Composition and discrete convergence

Jaroslav Šupina
joint work with Dávid Uhrik

Institute of Mathematics
Faculty of Science
P.J. Šafárik University in Košice

4^{th} of July 2018

Šupina J., *On Ohta–Sakai's properties of a topological space,* Topology Appl. **190** (2015), 119–134.

Let $\mathcal{F}, \mathcal{G}$ be families of real-valued functions on a set X.

We say that X has **a property DL**$(\mathcal{F}, \mathcal{G})$ if any function from $\mathcal{F}$ is a discrete limit of a sequence of functions from $\mathcal{G}$.

Šupina J., *On Ohta–Sakai's properties of a topological space,* Topology Appl. **190** (2015), 119–134.

Let $\mathcal{F}, \mathcal{G}$ be families of real-valued functions on a set X.

We say that X has **a property DL($\mathcal{F}, \mathcal{G}$)** if any function from $\mathcal{F}$ is a discrete limit of a sequence of functions from $\mathcal{G}$.

Convergence of $\langle f_n : n \in \omega \rangle$, $\quad f_n, f : X \to \mathbb{R}$

Pointwise convergence $\qquad f_n \to f$

$$(\forall x \in X)(\forall \varepsilon > 0)(\exists n_0)(\forall n \in \omega)(n \geq n_0 \to |f_n(x) - f(x)| < \varepsilon)$$

Monotone convergence $\qquad f_n \nearrow f \qquad\qquad f_n \searrow f$

$$f_n \nearrow f \Leftrightarrow f_n \to f \wedge (\forall n \in \omega)\ f_n \leq f_{n+1}$$
$$f_n \searrow f \Leftrightarrow f_n \to f \wedge (\forall n \in \omega)\ f_n \geq f_{n+1}$$

Quasi-normal (equal) convergence QN $\qquad f_n \xrightarrow{\text{QN}} f$
there exists $\langle \varepsilon_n : n \in \omega \rangle$ converging to 0 such that

$$(\forall x \in X)(\exists n_0)(\forall n \in \omega)(n \geq n_0 \to |f_n(x) - f(x)| < \varepsilon_n)$$

Discrete convergence D $\qquad f_n \xrightarrow{\text{D}} f$

$$(\forall x \in X)(\exists n_0)(\forall n \in \omega)(n \geq n_0 \to f_n(x) = f(x))$$

Šupina J., *On Ohta–Sakai's properties of a topological space,* Topology Appl. **190** (2015), 119–134.

Let $\mathcal{F}, \mathcal{G}$ be families of real-valued functions on a set X.

We say that X has **a property DL$(\mathcal{F}, \mathcal{G})$** if any function from $\mathcal{F}$ is a discrete limit of a sequence of functions from $\mathcal{G}$.

${}^{X}\mathbb{R}$	the family of all real-valued functions on X
${}^{X}[0,1]$	the family of all functions on X with values in $[0,1]$
$\mathcal{B}$	the family of all Borel functions on X
$\mathcal{B}_1$	the family of all first Baire class functions on X
$\mathrm{M}\boldsymbol{\Delta}^0_2$	the family of all $\boldsymbol{\Delta}^0_2$-measurable functions on X
$\mathcal{U}$	the family of all upper semicontinuous functions on X
$\mathcal{L}$	the family of all lower semicontinuous functions on X
$\mathrm{C}(X)$	the family of all continuous functions on X

$$\mathcal{F} \subseteq {}^{X}\mathbb{R} \qquad \widetilde{\mathcal{F}} = \mathcal{F} \cap {}^{X}[0,1]$$

Bukovská Z., *Quasinormal convergence,* Math. Slovaca **41** (1991), 137–146.

Császár Á. and Laczkovich M., *Some remarks on discrete Baire classes,* Acta Math. Acad. Sci. Hungar. **33** (1979), 51–70.

Theorem

Let X be a normal space, $f : X \to \mathbb{R}$. The following are equivalent.

(1) *f is a discrete limit of a sequence of continuous functions on X.*

(2) *f is a quasi-normal limit of a sequence of continuous functions on X.*

(3) *There is a sequence $\langle F_n : n \in \omega \rangle$ of closed subsets of X such that $f|F_n$ is continuous on F_n for any $n \in \omega$ and $X = \bigcup_{n\in\omega} F_n$.*

Jayne J.E. and Rogers C.A., *First level Borel functions and isomorphisms,* J. Math. Pures et Appl. **61** (1982), 177–205.

Theorem

If A is an analytic subset of a Polish space then A has $\mathrm{DL}(\mathrm{M}\boldsymbol{\Delta}^0_2, \mathrm{C}(X))$.

Bukovský L., Recław I. and Repický M., *Spaces not distinguishing convergences of real–valued functions,* Topology Appl. **112** (2001), 13–40.

