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Pinning Down versus Density

Joint work with I. Juhász, Z. Szentmiklóssy, J. van Mill

The *pinning down number* $pd(X)$ of a topological space X is the smallest cardinal κ such that for any neighborhood assignment $U : X \to \tau_X$ there is a set $A \in [X]^\kappa$; with $A \cap U(x) \neq \emptyset$ for all $x \in X$. Clearly, $\mathsf{c}(X) \leq pd(X) \leq d(X)$.

In a joint paper with Juhasz and Szentmiklossy we proved that the following statements are equivalent:

(1) $2^\kappa < \kappa^{+\omega}$ for each cardinal κ;
(2) $d(X) = pd(X)$ for each Hausdorff space X;
(3) $d(X) = pd(X)$ for each 0-dimensional Hausdorff space X.

This answersed two questions of Banakh and Ravsky.

The *dispersion character* $\Delta(X)$ of a space X is the smallest cardinality of a non-empty open subset of X. We also showed that the following three statements are *equiconsistent*:

(i) There is a singular cardinal λ with $pp(\lambda) > \lambda^+$, i.e. Shelah's Strong Hypothesis fails;
(ii) there is a 0-dimensional Hausdorff space X such that $|X| = \Delta(X)$ is a regular cardinal and $pd(X) < d(X)$;
(iii) there is a topological space X such that $|X| = \Delta(X)$ is a regular cardinal and $pd(X) < d(X)$.

We also discuss some recent results concerning the pinning down numbers of connected and homogeneous spaces.

[1] I. Juhász, L. Soukup, Z. Szentmiklóssy: Pinning Down versus Density, Israel J. Math, to appear.