

Global Chang's Conjecture and singular cardinals

Joint work with Yair Hayut

Foreman [1] asked to what extent a global version of Chang's Conjecture can hold. In [2], the authors proved that, relative to a huge cardinal, ZFC is consistent with the statement that for every regular κ and every $\mu < \kappa$, $(\kappa^+, \kappa) \twoheadrightarrow (\mu^+, \mu)$. In light of constraints imposed by GCH, we asked whether a maximal global Chang's Conjecture is consistent, which says that whenever $\text{cf}(\kappa) \geq \text{cf}(\mu)$, $(\kappa^+, \kappa) \twoheadrightarrow (\mu^+, \mu)$. We show here that it is inconsistent. On the other hand, we show it is consistent relative to a Shelah-for-supercompactness cardinal that for all $\alpha < \beta < \omega^\omega$ of countable cofinality, $(\aleph_{\beta+1}, \aleph_\beta) \twoheadrightarrow (\aleph_{\alpha+1}, \aleph_\alpha)$.

- [1] Foreman, Matthew. Ideals and generic elementary embeddings. Handbook of Set Theory, vol. 2, Springer Dordrecht, 2010, pp. 885–1147.
- [2] Eskew, Monroe and Hayut, Yair. Trans. Amer. Math. Soc. 370 (2018), no. 4, 2879–2905.