Foreword

Novi Sad Conference in Set theory and General Topology (SETTOP) is organized biannually by the Department of Mathematics of the Faculty of Science at the University of Novi Sad. This is its fouth edition, succeeding LogTop 2012, organized as one of three conferences celebrating 50 years of the Seminar for Analysis and Foundation of Mathematics, led by Professor Bogoljub Stanković, SETTOP 2014 and SETTOP 2016.

This event came as a natural consequence of the growing number of people on the Department of Mathematics interested in set theory and general topology, and their wish to become more connected to scientists around the world working in the same areas.

The main topics of this year's conference are set theory, model theory and general topology.

This year, after the previous conference held at Irški venac, at Fruška gora, we are back to Novi Sad. The plan in the following years is to alternate those two locations. This year all lectures will take place at the new central building of the University of Novi Sad, situated at the small park at the edge of the University campus, near the Danube river.

The Organizing Committee

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David Asperó (Norwich)
David Chodounský (Prague)
Natasha Dobrinen (Denver)
Vera Fischer (Vienna)
Heike Mildenberger (Freiburg)
Dilip Raghavan (Singapore)
Philipp Schlicht (Bristol)
Stevo Todorčević (Toronto, Paris)

PARTICIPANTS:

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Monroe Eskew (Vienna)
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Jaroslav Šupina (Košice)
Thilo Weinert (Vienna)
Teruyuki Yorioka (Shizuoka)
Jing Zhang (Pittsburgh)

ABSTRACTS

David Aspero

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Side conditions, adding few reals, and trees

I will present a method for building forcing iterations with small support and at the same time preserving (some initial segment of) GCH. The focus will be on a recent application to trees on ω_2.

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Borel complexity in hyperspaces up to equivalence

Joint work with Jozef Bobok, Pavel Pyrih, and Benjamin Vejnar

We say that two classes $\mathcal{C}$ and $\mathcal{D}$ of topological spaces are *equivalent* if every space in $\mathcal{C}$ is homeomorphic to a space in $\mathcal{D}$ and vice versa. For a class of metrizable compacta $\mathcal{C}$ we consider the collection of all families $\mathcal{F} \subseteq \mathcal{K}([0,1]^\omega)$ equivalent to $\mathcal{C}$, and we denote this collection by $[\mathcal{C}]$.

Usually, complexity of such class $\mathcal{C}$ means the complexity of the saturated family $\max([\mathcal{C}]) \subseteq \mathcal{K}([0,1]^\omega)$. There are many results of this type. We are rather interested in the lowest complexity among members of $[\mathcal{C}]$. This is rarely the complexity of the saturated family. We study this Borel complexity up to the equivalence because of its connection with our notion of *compactifiable classes*. We have shown [1] that every analytic family in $\mathcal{K}([0,1]^\omega)$ is equivalent to a G_δ family and that these correspond to *strongly Polishable classes*. Similarly, closed families correspond to *strongly compactifiable classes*. It is natural to ask about the other complexities – clopen, open, and F_σ

In the talk we give an overview of the theory and used notions, and we formulate our new results regarding open and F_σ classes.

[1] Bartoš, A., Bobok, J., Pyrih, P., Vejnar, B., Compactifiable classes of compacta. arXiv:1801.01826.

Jeffrey Bergfalk

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Walks and chain conditions of higher order

We conceive this talk as a sequel to our 2016 SETTOP talk. In that talk, we described higher-order principles of nontrivial coherence generalizing those associated to Todorcevic's method of (two-dimensional) walks on ordinals. These derived from homological arguments which seemed to herald the ZFC existence of $(n+1)$-dimensional walks on the ordinals of ω_n. In this talk, we describe those walks. Time permitting, we'll describe as well a higher-order chain condition satisfied by generic approaches to higher nontrivial coherence.

