SETTOP 2024 NOVI SAD

Foreword

Novi Sad Conference in Set theory and General Topology (SETTOP) has been organized biannually by the Department of Mathematics and Informatics of the Faculty of Sciences at the University of Novi Sad. This is its the sixth edition, succeeding LogTop 2012, organized as one of three conferences celebrating 50 years of the Seminar for Analysis and Foundation of Mathematics, led by Professor Bogoljub Stanković, SETTOP 2014, SETTOP 2016 and SETTOP 2018. Unfortunately, due to the well known reasons, we were not in position to organize the conference in 2020 and 2021. But, in 2022, together with SETTOP 2022, we organized Young Set Theory Workshop - YSTW.

Traditionally, the main topics of this year's conference are set theory, model theory and general topology.

This year's edition has the largest number of invited speakers so far.

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ABSTRACTS

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An unexpected connection between a prisoners-and-hats puzzle and set theory

There are many problems/puzzles in combinatorics, coding theory, and also recreational mathematics, posed in the setting with a warden gathering a number of prisoners and playing a kind of a game with them, where each prisoner will get a hat of a certain color, and then each of them has to guess (under certain rules) the color of his hat. Two decades ago, one of such games—n prisoners are standing in a line (each prisoner sees all the hats in front of him, but none of the hats behind him) and then each of them guesses (one by one) whether the hat on his head is white or black, aiming to maximize the total number of correct guesses—turned out to have a version that can be presented as a puzzle in set theory, as solving the same problem with infinitely many prisoners instead of n requires (maybe unexpectedly) invoking the axiom of choice in a sneaky manner.

In this talk another puzzle with prisoners and hats will be presented, together with its (easy) solution. Then we shall consider a variation in which only one parameter is slightly changed. Intuitively, one would probably guess that this modification, although it might change the answer, still does not change the purely combinatorial nature of the problem. However, we shall see that this is far from truth, as for the solution of this new puzzle an excursion a little bit deeper in the realms of set theory is ineluctable.

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Elementary self-extensions of the universe and applications in Ramsey Theory

By a well-known classic result of K. Kunen, there are no elementary extensions $j : \mathbb{V} \prec \mathbb{V}$ of the universe of ZFC in itself. However, dropping the axiom of Foundation (and without the use of large cardinals), one can consistently produce elementary extensions $j : \mathbb{V} \prec \mathbb{T}$ onto transitive classes $\mathbb{T}$, where $j(\omega)$ properly extends ω. By composing with inclusion, we obtain "bounded quantifier" elementary self-embeddings $* : \mathbb{V} \prec_b \mathbb{V}$ of the universe in which ${}^*A \neq \{{}^*a \mid a \in A\}$ for every infinite set A. Such embeddings were studied several years ago to provide foundations to the methods of nonstandard analysis, and gave rise the so-called "nonstandard set theories".

Recently, the iterated use of nonstandard extensions of the natural numbers ${}^*\mathbb{N}, {}^{**}\mathbb{N}, {}^{***}\mathbb{N}, \ldots$ proved to be useful in Arithmetic Ramsey Theory, as they are suitable for formalizing the intuitive idea of "different levels of infinity". Clearly, the above self-embeddings $* : \mathbb{V} \prec_b \mathbb{V}$ can naturally accommodate such arguments. To illustrate the idea, we will present (short) nonstandard proofs of some results in the area, including the following recent one:

- In every finite coloring $\mathbb{N} = C_1 \cup \ldots \cup C_r$ there exists a monochromatic exponential triple $a, b, b^a \in C_i$.

We will also present a new alternative proof of the classic van der Waerden's Theorem, where the use of different levels of infinity is clearly evident.

- For every finite coloring $\mathbb{N} = C_1 \cup \ldots \cup C_r$ and for every ℓ there exists a monochromatic ℓ-term arithmetic progression

$$a, a + d, \ldots, a + (\ell - 1)d \in C_i.$$

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Failure of an higher analogue of Mho

Justin Moore's weak club-guessing principle $\mho$ admits various possible generalizations to the second uncountable cardinal. One of them was shown to hold in ZFC by Shelah. A stronger version was shown to follow from several consequences of the continuum hypothesis by Inamdar and Rinot. In this short talk, we shall sketch a proof that the stronger one may consistently fail. More specifically, starting with a supercompact cardinal and an inaccessible cardinal above it, we devise a notion of forcing consisting of finite working parts and finitely many two types of (virtual) models as side conditions, to violate this analog of $\mho$ at the second uncountable cardinal.