Tsaban B. and Zdomskyy L., *Hereditary Hurewicz spaces and Arhangel'skiĭ sheaf amalgamations,* J. Eur. Math. Soc. (JEMS), **14** (2012), 353–372.

Proposition

Any perfectly normal QN-space has $\mathrm{DL}(\mathrm{M}\boldsymbol{\Delta}^0_2, \mathrm{C}(X))$.

Theorem

A perfectly normal space X is a QN-space if and only if X has Hurewicz property and $\mathrm{DL}(\mathcal{B}_1, \mathrm{C}(X))$.

Jayne J.E. and Rogers C.A., *First level Borel functions and isomorphisms,* J. Math. Pures et Appl. **61** (1982), 177–205.

Theorem

If A is an analytic subset of a Polish space then A has $\mathrm{DL}(\mathrm{M}\boldsymbol{\Delta}^0_2, \mathrm{C}(X))$.

Bukovský L., Recław I. and Repický M., *Spaces not distinguishing convergences of real–valued functions,* Topology Appl. **112** (2001), 13–40.

Tsaban B. and Zdomskyy L., *Hereditary Hurewicz spaces and Arhangel'skiĭ sheaf amalgamations,* J. Eur. Math. Soc. (JEMS), **14** (2012), 353–372.

Proposition

Any perfectly normal QN-space has $\mathrm{DL}(\mathrm{M}\boldsymbol{\Delta}^0_2, \mathrm{C}(X))$.

Theorem

A perfectly normal space X is a QN-space if and only if X has Hurewicz property and $\mathrm{DL}(\mathcal{B}_1, \mathrm{C}(X))$.

Jayne J.E. and Rogers C.A., *First level Borel functions and isomorphisms,* J. Math. Pures et Appl. **61** (1982), 177–205.

Theorem

If A is an analytic subset of a Polish space then A has $\mathrm{DL}(\mathrm{M}\mathbf{\Delta}^0_2, \mathrm{C}(X))$.

Bukovský L., Recław I. and Repický M., *Spaces not distinguishing convergences of real–valued functions,* Topology Appl. **112** (2001), 13–40.

Tsaban B. and Zdomskyy L., *Hereditary Hurewicz spaces and Arhangel'skiĭ sheaf amalgamations,* J. Eur. Math. Soc. (JEMS), **14** (2012), 353–372.

Proposition

Any perfectly normal QN-space has $\mathrm{DL}(\mathrm{M}\mathbf{\Delta}^0_2, \mathrm{C}(X))$.

Theorem

A perfectly normal space X is a QN-space if and only if X has Hurewicz property and $\mathrm{DL}(\mathcal{B}_1, \mathrm{C}(X))$.

Convergence of $\langle f_n : n \in \omega \rangle$, $\quad f_n, f : X \to \mathbb{R}$

Pointwise convergence $\quad f_n \to f$

$$(\forall x \in X)(\forall \varepsilon > 0)(\exists n_0)(\forall n \in \omega)(n \geq n_0 \to |f_n(x) - f(x)| < \varepsilon)$$

Monotone convergence $\quad f_n \nearrow f \qquad f_n \searrow f$

$$f_n \nearrow f \Leftrightarrow f_n \to f \wedge (\forall n \in \omega)\ f_n \leq f_{n+1}$$
$$f_n \searrow f \Leftrightarrow f_n \to f \wedge (\forall n \in \omega)\ f_n \geq f_{n+1}$$

Quasi-normal (equal) convergence QN $\quad f_n \xrightarrow{\text{QN}} f$

there exists $\langle \varepsilon_n : n \in \omega \rangle$ converging to 0 such that

$$(\forall x \in X)(\exists n_0)(\forall n \in \omega)(n \geq n_0 \to |f_n(x) - f(x)| < \varepsilon_n)$$

Discrete convergence D $\quad f_n \xrightarrow{\text{D}} f$

$$(\forall x \in X)(\exists n_0)(\forall n \in \omega)(n \geq n_0 \to f_n(x) = f(x))$$

Bukovský L., Recław I. and Repický M., *Spaces not distinguishing pointwise and quasinormal convergence of real functions,* Topology Appl. **41** (1991), 25–40.

A topological space X is a QN-space if each sequence of continuous real-valued functions converging to zero on X is converging quasi-normally.

- Tychonoff QN-space is zero-dimensional
- any QN-subset of a metric separable space is perfectly meager
- perfectly normal QN-space has Hurewicz property
- non$(\mathrm{QN\text{-}space}) = \mathfrak{b}$
- $\mathfrak{b}$-Sierpiński set is a QN-space (exists under $\mathfrak{b} = \mathrm{cov}(\mathcal{N}) = \mathrm{cof}(\mathcal{N})$)

Recław I., *Metric spaces not distinguishing pointwise and quasinormal convergence of real functions,* Bull. Acad. Polon. Sci. **45** (1997), 287–289.

