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On subsets of ℓ_∞ deciding the norm convergence of sequences in ℓ_1

Recall that ℓ_1 and ℓ_∞ denote the space of all summable scalar-valued sequences and the space of all bounded scalar-valued sequences, respectively:

$$\ell_1 = \Big\{(\alpha_n)_{n\in\omega}:\ \big\|(\alpha_n)_{n\in\omega}\big\|_1 = \sum_{n\in\omega}|\alpha_n| < \infty\Big\},$$

$$\ell_\infty = \Big\{(\beta_n)_{n\in\omega}:\ \big\|(\beta_n)_{n\in\omega}\big\|_\infty = \sup_{n\in\omega}|\beta_n| < \infty\Big\}.$$

If $x = (\alpha_n)_{n\in\omega} \in \ell_1$ and $y = (\beta_n)_{n\in\omega} \in \ell_\infty$, then we define $y(x) = \sum_{n\in\omega}\alpha_n \cdot \beta_n$.

The classical Schur theorem states that if a sequence $(x_n)_{n\in\omega}$ of elements of ℓ_1 converges weakly to 0, i.e. $\lim_{n\to\infty} y\big(x_n\big) = 0$ for every $y \in \ell_\infty$, then it converges to 0 in norm, i.e. $\lim_{n\to\infty}\big\|x_n\big\|_1 = 0$.

During my talk I will show that the Schur theorem can be strenghtened — I will show in ZFC that there exists a subset $D \subset \ell_\infty$ of cardinality equal to the cofinality cof($\mathcal{N}$) of the Lebesgue null ideal $\mathcal{N}$ such that if a sequence $(x_n)_{n\in\omega}$ of elements of ℓ_1 is such that $\lim_{n\to\infty} y\big(x_n\big) = 0$ for every $y \in D$, then it converges to 0 in norm. The cardinality of such D cannot be less than the pseudo-intersection number $\mathfrak{p}$. This implies that the existence of such D of cardinality strictly smaller than the continuum $\mathfrak{c}$ is undecidable in ZFC+$\neg$CH.