overline{\nabla}$ is a metric connection, from (2.4), (4.7), we get (4.5). This proves the theorem. □

Theorem 4.4. *Let (M, g) be a totally umbilical screen slant lightlike submanifold of a golden semi-Riemannian manifold $(\overline{M}, \overline{g}, \tilde{P})$. Then M is totally geodesic if $\overline{g}(Y, A_{\tilde{P}V}X) = 0$, for any $Y \in \Gamma(S(TM))$, $X \in \Gamma(TM)$ and $V \in \Gamma(\nu)$.*

Proof. Let M be a totally umbilical screen slant lightlike submanifold of a golden semi-Riemannian manifold $\overline{M}$. Since $\overline{\nabla}$ is a metric connection, then $(\overline{\nabla}_X \overline{g})(\tilde{P}\xi, Z) = 0$, for any $\xi \in \Gamma Rad(TM)$, $Z \in \Gamma F(S(TM))$ and for all $X \in \Gamma(TM)$. From (2.2) and (2.3), we get

$$(\overline{\nabla}_X \xi, \tilde{P}Z) = -\overline{g}(\tilde{P}\xi, \overline{\nabla}_X Z), \tag{4.9}$$

Now from (2.11), (2.13), (2.17), (3.9) and (4.9), we get

$$-\overline{g}(A^*_\xi X, BZ) + \overline{g}(h^s(X, \xi), CZ) = -\overline{g}(\tilde{P}\xi, D^l(X, Z)). \tag{4.10}$$

Using (2.14), (2.18) and (2.21) in (4.10), we obtain
$\overline{g}(h^l(X, BZ), \xi) + \overline{g}(h^s(X, \tilde{P}\xi), Z) = 0$, and by using (2.21) in this equation, we get

$$\overline{g}(h^l(X, BZ), \xi) = 0, \tag{4.11}$$

which implies $h^l(X, BZ) = 0$. Thus from (2.21), $H^l = 0$.
Let $\overline{g}(Y, \tilde{P}V) = 0$, for all $Y \in \Gamma S(TM)$ and $V \in \Gamma(\nu)$, from (2.2), we get $\overline{g}(\tilde{P}Y, V) = 0$ for any $Y \in \Gamma S(TM)$ and $V \in \Gamma(\nu)$. Since $\overline{\nabla}$ is a metric

connection, then $(\overline{\nabla}_X \overline{g})(\tilde{P}Y, V) = 0$, for any $Y \in \Gamma S(TM)$, $V \in \Gamma(\nu)$ and for all $X \in \Gamma(TM)$. From (2.2) and (2.3), we obtain

$$\overline{g}(\overline{\nabla}_X Y, \tilde{P}V) = -\overline{g}(Y, \overline{\nabla}_X \tilde{P}V). \tag{4.12}$$

Also, from (2.11), (2.13) and (4.12), we obtain

$$\overline{g}(h^s(X, Y), \tilde{P}V) = \overline{g}(Y, A_{\tilde{P}V} X). \tag{4.13}$$

Now, if $\overline{g}(Y, A_{\tilde{P}V} X) = 0$, then $h^s(X, Y) = 0$, then from (2.21), we get $H^s = 0$. Hence M is totally geodesic if $\overline{g}(Y, A_{\tilde{P}V} X) = 0$. This completes the proof. □

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