Miller A.W., *On the length of Borel hierarchies,* Ann. Math. Logic **16** (1979), 233–267.

- perfectly normal QN-space is a σ-set
- the theory **ZFC** $+$ “any QN-space is countable” is consistent

Bukovský L., Recław I. and Repický M., *Spaces not distinguishing convergences of real–valued functions,* Topology Appl. **112** (2001), 13–40.

Proposition

Let X be a perfectly normal space. The following are equivalent.

(1) X *is a* σ*-set with* $\mathrm{DL}(\mathrm{M}\boldsymbol{\Delta}^0_2, \mathrm{C}(X))$.

(2) X *possesses* $\mathrm{DL}(\mathcal{B}_1, \mathrm{C}(X))$.

(3) X *possesses* $\mathrm{DL}(\mathcal{B}, \mathrm{C}(X))$.

Cichoń J. and Morayne M., *Universal functions and generalized classes of functions,* Proc. Amer. Math. Soc. **102** (1988), 83–89.

Cichoń J., Morayne M., Pawlikowski J. and Solecki S. *Decomposing Baire functions,* J. Symbolic Logic **56** (1991), 1273–1283.

Let $\mathcal{F}, \mathcal{G}$ be families of real-valued functions on a set X.

dec($\mathcal{F}$, $\mathcal{G}$) denotes the minimal cardinal κ such that any function from $\mathcal{F}$ can be decomposed into κ many functions from $\mathcal{G}$.

X is usually a Polish space.

Cichoń J. and Morayne M., *Universal functions and generalized classes of functions,* Proc. Amer. Math. Soc. **102** (1988), 83–89.

Cichoń J., Morayne M., Pawlikowski J. and Solecki S. *Decomposing Baire functions,* J. Symbolic Logic **56** (1991), 1273–1283.

Let $\mathcal{F}, \mathcal{G}$ be families of real-valued functions on a set X.

dec($\mathcal{F}$, $\mathcal{G}$) denotes the minimal cardinal κ such that any function from $\mathcal{F}$ can be decomposed into κ many functions from $\mathcal{G}$.

X is usually a Polish space.

Cichoń J. and Morayne M., *Universal functions and generalized classes of functions,* Proc. Amer. Math. Soc. **102** (1988), 83–89.

Cichoń J., Morayne M., Pawlikowski J. and Solecki S. *Decomposing Baire functions,* J. Symbolic Logic **56** (1991), 1273–1283.

Let $\mathcal{F}, \mathcal{G}$ be families of real-valued functions on a set X.

dec($\mathcal{F}$, $\mathcal{G}$) denotes the minimal cardinal κ such that any function from $\mathcal{F}$ can be decomposed into κ many functions from $\mathcal{G}$.

X is usually a Polish space.

Theorem

(a) *Let X be a topological space. Then*

$$\mathrm{DL}(\mathcal{U}, \mathrm{C}(X)) \equiv \mathrm{DL}(\mathcal{L}, \mathrm{C}(X)) \equiv \mathrm{DL}(\widetilde{\mathcal{U}}, \mathrm{C}(X)) \equiv \mathrm{DL}(\widetilde{\mathcal{L}}, \mathrm{C}(X)) \equiv (\forall Y \subseteq X)\ \mathrm{DL}(\mathcal{U}, \mathrm{C}(Y)) \equiv (\forall Y \subseteq X)\ \mathrm{DL}(\mathcal{L}, \mathrm{C}(Y)).$$

(b) *Let X be a perfectly normal space. Then*

$$\mathrm{DL}(\mathcal{U}, \mathcal{L}) \equiv \mathrm{DL}(\mathcal{L}, \mathcal{U}) \equiv \mathrm{DL}(\mathcal{L}, \mathrm{C}(X)) \equiv \mathrm{DL}(\mathcal{B}_1, \mathrm{C}(X)) \equiv \mathrm{DL}(\mathcal{B}, \mathrm{C}(X)).$$

Theorem

(a) *Let X be a topological space. Then*

$$\mathrm{DL}(\mathcal{U}, \mathrm{C}(X)) \equiv \mathrm{DL}(\mathcal{L}, \mathrm{C}(X)) \equiv \mathrm{DL}(\widetilde{\mathcal{U}}, \mathrm{C}(X)) \equiv \mathrm{DL}(\widetilde{\mathcal{L}}, \mathrm{C}(X)) \equiv$$
$$(\forall Y \subseteq X)\ \mathrm{DL}(\mathcal{U}, \mathrm{C}(Y)) \equiv (\forall Y \subseteq X)\ \mathrm{DL}(\mathcal{L}, \mathrm{C}(Y)).$$