[1] Feldman Failure of an higher analogue of Mho. Arxiv version (2024).

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Concentrated sets and γ-sets in the Miller model

Joint work with Piotr Szewczak and Lyubomyr Zdomskyy

Bartoszyński and Halbeisen conjectured that in the Miller model there exists a concentrated set of reals of size $\mathfrak{c} = \omega_2$. Let us recall that a set $X \subseteq 2^\omega$ is concentrated if there exists a countable $Q \subseteq X$ such that $|X \setminus U| \leq \omega$ for every open set $U \subseteq 2^\omega$ with $Q \subseteq U$. In our talk we shall present the main ideas of the proof that this conjecture is false. Concentrated sets are canonical examples of Rothberger spaces of reals. We want to analyse the possible cardinalities of sets of reals satisfying selection principles in the Miller model. To avoid triviality we are interested in the totally imperfect cases, i.e. spaces that do not contain a copy of the Cantor space. Note that since $\mathfrak{d}$-concentrated sets are totally imperfect Menger spaces, there are such spaces of size continuum (since $\mathfrak{d} = \mathfrak{c}$). Moreover, we will see that for the strongest selection principle, the γ-set property, only cardinality atmost ω_1 is possible. We hope that the tools of our results can be used as a prototype for the non-existence of Rothberger sets of reals with cardinality $\mathfrak{c}$. The goal would be to prove the same for Hurewicz totally imperfect sets of reals, the latter being a weaker property than Rothberger in the Miller model.

[1] Bartoszyński, T.; Halbeisen, L., *On a theorem of Banach and Kuratowski and K-Lusin sets*. Rocky Mountain J. Math. **33** (2003), 1223–1231.

[2] Haberl, V.; Szewczak, P.; Zdomskyy, L., *Concentrated sets and γ-sets in the Miller model*. https://arxiv.org/pdf/2310.03864, to appear in Topology and its Applications.

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An ordinal rank measuring universality

Joint work with S. Shelah

We shall discuss a natural ordinal rank on mathematical structures that measures how complicated the object is. The main ingredient is an abstract concept of one-point extension which we call a transition. In natural cases, namely, categories leading to universal homogeneous objects (e.g. graphs, orderings, etc.) the rank behaves as one would expect: It is either a countable ordinal or $+$infinity – precisely when the object is universal. We shall also discuss this rank in metric-enriched categories, where approximate commutativity plays a significant role.

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On generic topological embeddings

Joint work with Wiesław Kubiś and Sławomir Turek

Applying Fraïssé theory we have obtained some results about generic topological embeddings, extending homeomorphism and retracts of some subsets of the Čech-Stone remainder ω^* and ultrametric spaces (generalized Baire spaces) κ^λ, where κ, λ are regular cardinals such that $\omega \leq \lambda \leq \kappa$. Results on ultrametric spaces will be presented.

[1] W. Kubiś, A. Kucharski, S. Turek, *On generic topological embeddings*, arXiv:2310.05043.

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Simultaneously nonvanishing higher derived limits