William Chen

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Cardinal characteristics of ω_1

Joint work with Geoff Galgon

The study of cardinal invariants at uncountable cardinals has progressed rapidly in recent years with many surprising results. In this talk, we focus on invariants of a slightly different flavor, which are connected with the combinatorics of ω_1 and of the continuum. Following suggestions of Thilo Weinert, we are especially interested in such invariants which take an ordinal parameter.

For example, the stick number is the least size of a family of infinite subsets of ω_1 so that every uncountable subset of ω_1 contains a member of the family as a subset. This definition can be modified to demand that every member of the family has order-type at least γ, where γ is a fixed countable ordinal.

The definition of the stick number can also be modified in another way, so that we only demand that every uncountable subset of ω_1 intersects a member of the family in an infinite set, and we can further modify this definition to demand that every member of the family has order-type at most γ.

We show that these modifications lead to new cardinal invariants, compute them in certain models of set theory, and demonstrate their relationships with other previously-studied quantities.

[1] Chen, William, Variations of the stick principle. European Journal of Mathematics, Vol. 3 No. 3 (2017), 650–658.

[2] Chen, William, Galgon, Geoff, Antichains, the stick principle, and a matching number. preprint.

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Killing P-points made simple

In [1] the authors proved that adding a Silver real kills P-points. The main applications of this fact is the existence of canonical models without P-points, and models without P-points with the continuum arbitrarily large. The technical details of the argument in [1] are somewhat complicated. I will demonstrate that the ω-product of Silver posets kills P-points. This proof uses the same core idea as the corresponding result for the Silver poset and avoids most of the technical details. Although the result is weaker than the original one, it is still sufficient for the mentioned applications.

[1] Chodounský, D., Guzmán, O., There are no P-points in Silver extensions, preprint.

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Big Ramsey degrees of the universal triangle-free graph

It is a central question in the theory of homogeneous relational structures as to which structures have finite big Ramsey degrees. This question, of interest for several decades, has gained recent momentum as it was brought into focus by Kechris, Pestov, and Todorcevic in their 2005 paper, in which they proved a deep correspondence between Ramsey theory of Fraïssé limits and topological dynamics. An infinite structure $\mathbf{S}$ is **homogeneous** if any isomorphism between two finitely generated substructures of S can be extended to an automorphism of $\mathbf{S}$. A homogeneous structure $\mathbf{S}$ is said to have **finite big Ramsey degrees** if for each finite substructure A of $\mathbf{S}$, there is a number $n(\mathrm{A})$ such that any coloring of the copies of A in $\mathbf{S}$ into finitely many colors can be reduced down to no more than n colors on some substructure $\mathbf{S}'$ isomorphic to $\mathbf{S}$. This is interesting not only as a Ramsey property for infinite structures, but also because of its implications for topological dynamics, following recent work of Zucker.

Prior to work of the speaker, finite big Ramsey degrees had been proved for a handful of homogeneous structures including the rationals (Devlin 1979), the Rado graph (Sauer 2006), ultrametric spaces (Nguyen Van Thé 2008), and enriched versions of the rationals and related circular directed graphs (Laflamme, Nguyen Van Thé, and Sauer 2010). According to Nguyen Van Thé, Sauer, and Todorcevic, the lack of tools to represent ultrahomogeneous structures with forbidden configurations, particularly the lack of any analogue of Milliken's Ramsey theorem for strong trees applicable to such structures, was a major obstacle towards a better understanding of their infinite partition properties.

The universal triangle-free graph, constructed by Henson in 1971 and denoted $\mathbf{H}_3$, is the simplest homogeneous structure with a forbidden configuration. Prior to my work, Komjáth and Rödl had proved in 1986 that vertex colorings have big Ramsey degree 1, and

Sauer had proved in 1998 that edge colorings have big Ramsey degree 2. It was a major open problem in the Ramsey theory of homogeneous structures whether or not each finite triangle has a finite big Ramsey degree. Recently, I solved this problem in [1]. This work involved developing a new notion of trees coding $\mathbf{H}_3$; using forcing techniques to prove, in ZFC, Ramsey theorems for these trees, proving new Halpern-Läuchli and Milliken-style theorems; and deducing bounds from the tree structures. Work in progress is the theorem that all Henson graphs, the universal k-clique-free graphs, have finite big Ramsey degrees. The methods developed seem robust enough that correct modifications should likely apply to a large class of homogeneous structures with forbidden configurations.