(b) *Let X be a perfectly normal space. Then*

$$\mathrm{DL}(\mathcal{U}, \mathcal{L}) \equiv \mathrm{DL}(\mathcal{L}, \mathcal{U}) \equiv \mathrm{DL}(\mathcal{L}, \mathrm{C}(X)) \equiv \mathrm{DL}(\mathcal{B}_1, \mathrm{C}(X)) \equiv \mathrm{DL}(\mathcal{B}, \mathrm{C}(X)).$$

- F_σ-measurable function $f : X \to [0,1]$
- Lindenbaum's Theorem: there are lower semicontinuous functions $g : [0,1] \to [0,1]$, $h : X \to [0,1]$ such that $f = g \circ h$
- for h: $\langle F_n : n \in \omega \rangle$ of closed subsets of X, $h|F_n$ is continuous on F_n, $X = \bigcup_{n\in\omega} F_n$
- $f|F_n = g \circ h|F_n$ is lower semicontinuous on F_n
- for g: $\langle F_{n,m} : m \in \omega \rangle$ of closed subsets of X, $f|F_{n,m} = g \circ h|F_{n,m}$ is continuous on $F_{n,m}$, $F_n = \bigcup_{m\in\omega} F_{n,m}$

- F_σ-measurable function $f : X \to [0,1]$
- Lindenbaum's Theorem: there are lower semicontinuous functions $g : [0,1] \to [0,1]$, $h : X \to [0,1]$ such that $f = g \circ h$
- for h: $\langle F_n : n \in \omega \rangle$ of closed subsets of X, $h|F_n$ is continuous on F_n, $X = \bigcup_{n\in\omega} F_n$
- $f|F_n = g \circ h|F_n$ is lower semicontinuous on F_n
- for g: $\langle F_{n,m} : m \in \omega \rangle$ of closed subsets of X, $f|F_{n,m} = g \circ h|F_{n,m}$ is continuous on $F_{n,m}$, $F_n = \bigcup_{m\in\omega} F_{n,m}$

- F_σ-measurable function $f : X \to [0,1]$

- Lindenbaum's Theorem: there are lower semicontinuous functions $g : [0,1] \to [0,1]$, $h : X \to [0,1]$ such that $f = g \circ h$

- for h: $\langle F_n : n \in \omega \rangle$ of closed subsets of X, $h|F_n$ is continuous on F_n, $X = \bigcup_{n\in\omega} F_n$

- $f|F_n = g \circ h|F_n$ is lower semicontinuous on F_n

- for g: $\langle F_{n,m} : m \in \omega \rangle$ of closed subsets of X, $f|F_{n,m} = g \circ h|F_{n,m}$ is continuous on $F_{n,m}$, $F_n = \bigcup_{m\in\omega} F_{n,m}$

- F_σ-measurable function $f : X \to [0,1]$
- Lindenbaum's Theorem: there are lower semicontinuous functions $g : [0,1] \to [0,1]$, $h : X \to [0,1]$ such that $f = g \circ h$
- for h: $\langle F_n : n \in \omega \rangle$ of closed subsets of X, $h|F_n$ is continuous on F_n, $X = \bigcup_{n\in\omega} F_n$
- $f|F_n = g \circ h|F_n$ is lower semicontinuous on F_n
- for g: $\langle F_{n,m} : m \in \omega \rangle$ of closed subsets of X, $f|F_{n,m} = g \circ h|F_{n,m}$ is continuous on $F_{n,m}$, $F_n = \bigcup_{m\in\omega} F_{n,m}$

- F_σ-measurable function $f : X \longrightarrow [0, 1]$
- Lindenbaum's Theorem: there are lower semicontinuous functions $g : [0, 1] \longrightarrow [0, 1]$, $h : X \longrightarrow [0, 1]$ such that $f = g \circ h$
- for h: $\langle F_n : n \in \omega \rangle$ of closed subsets of X, $h|F_n$ is continuous on F_n, $X = \bigcup_{n\in\omega} F_n$
- $f|F_n = g \circ h|F_n$ is lower semicontinuous on F_n
- for g: $\langle F_{n,m} : m \in \omega \rangle$ of closed subsets of X, $f|F_{n,m} = g \circ h|F_{n,m}$ is continuous on $F_{n,m}$, $F_n = \bigcup_{m\in\omega} F_{n,m}$

Lindenbaum A., *Sur les superpositions des fonctions représentables analytiquement,* Fund. Math. **23** (1934), 15–37.

9. *Théorème.* *Il existe une fonction λ semi-continue inférieurement, telle que, pour tout nombre ordinal β $(0 < \beta < \Omega)$, chaque fonction f de classe $\mathfrak{L}_{\beta+1}$ peut être representée sous la forme:*

$$f = \lambda \bigcirc g,$$

la fonction g (ne prenant que des valeurs irrationnelles) étant convenablement choisie dans $\mathfrak{L}_{\beta}$.

