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Alternatives to Halpern and Läuchli's theorem

It is shown that the fact that BPI holds in Cohen's symmetric model can be used as an equal substitute for Halpern and Läuchli's theorem in proofs where it is used. Also, some alternatives to Halpern and Läuchli's theorem are given in the form of absoluteness theorems for a certain class of statements. The article also contains a new proof of Halpern and Läuchli's theorem.

Theorem. *The Cohen's symmetric model satisfies the following principle* Z'*: For any finite partitioning of* K^d*, where* K *is Cantor's set and* d *is positive integer, at least one of parts has subset which is Cartesian product of somewhere dense subsets. Therefore,* $\mathrm{ZF} + \mathrm{BPI} + \mathrm{Z}'$ *is consistent theory.*

Theorem. *Let* F *be a formula of the form*

$$(\forall x)(\exists y \in \mathrm{tc}(\{x\}))(\forall z)R(x, y, z), \tag{1}$$

where the formula $R(x, y, z)$ *is absolute for transition between ground model and Cohen's symmetric model as it's extension. Then, it holds*

$$(\mathrm{ZF} + \mathrm{BPI} + \mathrm{Z}' \vdash F) \Rightarrow (\mathrm{ZFC} \vdash F).$$

Theorem. *Denote we by* $\mathrm{Z}''(\kappa)$ *the generalization of the* $(\forall d)DDF(d)$ *principle from [1] to a partitioning of cardinality not greater than* κ*. Let* M *be any model of* ZFC *and* κ *is its infinite cardinal. Then there is the a regular cardinal* λ *of the model* M *such that by adding* λ *Cohen reals we obtain the model satisfying* $\mathrm{Z}''(\kappa)$*. Therefore, the statements like* $\mathrm{Z}''(\aleph_{\omega_5})$ *are consistent with* ZFC*.*

Theorem. *If F is a formula of the form (1), where R is the formula which is absolute for a ccc forcing extensions, then the statements like the following*

$$(\mathrm{ZFC} + \mathrm{Z}''(\aleph_{\omega_5}) \vdash F) \Rightarrow (\mathrm{ZFC} \vdash F)$$

hold. Note that the single dimensional case of the $\mathrm{Z}''(\kappa)$ is the generalization of the Baire's category theorem for partitioning of the space $\mathbb{R}^n$ of cardinality not greater than κ.

[1] Zucker, A., A new proof of the 2-dimensional Halpern–Läuchli Theorem, `https://www.math.cmu.edu/~andrewz/HL2d.pdf` (2017)