[1] Dobrinen, N., The Ramsey theory of the universal homogeneous triangle-free graph. 48 pp. Submitted.

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Global Chang's Conjecture and singular cardinals

Joint work with Yair Hayut

Foreman [1] asked to what extent a global version of Chang's Conjecture can hold. In [2], the authors proved that, relative to a huge cardinal, ZFC is consistent with the statement that for every regular κ and every $\mu < \kappa$, $(\kappa^+, \kappa) \twoheadrightarrow (\mu^+, \mu)$. In light of constraints imposed by GCH, we asked whether a maximal global Chang's Conjecture is consistent, which says that whenever cf$(\kappa) \geq$ cf(μ), $(\kappa^+, \kappa) \twoheadrightarrow (\mu^+, \mu)$. We show here that it is inconsistent. On the other hand, we show it is consistent relative to a Shelah-for-supercompactness cardinal that for all $\alpha < \beta < \omega^\omega$ of countable confinality, $(\aleph_{\beta+1}, \aleph_\beta) \twoheadrightarrow (\aleph_{\alpha+1}, \aleph_\alpha)$.

[1] Foreman, Matthew. Ideals and generic elementary embeddings. Handbook of Set Theory, vol. 2, Springer Dordrecht, 2010, pp. 885–1147.

[2] Eskew, Monroe and Hayut, Yair. Trans. Amer. Math. Soc. 370 (2018), no. 4, 2879–2905.

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Killing ideals softly

Joint work with Lyubomyr Zdomskyy.

We say that a forcing notion $\mathbb{P}$ $+$-*destroys* a Borel ideal $\mathcal{J}$ if $\mathbb{P}$ adds an $\dot{H} \in \mathcal{J}^+ = \mathcal{P}(\omega) \setminus \mathcal{J}$ such that $|A \cap \dot{H}| < \omega$ for every $A \in \mathcal{J}^V$.

In this talk, I will discuss (a) which ideals can be $+$-destroyed, (b) examples of ideals which can be destroyed (that is, when we require only $\dot{H} \in [\omega]^\omega$) without being $+$-destroyed, also examples when destruction implies $+$-destruction, and (c) the cardinal invariants associated to $+$-destruction of ideals.

Furthermore, I will present a combinatorial characterization of $+$-destructibility of ideals by forcing notions of the form $\mathbb{P}_I$.

Vera Fischer

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The spectrum of independence

The set of possible sizes of maximal independent families is referred to as the spectrum of independence and denoted $\mathsf{Sp}(\mathfrak{i})$. We will show that:

- Whenever $\{\kappa_i\}_{i=1}^n$ are regular uncountable cardinals, it is consistent that $\{\kappa_i\}_{i=1}^n \subseteq \mathsf{Sp}(\mathfrak{i})$.
- Whenever κ has uncountable cofinality, it is consistent that $\mathsf{Sp}(\mathfrak{i}) = \{\aleph_1, \kappa = \mathfrak{c}\}$.
- Assuming that $\kappa_1 < \cdots < \kappa_n$ are measurable cardinals, it is consistent that

$$\{\kappa_i\}_{i=1}^n \subseteq \mathsf{Sp}(\mathfrak{i}) \text{ and } \Big(\bigcup_{i=1}^{n-1}(\kappa_i, \kappa_{i+1})\Big) \cap \mathsf{Sp}(\mathfrak{i}) = \emptyset.$$

In addition, to any independent family, we will associate two ideals on ω and define a class of maximal independent families for which Sacks indestructibility can be naturally characterized in terms of these ideals.