Cichoń J., Morayne M., Pawlikowski J. and Solecki S. *Decomposing Baire functions,* J. Symbolic Logic **56** (1991), 1273–1283.

THEOREM 4.4 (A. Lindenbaum). *There exists $g \in L_0$ such that $L_{\alpha+1}(Z) = \{g \circ h: h \in U_\alpha(Z)\}$ for every $\alpha < \omega_1$ and every Polish space Z.*

Convergence of $\langle f_n : n \in \omega \rangle$, $\quad f_n, f : X \to \mathbb{R}$

Pointwise convergence $\qquad f_n \to f$

$$(\forall x \in X)(\forall \varepsilon > 0)(\exists n_0)(\forall n \in \omega)(n \geq n_0 \to |f_n(x) - f(x)| < \varepsilon)$$

Monotone convergence $\qquad f_n \nearrow f \qquad\qquad f_n \searrow f$

$$f_n \nearrow f \Leftrightarrow f_n \to f \wedge (\forall n \in \omega)\ f_n \leq f_{n+1}$$
$$f_n \searrow f \Leftrightarrow f_n \to f \wedge (\forall n \in \omega)\ f_n \geq f_{n+1}$$

Quasi-normal (equal) convergence QN $\qquad f_n \xrightarrow{\text{QN}} f$
there exists $\langle \varepsilon_n : n \in \omega \rangle$ converging to 0 such that

$$(\forall x \in X)(\exists n_0)(\forall n \in \omega)(n \geq n_0 \to |f_n(x) - f(x)| < \varepsilon_n)$$

Discrete convergence D $\qquad f_n \xrightarrow{\text{D}} f$

$$(\forall x \in X)(\exists n_0)(\forall n \in \omega)(n \geq n_0 \to f_n(x) = f(x))$$

Baire 1899:

$$\mathbf{B}_0(X) = \mathrm{C}(X)$$

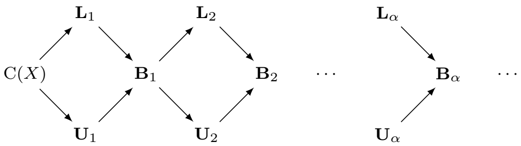
$$\mathbf{B}_\alpha(X) = \left\{ f : f_n \to f \wedge f_n \in \bigcup_{\beta<\alpha} \mathbf{B}_\beta(X) \right\}$$

Young 1910:

$$\mathbf{L}_0(X) = \mathbf{U}_0(X) = \mathrm{C}(X)$$

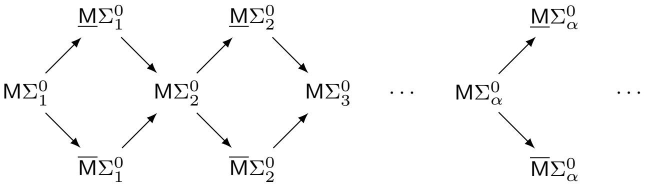
$$\mathbf{L}_\alpha(X) = \left\{ f : f_n \nearrow f \wedge f_n \in \bigcup_{\beta<\alpha} \mathbf{U}_\beta(X) \right\}$$

$$\mathbf{U}_\alpha(X) = \left\{ f : f_n \searrow f \wedge f_n \in \bigcup_{\beta<\alpha} \mathbf{L}_\beta(X) \right\}$$



$$\mathrm{M}\Sigma^0_\alpha(X) = \{f : (\forall U \text{ open in } [0,1])\ f^{-1}(U) \in \Sigma^0_\alpha(X)\}$$
$$\underline{\mathrm{M}}\Sigma^0_\alpha(X) = \{f : (\forall r \in [0,1])\ f^{-1}((r,1]) \in \Sigma^0_\alpha(X)\}$$
$$\overline{\mathrm{M}}\Sigma^0_\alpha(X) = \{f : (\forall r \in [0,1])\ f^{-1}([0,r)) \in \Sigma^0_\alpha(X)\}$$

$$\mathbf{B}_\alpha(X) = \mathrm{M}\Sigma^0_{\alpha+1}(X)$$
$$\mathbf{L}_\alpha(X) = \underline{\mathrm{M}}\Sigma^0_\alpha(X)$$
$$\mathbf{U}_\alpha(X) = \overline{\mathrm{M}}\Sigma^0_\alpha(X)$$



CSÁSZÁR, Á., LACZKOVICH, M.: *Some remarks on discrete Baire classes*, Acta Math. Acad. Sci. Hung. 33 (1979), 51-70.

SIKORSKI, R.: *Funkcje Rzeczywiste I*, Panstwowe Wydawnictwo Naukowe 1958.

Uhrik D., *Compositions of discontinuous functions,* Master Thesis, UPJŠ, Košice, 2018.