Joint work with Matteo Casarosa

We present new results concerning the derived limits of inverse systems of the form $\mathbf{A}[H]$, where H is an arbitrary abelian group. These inverse systems are indexed by the Baire space ${}^{\omega}\omega$, and their derived limits show up in foundational questions in various mathematical settings, including the study of strong homology and condensed mathematics. It has been known since at least the 1980s that these derived limits are sensitive to set-theoretic assumptions, and the last five years has seen considerable progress in our understanding of the interplay between combinatorial set theory and their behavior, particularly in degrees 2 and higher. For example, by recent work of Bergfalk, Hrušák, and the speaker [2], and further work by Bannister [1], we know that if one adds $\beth_\omega$-many Cohen reals to any model of ZFC, then $\lim^n \mathbf{A}[H] = 0$ for every $n > 0$ and every abelian group H. In this talk, we discuss some recent complementary results. In particular, we show that, if the dominating number equals $\aleph_n$ for some $n < \omega$, then there is an abelian group H for which $\lim^n \mathbf{A}[H] \neq 0$, thus showing that the simultaneous vanishing of $\lim^n \mathbf{A}[H]$ for all n and H is incompatible with any value of the continuum below $\aleph_\omega$. In addition, we show that conjunctions of square principles and weak diamonds yield nontrivial derived limits and use this to exhibit a model of ZFC in which $\lim^n \mathbf{A}[H] \neq 0$ simultaneously for every $n \in [2, \omega)$ and every nontrivial abelian group H. This builds on earlier work of Veličković and Vignati [4] and of Casarosa [3].

[1] Bannister, N., Additivity of derived limits in the Cohen model. arXiv preprint, arXiv:2302.07222 (2023).

[2] Bergfalk, J., Hrušák, M., Lambie-Hanson, C., Simultaneously vanishing higher derived limits without large cardinals. Journal of Mathematical Logic, Vol. 23 No. 1 (2023), 2250019.

[3] Casarosa, M., Nonvanishing derived limits without scales. arXiv preprint, arXiv:2404.08983 (2024)

[4] Veličković, B., Vignati, A., Non-vanishing higher derived limits. Communications in Contemporary Mathematics, Vol. 26 No. 07 (2024), 2350031.

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Fraïssé's Conjecture and big Ramsey degrees of structures admitting finite monomorphic decomposition[1]

Joint work with Veljko Toljić

Monomorphic structures (structures with only one kind of n-element substructures, for each n) were introduced and studied by R. Fraïssé as natural generalizations of chains (= linear orders). This notion was later generalized by Pouzet and Thierý to structures admitting a finite monomorphic decomposition. In this paper we characterize countable structures admitting a finite monomorphic decomposition which have finite big Ramsey degrees. The necessary prerequisite for that is the characterization of monomorphic structures with finite big Ramsey degrees. Interestingly, both characterizations require deep structural properties of chains. Fraïssé's Conjecture (actually, its positive resolution due to Laver) is instrumental in the characterization of monomorphic structures with finite big Ramsey degrees, while the analysis of big Ramsey combinatorics of structures admitting a finite monomorphic decomposition requires a product Ramsey theorem for big Ramsey degrees of chains. We find this last result particularly intriguing because big Ramsey degrees misbehave notoriously when it comes to general product statements. As a spin-off of the product Ramsey theorem we provide an alternative proof of Hubička's result that the generic partial order has finite big Ramsey degrees.

[1]This research was supported by the Science Fund of the Republic of Serbia, Program IDEAS, Grant No. 7750027: Set-theoretic, model- theoretic and Ramsey-theoretic phenomena in mathematical structures: similarity and diversity–SMART.

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Cofinal types of topological groups

Joint work with Boriša Kuzeljević

We present some basic results about cofinal types of topological groups. Tukey order is used to compare topological groups and directed sets. In this language, we give some additional conditions which ensure that a countably tight topological group is metrizable. Also, we study which properties are preserved by Tukey order.

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Weakly o-minimal types[2]

Joint work with Predrag Tanović

A complete type $p(x)$ is weakly o-minimal (wom) if there exists a relatively definable linear order $<$ on its locus $p(\mathfrak{C})$ such that every relatively definable subset of the locus is a union of finitely many $<$-convex parts. In the talk, I will discuss several problems concerning this notion. These include a description of all relatively definable linear orders on $p(\mathfrak{C})$, appropriate Monotonicity theorems for functions relatively definable on $p(\mathfrak{C})$, and, if time permits, properties of the forking-dependence relation.

[2]This research was supported by the Science Fund of the Republic of Serbia, Program IDEAS, Grant No. 7750027: Set-theoretic, model- theoretic and Ramsey-theoretic phenomena in mathematical structures: similarity and diversity–SMART.

