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Proper forcing and spaces of countable tightness

Abstract: If a space is countably compact non-compact and has countable tightness, then there is a proper poset that forces the space to contain a copy of ω_1. it therefore follows from PFA that such a space will carry an algebraic free sequence [Todorcevic]. Moreover [Balogh] compact spaces of countable tightness will be sequential and C-closed (countably compact subsets are closed). Also [Eisworth] PFA implies $\aleph_0$-bounded non-compact spaces of countable tightness will actually contain copies of ω_1. Sapirovskii proved that countable tightness is equivalent to having hereditary countable pi-character in the class of compact spaces. The classical Ostaszewski and Fedorchuk spaces under diamond give examples of compact spaces of countable tightness that are not C-closed. Eisworth noticed that the stronger assumption of hereditary countable pi-character was a big help in this area. This led to the independence from CH of the Moore-Mrowka problem by establishing the consistency with CH of the statement "every regular space with hereditary countable pi-character is C-closed". We discuss this and more results about the C-closed property and uncountable free sequences in the class of spaces with hereditary countable pi-character.