Nenad Morača

University of Novi Sad, Serbia
nenad.moraca@dmi.uns.ac.rs

Reversible sequences of natural numbers and reversibility of some disconnected binary structures

Joint work with Miloš S. Kurilić

A relational structure $\mathbb{X}$ is said to be reversible iff every bijective endomorphism $f : \mathbb{X} \to \mathbb{X}$ is an automorphism. Since equivalence relations are, up to isomorphism, characterized by the sequence of cardinalities of its connectivity components, their reversibility can be regarded as a property of the corresponding sequence of cardinals (called reversibility as well). We first show that a sequence $\langle \kappa_i : i \in I \rangle$ of cardinals is reversible iff it is a finite-to-one sequence, or a reversible sequence of natural numbers. Next, we characterize reversible sequences $\langle n_i : i \in I \rangle$ of natural numbers: either it is a finite-to-one sequence, or $K = \{m \in \mathbb{N} : n_i = m$ for infinitely many $i \in I\}$ is a nonempty independent set and $\gcd(K)$ divides at most finitely many elements of the set $\{n_i : i \in I\}$. We isolate a class of disconnected binary structures (containing equivalence relations) such that a structure from the class is reversible iff the sequence of cardinalities of its connectivity components is reversible. In addition, we isolate a class of disconnected binary structures (containing posets that are disjoint unions of chains) such that, for a structure from that class, reversibility of the sequence of cardinalities of its components implies reversibility of the structure. In that way, we can detect numerous examples of reversible posets, as well as reversible topological spaces. Finally, using the characterization of reversible sequences of natural numbers, we characterize reversible posets that are disjoint unions of ordinals and their inverses.