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Borel ideals

Joint work with M.Hrusak and with C.Uzcategui

We say that an ideal $\mathcal{I}$ on ω is tall if for every infinite $x \subseteq \omega$ there is an infinite $y \subseteq x$ such that $y \in \mathcal{I}$. I will present a result from [1] which states that the set of all tall F_σ-ideals is Π^1_2-complete. Consequently we have that there is no analytic tall ideal that is below all Borel tall ideals in the Katětov order, which answers a question of M. Hrusak. Next I present a result from [2] where we used the complexity result from [1] to show that there is a tall F_σ-ideal such that no Borel function can witness its tallness. This has some conseqeunce on possible uniform versions of Galvin's and Nash-Williams's theorems from infinite-dimensional Ramsey theory.

[1] Grebik, J., Hrusak, M., *No minimal tall Borel ideal in the Katětov order*, preprint, 2017.

[2] Grebik, J., Uzcategui, C., *Bases and Borel selectors for tall families*, preprint, 2017.

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Nonamalgamation in the generic multiverse

Joint work with Joel David Hamkins, Lukas Daniel Klausner, Jonathan Verner, and Kameryn Williams

Fix a countable transitive model M of ZFC and consider its generic multiverse, the family of all forcing extensions of M. If we order the generic multiverse by inclusion, the resulting structure has interesting universality properties. Mostowski showed that any finite poset embeds into the generic multiverse in a way that also preserves *nonamalgamability*, i.e. the nonexistence of upper bounds. His embedding actually only used extensions adding Cohen reals. I will present some results from a joint project in which we extended Mostowski's result in order to embed several infinite posets into the generic multiverse. In addition to nonamalgamability, our method also ensures that greatest lower bounds are preserved. In particular, any finite meet-semilattice embeds as such into the generic multiverse. Further variations on the proof also allow us to realize these embeddings with a wide variety of forcing notions beyond just Cohen forcing.

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Basis and antibasis results for actions of locally compact groups

Joint work with Benjamin Miller

Given any locally compact, separable, and non-compact group G we construct contiunuum many free actions of G on locally compact Polish spaces with non-smooth induced orbit equivalence relation that form a basis under embedibility of all such actions of G on Polish spaces. Simultaneously, we show that any such basis must be of size at least continuum.

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The Banach–Mazur game and the strong Choquet game in domain theory

We prove that a player α has a winning strategy in the Banach–Mazur game on a space X if and only if X is F-Y countably π-domain representable. We show that Choquet complete spaces are F-Y countably domain representable.

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P-ideal dichotomy and some versions of the Souslin's Hypothesis

Joint work with Stevo Todorčević

P-ideal dichotomy is a combinatorial set theoretic principle which has many consequences on the universe of set theory. For example, it implies the Souslin's Hypothesis, Singular Cardinals Hypothesis, and Projective Determinacy. In this talk we will analyze a relationship between the P-ideal dichotomy and the statement that all Aronszajn trees can be embedded into the rational line.

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Unbounded colorings and the C-sequence number

Joint work with Assaf Rinot

Motivated by questions about the infinite productivity of strong chain conditions, we introduce and analyze a coloring principle asserting the existence of certain strongly unbounded functions. We use this principle to show, for instance, that the κ-Knaster property is not infinitely productive for any successor cardinal κ. We also introduce a cardinal invariant, the *C-sequence number*, that is deeply connected to our coloring principle and can be seen as a way of measuring the compactness of an uncountable cardinal. We then present a number of ZFC theorems and independence results concerning the C-sequence number and linking it to various large cardinal notions.

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Finite big Ramsey degrees in countable universal structures

Let F be a countable ultrahomogeneous relational structure, and let $\mathrm{Age}(F)$ denote the class of all the finite structures that F embeds. A positive integer n is a *big Ramsey degree* of a finite structure $A \in \mathrm{Age}(F)$ in F if for every $k \geq 2$ and every coloring of copies of A in F with k colors, there is a copy F' of F inside F such that the copies of A that sit inside F' attain at most n colors in this coloring. In this case we say that A has *finite big Ramsey degree in* F. For example, Glavin proved in 1968/9 that finite chains have finite big Ramsey degrees in $\mathbb{Q}$, Sauer proved in 2006 that finite graphs have finite big Ramsey degrees in the Rado graph, and Dobrinen has just recently proved that finite triangle-free graphs have finite big Ramsey degrees in the Henson graph H_3.