Theorem (A. Lindenbaum)

Let X *be a perfectly normal topological space,* $\alpha < \omega_1$*. Then there exists a* $g \in \mathbf{L}_1([0,1])$ *such that*

$$\mathbf{L}_{\alpha+1}(X) = g \circ \mathbf{U}_\alpha(X, [0,1]).$$

Corollary

Let X *be a perfectly normal topological space,* $\alpha < \omega_1$*. Then we have*

$$\mathbf{L}_{\alpha+1} = \mathbf{L}_1 \circ \mathbf{U}_\alpha = \mathbf{L}_1 \circ \mathbf{L}_\alpha = \mathbf{L}_1 \circ \mathbf{B}_\alpha$$

$$\mathbf{U}_{\alpha+1} = \mathbf{U}_1 \circ \mathbf{L}_\alpha = \mathbf{U}_1 \circ \mathbf{U}_\alpha = \mathbf{U}_1 \circ \mathbf{B}_\alpha$$

Theorem (A. Lindenbaum)

Let X *be a perfectly normal topological space,* $0 < \alpha, \beta < \omega_1$*. Then*

$$\mathbf{L}_{\beta+\alpha}(X) = \mathbf{L}_\alpha([0,1]) \circ \mathbf{L}_\beta(X, [0,1]).$$

Uhrik D., *Compositions of discontinuous functions,* Master Thesis, UPJŠ, Košice, 2018.

Theorem (A. Lindenbaum)

Let X be a perfectly normal topological space, $\alpha < \omega_1$. Then there exists a $g \in \mathbf{L}_1([0,1])$ such that

$$\mathbf{L}_{\alpha+1}(X) = g \circ \mathbf{U}_\alpha(X, [0,1]).$$

Corollary

Let X be a perfectly normal topological space, $\alpha < \omega_1$. Then we have

$$\mathbf{L}_{\alpha+1} = \mathbf{L}_1 \circ \mathbf{U}_\alpha = \mathbf{L}_1 \circ \mathbf{L}_\alpha = \mathbf{L}_1 \circ \mathbf{B}_\alpha$$
$$\mathbf{U}_{\alpha+1} = \mathbf{U}_1 \circ \mathbf{L}_\alpha = \mathbf{U}_1 \circ \mathbf{U}_\alpha = \mathbf{U}_1 \circ \mathbf{B}_\alpha$$

Theorem (A. Lindenbaum)

Let X be a perfectly normal topological space, $0 < \alpha, \beta < \omega_1$. Then

$$\mathbf{L}_{\beta+\alpha}(X) = \mathbf{L}_\alpha([0,1]) \circ \mathbf{L}_\beta(X, [0,1]).$$

Uhrik D., *Compositions of discontinuous functions*, Master Thesis, UPJŠ, Košice, 2018.

Theorem (A. Lindenbaum)

Let X be a perfectly normal topological space, $\alpha < \omega_1$. Then there exists a $g \in \mathbf{L}_1([0,1])$ such that

$$\mathbf{L}_{\alpha+1}(X) = g \circ \mathbf{U}_\alpha(X, [0,1]).$$

Corollary

Let X be a perfectly normal topological space, $\alpha < \omega_1$. Then we have

$$\mathbf{L}_{\alpha+1} = \mathbf{L}_1 \circ \mathbf{U}_\alpha = \mathbf{L}_1 \circ \mathbf{L}_\alpha = \mathbf{L}_1 \circ \mathbf{B}_\alpha$$
$$\mathbf{U}_{\alpha+1} = \mathbf{U}_1 \circ \mathbf{L}_\alpha = \mathbf{U}_1 \circ \mathbf{U}_\alpha = \mathbf{U}_1 \circ \mathbf{B}_\alpha$$

Theorem (A. Lindenbaum)

Let X be a perfectly normal topological space, $0 < \alpha, \beta < \omega_1$. Then

$$\mathbf{L}_{\beta+\alpha}(X) = \mathbf{L}_\alpha([0,1]) \circ \mathbf{L}_\beta(X, [0,1]).$$

Cichoń J., Morayne M., Pawlikowski J. and Solecki S., *Decomposing Baire functions*, J. Symbolic Logic **56** (1991), 1273–1283.

Set $\mathbf{S} \subseteq [0,1]$, dense in $[0,1]$, $\phi : {}^{\omega}\mathbf{S} \to \mathbf{T} \subseteq [0,1]$ is a homeomorphism.

Definition

Let X be a topological space, $(f_i)_{i<\omega}$ be a sequence of functions from ${}^{X}[0,1]$. The *coding function* of $(f_i)_{i<\omega}$ from X to ${}^{\omega}[0,1]$ is defined by

$$\mathbf{f}(x) = (f_i(x))_{i<\omega} .$$

Proposition

Let X be a perfectly normal topological space, $\alpha < \omega_1$, $(f_i)_{i<\omega}$ be a sequence of functions from $\mathbf{L}_\alpha(X, \mathbf{S})$. Then $\phi \circ \mathbf{f} \in \mathbf{L}_\alpha(X)$.