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Polish ultrametric spaces, their isometry groups, and generalized wreath products

Joint work with R. Camerlo and A. Marcone

We address Krasner's problem (1956) for Polish ultrametric spaces and present some recent results towards its solution. In particular, we discuss a correspondence between the isometry groups of Polish ultrametric spaces belonging to some natural subclasses and various collections of generalized wreath products proposed in the literature.

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Characterizing XY-homogeneity

Joint work with Andrés Aranda, Bojana Pavlica and Christian Pech

The notion of homomorphism homogeneity was introduced in 2002 by Cameron and Nešetřil. A relational structure $\mathbf{U}$ is called *homomorphism homogeneous* if every local homomorphism of $\mathbf{U}$ can be extended to an endomorphism of $\mathbf{U}$. The notion of XY-homogeneity introduced in 2014 by Lockett and Truss generalizes the notion of homomorphism homogeneity. A relational structure $\mathbf{U}$ is called *XY-homogeneous* if every homomorphism of type X between finite structures of $\mathbf{U}$ can be extended to an endomorphism of $\mathbf{U}$ of type Y. Here X and Y refer to certain standard classes of morphisms such as homomorphisms, monomorphisms, endomorphisms, etc.

In this talk we use a notion of morphism classes X and Y that goes beyond the classification by Lockett and Truss. Our main result are two Fraïssé-type theorems for XY-homogeneous structures, that extend and generalize the results by Coleman, Evans and Gray.

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The Universally Baire Sets

In this talk we will explain recent developments in the study of the Universally Baire Sets.

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Intermediate models and Kinna-Wagner degrees

The intermediate model theorem states that whenever G is generic over V and $V \subseteq M \subseteq V[G]$ are models of ZFC, then M is also a forcing extension of V. Unfortunately, this fails completely if we only assume ZF instead. Can more can be said?

The goal of our talk is to present a generalization of the above theorem that works for ZF and talk about some of the recent progress made in the theory of symmetric extensions. In doing so, we prove Karagila's Kinna-Wagner conjecture that relates the structure of intermediate models of ZF to certain choice principles. We also provide a generalization to the fact that every set of ordinals is generic over HOD. This is joint with A. Karagila.

[1] Karagila, A., Schilhan, J., Kinna-Wagner Principles in intermediate models of generic extensions. In preparation.

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More minimal non-σ-scattered linear orders

Assuming an instance of the Brodsky-Rinot proxy principle holding at a regular uncountable cardinal κ, we construct 2^κ-many pairwise non-embeddable minimal non-σ-scattered linear orders of size κ. In particular, in Gödel's constructible universe L, these linear orders exist for any regular uncountable cardinal κ that is not weakly compact. This extends a recent result of Cummings, Eisworth and Moore that takes care of all the successor cardinals of L.

At the level of $\aleph_1$, their work answered an old question of Baumgartner by constructing from $\diamondsuit$ a minimal Aronszajn line that is not Souslin. Our use of the proxy principle yields the same conclusion from a weaker assumption which holds for instance in the generic extension after adding a single Cohen real to a model of CH.

[1] Shalev R. More minimal non-σ-scattered linear orders. Arxiv version (2023).

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Alternatives to the Halpern-Läuchli theorem, Part 2

At SETTOP 2022, some alternatives to the Halpern-Läuchli theorem were presented [2], such as Stefanović's continuation of Zucker's 2017 research [4], in which Zucker proved the DDF principle for two dimensions and finitely many colors, from which the Halpern-Läuchli theorem easily follows, as well as that the DDF principle in three dimensions for two colors is in conflict with the continuum hypothesis.

On that occasion, a new method with Cohen branches was presented, as well as a theorem on the consistency of the DDF principle for an arbitrary number of dimensions and uncountably many colors with ZFC axioms. Both results reproduce the Halpern-Läuchli theorem, where the second providing a new proof of the Halpern-Läuchli theorem.

Later, Stefanović on one side [3], and Andy Zucker and Chris Lambie-Hanson on the other [1], independently achieved better results with a stronger PG principle—Zucker and Lambie-Hanson for countably many colors, as part of a broader project investigating forcing with the addition of many Cohen reals, while Stefanović proved it for fewer than continuum many colors, which is the theoretical maximum. Both results provide proofs of the Halpern-Läuchli theorem.