In this talk we consider the context where F is a countable structure universal for a class of finite structures, but not necessarily an ultrahomogeneous one. For each of the following classes of structures:

- acyclic digraphs,
- finite permutations,
- a special class of finite posets with a linear order extending the poset relation, and
- a special class of metric spaces

we show that there exists a countably infinite universal structure S such that every finite structure from the class has finite big Ramsey degree in S. Although not apparent from the formulation of the results, the techniques we use heavily rely on the reinterpretation of Ramsey theoretic notions in terms of category theory.

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A Ramsey-theoretic notion of forcing

We introduce Gowers–Matet forcing with a finite sequence of pairwise non-isomorphic Ramsey ultrafilters over ω and Gowers-Milliken-Taylor ultrafilters over the k-valued blocks Fin_k. For evaluating the new forcings, we prove a strengthening of Gowers' theorem on colourings of Fin_k. We investigate Ramsey spaces in the forcing extension.

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Reversible sequences of natural numbers and reversibility of some disconnected binary structures

Joint work with Miloš S. Kurilić

A relational structure $\mathbb{X}$ is said to be reversible iff every bijective endomorphism $f : \mathbb{X} \to \mathbb{X}$ is an automorphism. Since equivalence relations are, up to isomorphism, characterized by the sequence of cardinalities of its connectivity components, their reversibility can be regarded as a property of the corresponding sequence of cardinals (called reversibility as well). We first show that a sequence $\langle \kappa_i : i \in I \rangle$ of cardinals is reversible iff it is a finite-to-one sequence, or a reversible sequence of natural numbers. Next, we characterize reversible sequences $\langle n_i : i \in I \rangle$ of natural numbers: either it is a finite-to-one sequence, or $K = \{m \in \mathbb{N} : n_i = m$ for infinitely many $i \in I\}$ is a nonempty independent set and $\gcd(K)$ divides at most finitely many elements of the set $\{n_i : i \in I\}$. We isolate a class of disconnected binary structures (containing equivalence relations) such that a structure from the class is reversible iff the sequence of cardinalities of its connectivity components is reversible. In addition, we isolate a class of disconnected binary structures (containing posets that are disjoint unions of chains) such that, for a structure from that class, reversibility of the sequence of cardinalities of its components implies reversibility of the structure. In that way, we can detect numerous examples of reversible posets, as well as reversible topological spaces. Finally, using the characterization of reversible sequences of natural numbers, we characterize reversible posets that are disjoint unions of ordinals and their inverses.

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Some results on cardinal invariants above the continuum

I will survey some of my work with Saharon Shelah on cardinal invariants at uncountable regular cardinals. The focus will be on the bounding, dominating, splitting, reaping, and almost disjointness numbers. The discussion will include both consistency results as well as a few unexpected ZFC theorems, which show that the situation at regular uncountable cardinals is fundamentally different from the situation at ω. The ZFC theorems rely on a variety of combinatorial techniques including some facts about PCF theory.

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Topological isomorphism of oligomorphic groups

The closed subgroups of the permutation group S_∞ of $\mathbb{N}$ coincide with the automorphism groups of structures on $\mathbb{N}$. There are well-known connections between model-theoretic properties of a structure and properties of its automorphism group. For instance, the automorphism groups of ω-categorical structures on $\mathbb{N}$ are precisely the oligomorphic closed subgroups of S_∞ (a permutation group is oligomorphic if for each k there are only finitely many k-orbits).

We study the complexity of topological isomorphism of oligomorphic closed subgroups of S_∞ in the setting of Borel reducibility. Previous work of Kechris, Nies and Tent (and independently Rosendal and Zielinski) showed that this equivalence relation is below graph isomorphism. We show that it is below a Borel equivalence relation with countable equivalence classes. This is a joint project with Andre Nies and Katrin Tent.