Proposition

The function $\mathrm{s} : {}^{\omega}[0,1] \to [0,1]$ *defined by*

$$\mathrm{s}((t_i)_{i<\omega}) = \sup\{t_i : i < \omega\}$$

is in $\mathbf{L}_1({}^{\omega}[0,1])$.

Cichoń J., Morayne M., Pawlikowski J. and Solecki S., *Decomposing Baire functions,* J. Symbolic Logic **56** (1991), 1273–1283.

Set $\mathbf{S} \subseteq [0,1]$, dense in $[0,1]$, $\phi : {}^{\omega}\mathbf{S} \to \mathbf{T} \subseteq [0,1]$ is a homeomorphism.

Definition

Let X be a topological space, $(f_i)_{i<\omega}$ be a sequence of functions from ${}^{X}[0,1]$. The *coding function* of $(f_i)_{i<\omega}$ from X to ${}^{\omega}[0,1]$ is defined by

$$\mathbf{f}(x) = (f_i(x))_{i<\omega} .$$

Proposition

Let X be a perfectly normal topological space, $\alpha < \omega_1$, $(f_i)_{i<\omega}$ be a sequence of functions from $\mathbf{L}_\alpha(X, \mathbf{S})$. Then $\phi \circ \mathbf{f} \in \mathbf{L}_\alpha(X)$.

Proposition

The function $\mathsf{s} : {}^{\omega}[0,1] \to [0,1]$ *defined by*

$$\mathsf{s}((t_i)_{i<\omega}) = \sup\{t_i : i < \omega\}$$

is in $\mathbf{L}_1({}^{\omega}[0,1])$.

Cichoń J., Morayne M., Pawlikowski J. and Solecki S., *Decomposing Baire functions,* J. Symbolic Logic **56** (1991), 1273–1283.

Set $\mathbf{S} \subseteq [0,1]$, dense in $[0,1]$, $\phi : {}^{\omega}\mathbf{S} \to \mathbf{T} \subseteq [0,1]$ is a homeomorphism.

Definition

Let X be a topological space, $(f_i)_{i<\omega}$ be a sequence of functions from ${}^{X}[0,1]$. The *coding function* of $(f_i)_{i<\omega}$ from X to ${}^{\omega}[0,1]$ is defined by

$$\mathbf{f}(x) = (f_i(x))_{i<\omega} \, .$$

Proposition

Let X be a perfectly normal topological space, $\alpha < \omega_1$, $(f_i)_{i<\omega}$ be a sequence of functions from $\mathbf{L}_\alpha(X, \mathbf{S})$. Then $\phi \circ \mathbf{f} \in \mathbf{L}_\alpha(X)$.

Proposition

The function $\mathsf{s} : {}^{\omega}[0,1] \to [0,1]$ *defined by*

$$\mathsf{s}((t_i)_{i<\omega}) = \sup \{t_i : i < \omega\}$$

is in $\mathbf{L}_1({}^{\omega}[0,1])$.

Cichoń J., Morayne M., Pawlikowski J. and Solecki S., *Decomposing Baire functions*, J. Symbolic Logic **56** (1991), 1273–1283.

Set $\mathbf{S} \subseteq [0,1]$, dense in $[0,1]$, $\phi : {}^{\omega}\mathbf{S} \to \mathbf{T} \subseteq [0,1]$ is a homeomorphism.

Definition

Let X be a topological space, $(f_i)_{i<\omega}$ be a sequence of functions from ${}^{X}[0,1]$. The *coding function* of $(f_i)_{i<\omega}$ from X to ${}^{\omega}[0,1]$ is defined by

$$\mathbf{f}(x) = (f_i(x))_{i<\omega}\,.$$

Proposition

Let X be a perfectly normal topological space, $\alpha < \omega_1$*,* $(f_i)_{i<\omega}$ *be a sequence of functions from* $\mathbf{L}_\alpha(X, \mathbf{S})$*. Then* $\phi \circ \mathbf{f} \in \mathbf{L}_\alpha(X)$*.*

