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Complicated Souslin trees

For a regular uncountable cardinal κ, a κ-**Aronszajn tree** is a downward-closed subcollection T of ${}^{<\kappa}2$ such that, for every $\alpha < \kappa$, its α^{th}-level $T_\alpha := T \cap {}^{\alpha}2$ has size $< \kappa$, and such that $(T, \subseteq)$ has no chains of size κ. It is a κ-**Souslin tree** if, in addition, it has no antichains of size κ.

If κ is a successor of a regular cardinal λ, then a standard combinatorial hypothesis such as $\mathrm{GCH} + \diamondsuit(E^{\lambda^+}_{\lambda})$ is sufficient for the construction of κ-Souslin trees of various sorts. In contrast, for κ a successor of a singular cardinal or an inaccessible, GCH and diamonds are not even sufficient for the construction of a κ-Aronszajn tree. To overcome this problem, Brodsky and Rinot introduced a *parameterized proxy principle* $P(\kappa, \ldots)$ that allows the construction of κ-Souslin trees regardless of the identity of κ, whether it is the case that $\kappa = \aleph_2$, $\kappa = \aleph_{\omega_1+1}$, or $\kappa = \aleph_\kappa$.

They showed that all previously known sufficient conditions for the construction of κ-Souslin trees imply an instance of the proxy principle $P(\kappa, \ldots)$ sufficient for the construction of a tree of the same sort. The construction of a plain κ-Souslin tree can be carried out assuming a very weak instance of $P(\kappa, \ldots)$, and more complicated trees (such as coherent Souslin trees, or Souslin trees with precise control over their reduced powers) can be obtained from stronger instances.

In this poster, we present new proxy-based constructions of κ-Souslin trees:

- We construct a κ-Souslin tree T such that $\Vdash_T$ "T is a κ-Kurepa tree".

- We construct a κ-Souslin tree such that each limit level T_α omits no more than one cofinal branch from $\bigcup_{\beta<\alpha} T_\beta$.

- For a prescribed (finite or infinite) cardinal $\chi < \kappa$, we construct a κ-Souslin tree such that for every injective choice $\langle t_i \mid i < \sigma \rangle$ of less than χ many nodes of some level of the tree T, the product of their cones $\bigotimes_{i<\sigma}(t_i)^{\uparrow}$ is again κ-Souslin, but the same fails badly for sequences of length $\sigma = \chi$.

- For $\chi < \kappa$ and a given κ-Souslin tree T, we construct a family $\mathcal{S}$ of 2^κ many κ-Souslin trees such that for any subfamily $\mathcal{S}' \subseteq \mathcal{S}$ of size less than χ, the product $T \times \bigotimes_{S\in\mathcal{S}'} S$ is again κ-Souslin.
- We also deal with the dual problem of making the products of the previous item non-Souslin in a very strong sense.