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Ladder system uniformization on trees

Uniformization properties of ladder system colourings have been extensively studied due to various connections to algebra, in particular to the Whitehead problem and its relatives [4, 7, 8], to topology [1, 3, 9, 13], and to fundamental questions in combinatorial set theory [5, 6, 10].

Our current interest lies in understanding a relatively new version of the uniformization property introduced by Justin Moore, a notion that played a key role in understanding minimal uncountable linear orders [6]. Given a tree T of height ω_1, we say that a ladder system colouring $(f_\alpha)_{\alpha\in\lim\omega_1}$ has a T-uniformization if there is a function φ defined on a pruned, downward closed subtree S of T so that for any $s \in S_\alpha$ of limit height and almost all $\xi \in \mathrm{dom} f_\alpha$, $\varphi(s \restriction \xi) = f_\alpha(\xi)$.

In this talk, I will first outline how variations of the diamond principle (weak and strong) imply that there are ladder system colourings without T-uniformizations; as expected, stronger diamonds imply that we can take care of more trees and find simple colourings without T-uniformizations. However, rather unexpectedly, we prove that whenever $\diamondsuit^+$ holds, for any ladder system $\mathbf{C}$ there is an Aronszajn tree T so that any monochromatic colouring of $\mathbf{C}$ (i.e. each f_α is some constant function) has a T-uniformization [11].

Second, we look at the existence of T-uniformizations for Suslin trees T and answering a question of J. Baumgartner [2], I outline why the existence of a Suslin tree does not necessarily imply that there are minimal uncountable linear orders other than ω_1 and its reverse [12]. It remains open from [2] if $\diamondsuit$ suffices to construct such minimal linear orders.

[1] Zoltán Balogh, Todd Eisworth, Gary Gruenhage, Oleg Pavlov, and Paul Szeptycki. Uniformization and anti-uniformization properties of ladder systems. *Fund. Math*, 181:189–213, 2004.

[2] James E Baumgartner. Order types of real numbers and other uncountable orderings. In *Ordered sets*, pages 239–277. Springer, 1982.

[3] Keith J Devlin and Saharon Shelah. A note on the normal moore space conjecture. *Canad. J. Math*, 31:241–251, 1979.

[4] Paul C Eklof, Alan H Mekler, and Saharon Shelah. Uniformization and the diversity of whitehead groups. *Israel Journal of Mathematics*, 80(3):301–321, 1992.

[5] Paul Larson and Paul McKenney. Automorphisms of $P(\lambda)/I_\kappa$. *Fund. Math*, 233:271–291, 2016.

[6] Justin T. Moore. ω_1 and $-\omega_1$ may be the only minimal uncountable linear orders. *Michigan Math. J*, 55(2):437–457, 2007.

[7] Saharon Shelah. Whitehead groups may be not free, even assuming CH, I. *Israel Journal of Mathematics*, 28(3):193–204, 1977.

[8] Saharon Shelah. Whitehead groups may not be free even assuming CH, II. *Israel Journal of Mathematics*, 35(4):257–285, 1980.

[9] Saharon Shelah. A consistent counterexample in the theory of collectionwise hausdorff spaces. *Israel Journal of Mathematics*, 65(2):219–224, 1989.

[10] Saharon Shelah. *Proper and improper forcing*, volume 5. Cambridge University Press, 2017.

[11] Daniel T. Soukup. Ladder system uniformization on trees. *2018, in preparation*.

[12] Daniel T. Soukup. A model with suslin trees but no minimal uncountable linear orders other than ω_1 and $-\omega_1$. *submitted to the Israel Journal of Mathematics, arXiv:1803.03583*.

[13] Stephen Watson. Locally compact normal meta-lindelöf spaces may not be paracompact: an application of uniformization and suslin lines. *Proceedings of the American Mathematical Society*, 98(4):676–680, 1986.