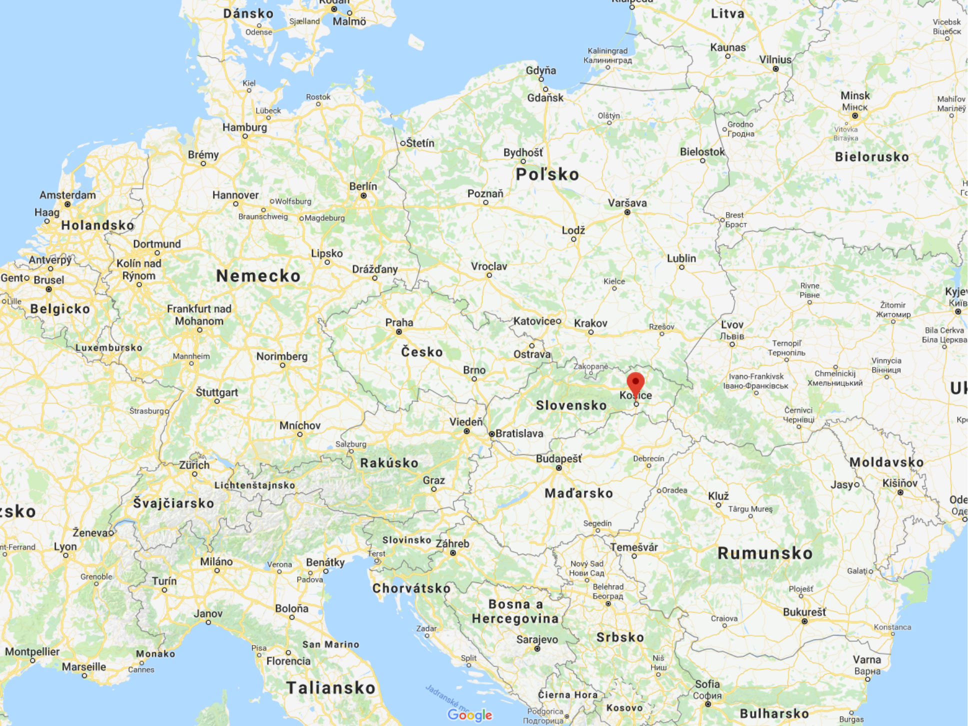
Proposition

The function $\mathsf{s} : {}^{\omega}[0,1] \to [0,1]$ *defined by*

$$\mathsf{s}((t_i)_{i<\omega}) = \sup\{t_i : i < \omega\}$$

is in $\mathbf{L}_1({}^{\omega}[0,1])$*.*

Thanks for Your attention!


Dánsko
Sjælland
Malmö
Odense
Litva
Kaunas
Vilnius
Vicebsk
Віцебск
Kaliningrad
Калининград
Gdyňa
Gdańsk
Minsk
Мінск
Mahiľov
Магілёў
Kiel
Rostok
Lübeck
Olštýn
Grodno
Гродна
Vitovka
Вітаўка
Hamburg
Štetín
Bydhošť
Bielostok
Bielorusko
Brémy
Poľsko
Amsterdam
Hannover
Wolfsburg
Berlín
Poznaň
Varšava
Haag
Braunschweig
Magdeburg
Brest
Брэст
Holandsko
Lodž
Dortmund
Antverpy
Kolín nad
Rýnom
Lipsko
Lublin
Gent
Brusel
Nemecko
Drážďany
Vroclav
Lille
Belgicko
Kielce
Rivne
Рівне
Frankfurt nad
Mohanom
Žitomir
Житомир
Kyjev
Київ
Praha
Katovice
Krakov
Rzešov
Ľvov
Львів
Luxembursko
Mannheim
Norimberg
Česko
Ostrava
Ternopiľ
Тернопіль
Bila Cerkva
Біла Церква
Zakopané
Brno
Vinnycia
Вінниця
Ivano-Frankivsk
Івано-Франківськ
Chmelnickij
Хмельницький
Stuttgart
Košice
Slovensko
Štrasburg
Černivci
Чернівці
Mníchov
Viedeň
Bratislava
Salzburg
Rakúsko
Budapešť
Debrecín
Zürich
Moldavsko
Graz
Lichtenštajnsko
Oradea
Jasy
Kišiňov
Maďarsko
Kluž
Švajčiarsko
Târgu Mureş
Ženeva
Segedín
Slovinsko
Záhreb
Temešvár
Lyon
Rumunsko
Miláno
Verona
Benátky
Terst
Nový Sad
Нови Сад
Galați
Grenoble
Padova
Turín
Chorvátsko
Belehrad
Београд
Ploješť
Bosna a
Hercegovina
Bukurešť
Janov
Boloňa
Craiova
Zadar
Sarajevo
Srbsko
Konstanca
San Marino
Montpellier
Pisa
Monako
Niš
Ниш
Marseille
Cannes
Florencia
Split
Varna
Варна
Sofia
София
Taliansko
Jadranské more
Čierna Hora
Kosovo
Podgorica
Подгорица
Bulharsko
Burgas
Google

Set-theoretic methods in topology and real functions theory, dedicated to 80th birthday of Lev Bukovsky

KOŠICE

9.9. - 13.9.2019

Scientific Committee

Chodounský David
Goldstern Martin
Hart Klaas Pieter
Holá Ľubica
Jech Thomas
Repický Miroslav
Sakai Masami
Zdomskyy Lyubomyr

Organizing Committee

Eliáš Peter
Repický Miroslav
Šupina Jaroslav