Jarosław Swaczyna

Institute of Mathematics of Łódź University of Technology, Poland
jaroslaw.swaczyna@p.lodz.pl

Continuity of coordinate functionals for filter Schauder basis

Joint work with Tomasz Kania and Noe de Rancourt

Given a filter of subsets of natural numbers $\mathcal{F}$ we say that a sequence (x_n) is $\mathcal{F}$-convergent to x if for every $\varepsilon > 0$ condition $\{n \in \mathbb{N} : d(x_n, x) < \varepsilon\} \in \mathcal{F}$ holds. We may use this notion to generalize the idea of Schauder basis, namely we say that a sequence (e_n) is an $\mathcal{F}$-basis if for every $x \in X$ there exists a unique sequence of scalars (α_n) s.t. $\sum_{n,F} \alpha_n e_n = x$, which means that the sequence of partial sums is F-convergent to x. Once such a notion is introduced it is natural to ask whenever corresponding coordinate functionals are continuous. Such a question was posed by V. Kadets during the 4th conference Integration, Vector Measures, and Related Topics held in 2011 in Murcia. There is an obstacle related to the lack of uniform boundedness of functionals related to F basis, due to which we can not use Functional Analysis techniques to mimic the proof of classical talk. Surprisingly, Set Theorey tools allowed us to give positive answer to Kadets' question. During my talk, I will discuss the problem and provide two proofs of continuity of considered functionals. This is joint work with Tomasz Kania and Noe de Rancourt.

[1] Kania T., Swaczyna J., Large cardinals and continuity of coordinate functionals of filter bases in Banach spaces. Bull. Lond. Math. Soc. 53 (2021), no. 1, 231–239.

[2] de Rancourt N., Kania T., Swaczyna J., Continuity of coordinate functionals of filter bases in Banach spaces. submitted, avaible online: https://arxiv.org/abs/2203.15123