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Open colorings, perfect sets and games on generalized Baire spaces

The generalized Baire space for an uncountable cardinal $\kappa = \kappa^{<\kappa}$ is the space ${}^{\kappa}\kappa$ of functions $\kappa \to \kappa$ equipped with the $<\kappa$-support topology. The study of the topology and descriptive set theory of these spaces is an active area of research today, with close connections to many other areas of set theory and to model theory.

The notions of perfectness, scatteredness and the Cantor-Bendixson hierarchy were generalized for the κ-Baire space ${}^{\kappa}\kappa$ by J. Väänänen [1] based on games of length $\leq \kappa$. Some different definitions of κ-perfectness for ${}^{\kappa}\kappa$ are also widely used. In the classical setting, these definitions correspond to equivalent notions, but this is no longer the case in the uncountable setting. In the first part of this talk, we detail connections between these concepts and their underlying games. For example, we show that Väänänen's Cantor-Bendixson theorem [1] is equivalent to the κ-perfect set property, and is therefore equiconsistent with the existence of an inaccessible cardinal above κ.

In the second part of this talk we introduce the uncountable analogue $\mathrm{OCA}_{\kappa}(X)$ of the Open Coloring Axiom for subsets X of the κ-Baire space, and also its κ-perfect set version $\mathrm{OCA}^{*}_{\kappa}(X)$. We show that $\mathrm{OCA}^{*}_{\kappa}(X)$ for all κ-analytic subsets $X \subseteq {}^{\kappa}\kappa$ is consistent relative to (and therefore equiconsistent with) the existence of an inaccessible cardinal above κ.

[1] Väänänen, J., A Cantor-Bendixson theorem for the space $\omega_1^{\omega_1}$. Fundamenta Mathematicae, Vol. 137 (1991), 187–199.

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Divisibility in ${}^{*}N$ and βN

We investigate properties of the extension ${}^{*}|$ of the divisibility relation to a set ${}^{*}N$ of nonstandard integers, as well as its extension $\widetilde{|}$ to the Stone-Čech compactification βN. Using the connection between ${}^{*}N$ and βN, we obtain a new equivalent condition for $\widetilde{|}$. This illuminates the situation on the lower levels of the $\widetilde{|}$-hierarchy of ultrafilters. We also use limits by ultrafilters to get some results on ultrafilters on the higher levels of this hierarchy.

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Relation between Ideal convergence and Sequence selection principles

Joint work with Jaroslav Šupina

The selection principle $\mathrm{S}_1(\Gamma,\Gamma)$ which has been introduced by M. Scheepers in 1996 is in our main interest. We are focused especially on space of all continuous functions on X, namely $C_p(X)$. By modification of mentioned principle we understand to change γ-cover to $\mathcal{I}$-γ-cover and in case of $C_p(X)$ to change classical convergence to $\mathcal{I}$-convergence. In our paper [1] we provide the overview of this problematics and we are adding our results which are focused particularly on description of critical cardinality of these spaces.

[1] Šottová V., Šupina J.: Principle $\mathrm{S}_1(\mathcal{P},\mathcal{R})$: Ideals and functions, preprint.

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Composition and discrete convergence

Joint work with Dávid Uhrik

A sequence of functions converges discretely if their values are equal to the limit function for all but finitely many indices. We are interested in topological space X such that any function from a chosen family of functions on X is a discrete limit of continuous functions. The considered families are the families of all upper or lower semicontinuous functions, all Borel measurable functions etc.

One of results has been shown using composition theorem by A. Lindenbaum [1] which was originally proved for real line. We show that it is valid in perfectly normal topological space as well.

[1] Lindenbaum A., *Sur les superpositions des fonctions représentables analytiquement,* Fund. Math. **23** (1934), 15–37.

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Universally meager sets and a conjecture of Galvin

Joint work with Dilip Raghavan

Using large cardinals we compute the 2-dimesional Ramsey degree of the topological copy of rationals inside a quite a large class of topological spaces that, in particular, include the class of all metrizable spaces.

