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Applications of the open dihypergraph dichotomy for generalized Baire spaces

Joint work with Philipp Schlicht

The open graph dichotomy for a given subset X of the Baire space ${}^{\omega}\omega$ states that any open graph on X either contains a large complete subgraph or admits a countable coloring. It is a definable version of the open coloring axiom for X and it generalizes the perfect set property. Miller, Carroy and Soukup showed that several well-known results regarding the second level of the Borel hierarchy follow from an infinite dimensional generalization of the open graph dichotomy.

We show that several of these applications, including the Hurewicz dichotomy and the Jayne-Rogers theorem, can be lifted to the generalized Baire space ${}^{\kappa}\kappa$, where κ is an uncountable cardinal such that $\kappa^{<\kappa} = \kappa$. We also obtain new applications, such as the determinacy of Väänänen's perfect set game for all subsets of ${}^{\kappa}\kappa$ and an asymmetric version of the Baire property. These results extend previous work of Lücke, Motto Ros, Schlicht, Väänänen and the author.