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Cardinal characteristics & partition relations

Joint work with separately Chris Lambie-Hanson and both William Chen and Shimon Garti

Many a partition relation has been proved assuming the Generalised Continuum Hypothesis. More precisely, many negative partition relations involving ordinals smaller than ω_2 have been proved assuming the Continuum Hypothesis. Some recent results in this vein for polarised partition relations came from Garti and Shelah. The talk will focus on ordinary partition relations. The negative relations $\omega_1\omega \not\longrightarrow (\omega_1\omega, 3)^2$ and $\omega_1^2 \not\longrightarrow (\omega_1\omega, 4)^2$ were both shown to follow from the Continuum Hypothesis, the former in 1971 by Erdős and Hajnal and the latter in 1987 by Baumgartner and Hajnal. The former relation was shown to follow from both the dominating number and the stick number being $\aleph_1$ in 1987 by Takahashi. In 1998 Jean Larson showed that simply the dominating number being $\aleph_1$ suffices for this. It turns out that the unbounding number and the stick number both being $\aleph_1$ yields the same result. Moreover, also the second relation follows both from the dominating number being $\aleph_1$ and from both the unbounding number and the stick number being $\aleph_1$ thus answering a question of Jean Larson.

[1] Chen, W., Garti, Sh. Weinert, T., Cardinal characteristics of the continuum and partitions. Submitted.

[2] Lambie-Hanson, Ch., Weinert, T., Partitions of subsets of generalised scattered orders. Journal of the Mathematical Society of Japan, to appear.

[3] Larson, J., An ordinal partition from a scale. In "Set theory (Curaçao,1995; Barcelona, 1996)", KluwerAcad. Publ., Dordrecht (1998), 109–125.

[4] Baumgartner, J., Hajnal, A., A remark on partition relations for infinite ordinals with an application to finite combinatorics. In "Logic and Combinatorics(Arcata, Calif., 1985)", Vol. 65 of Contemp. Math., (Amer. Math. Soc., Providence, RI1987), 157–167.

[5] Takahashi, J., Two negative partition relations. Periodica Mathematica Hungarica, Vol. 18 No. 1 (1987), 1–6.

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A fragment of Asperó-Mota's Finitely Proper Forcing Axiom and an entangled set of reals

We introduce a fragment $\mathsf{PFA}^{\text{s-fin}}(\omega_1)$ of Asperó-Mota's finitely proper forcing axiom $\mathsf{PFA}^{\text{fin}}(\omega_1)$. $\mathsf{PFA}^{\text{s-fin}}(\omega_1)$ implies some consequences of $\mathsf{PFA}^{\text{fin}}(\omega_1)$, for example $\mathsf{MA}_{\aleph_1}$, the failure of $\mho$, no weak club guessing ladder systems, and the assertion that every two Aronszajn trees are club-isomorphic. For each integer $k \geq 2$, it is consistent that $\mathsf{PFA}^{\text{s-fin}}(\omega_1)$ holds, there exists a k-entangled set of reals, and $2^{\aleph_0} = \aleph_2$. This extends Abraham-Shelah's theorem that Martin's axiom does not imply that every two $\aleph_1$-dense sets of reals are isomorphic.

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Some results on the Baire Rado Conjecture

Baire Rado's Conjecture (RC^b in short, introduced by Todorcevic as a weakening of the Rado's Conjecture) asserts that any non-trivial Baire tree of height ω_1 has a nonspecial subtree of size $\leq \aleph_1$. It is incompatible with MA. This work is motivated by the question "which fragment of forcing axioms is compatible with RC^b". A poset is Baire Indestructibly Proper (BIP) is it remains proper in any forcing extension by a Baire tree. We will show RC^b is compatible with $MA_{\omega_1}(BIP)$ and as a consequence RC^b does not imply RC. If time permits we will talk about the interaction of RC^b with other combinatorial principles like simultaneous stationary reflection, versions of weak squares and polarized partition relations.

[1] Jing Zhang, Rado's Conjecture and its Baire Version. preprint, https://arxiv.org/abs/1712.02455.