Lambie-Hanson and Zucker showed in [1] the smallest possible size of the continuum for the DDF principle with finitely many colors, and for the PG principle with countably many colors, and they proved the PG principle for two colors and two dimensions in ZFC.

Here we present a ZFC theorem for an arbitrary number of dimensions from which the Halpern-Läuchli theorem easily follows.

Theorem. *Let $T_1, \dots, T_d$ be trees of height ω, where each node has at least one but at most countably many immediate successors. If $T(l)$ denotes the set of all nodes of the tree T that are at height l, let f be an arbitrary mapping from the set $\bigcup_{l\in\omega}(T_1(l) \times \cdots \times T_d)$ into some finite set C, and let F_0 be an arbitrary F_σ filter on the set $\mathcal{P}(\omega)$ (in the topology of the set $\mathcal{P}(\omega)$ identified with 2^ω). For each i, let M_i be a countable union of nowhere dense sets of branches of the tree T_i.*

Then there exist sets $D_1, \dots, D_d, F$ such that the following holds:

- *For each i, D_i is a dense set of branches of the tree T_i.*
- *For each i, $D_i \cap M_i = \emptyset$.*
- *There exists a countable set $E \subseteq \mathcal{P}(\omega)$ such that F is a filter on ω generated by the set $F_0 \cup E$.*

- *The mapping* $f_F : D_1 \times \cdots \times D_d \longrightarrow C$ *where*

$$f_F(b_1, \ldots, b_d) = c \quad \Leftrightarrow \quad \{n \in \omega \mid f(b_1(n), \ldots, b_d(n)) = c\} \in F$$

  is well-defined.

- *The mapping* f_F *is locally constant.*

- F *is an* F_σ *filter on* ω*.*

- *If the filter* F_0 *is countably generated, then the filter* F *is also countably generated.*

[1] Lambie-Hanson, C., Zuker Polish Space Partition Pprinciples and the Halpern–Läuchli Theorem. The Journal of Symbolic Logic, First View, (2024) pp. 1-19, DOI: https://doi.org/10.1017/jsl.2024.4

[2] Stefanović, N., Alternatives for the Halpern-Läuchli theorem. SETTOP 2022, Annals of Pure and Applied Logic Volume 174, Issue 9, October–November 2023, 103313 (2023)

[3] Stefanović, N., The coloring principle for the product of polish spaces and the Halpern and Läuchli's theorem., arxiv, `https://arxiv.org/abs/2209.07768`, (2022)

[4] Zucker, A., A new proof of the 2-dimensional Halpern–L" auchli. `https://www.math.cmu.edu/~andrewz/HL2d.pdf`. (2017)

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Weak and Strong Forms of Baumgartner's Axiom for Polish Spaces

If κ is a cardinal and X a topological space a subset $A \subseteq X$ is called κ-*dense* if it has intersection size κ with every non-empty open subset of X. We denote by $\mathsf{BA}_\kappa(X)$ the statement that for every pair of κ-dense subsets of X, say A and B, there is an autohomeomorphism $h : X \to X$ so that $h``A = B$. It's easy to see that $\mathsf{BA}_{\aleph_1}(\mathbb{R})$ is equivalent to the celebrated *Baumgartner axiom*, BA: every pair of $\aleph_1$-dense subsets of $\mathbb{R}$ are isomorphic as linear orders. This was shown to be consistent by Baumgartner in 1970. There are many natural questions of the form "When does $\mathsf{BA}_\kappa(X)$ imply $\mathsf{BA}_\kappa(Y)$ for spaces X and Y?" and "What consequences do Baumgartner type axioms have e.g. on cardinal characteristic inequalities?". For instance, famously Todorčević showed that BA implies $\mathfrak{b} > \aleph_1$ while Steprāns and Watson showed that $\mathsf{BA}_{\aleph_1}(\mathbb{R}^n)$ does not imply BA for any $n > 1$. We are motivated in particular by two related open questions: Does BA imply $\mathsf{BA}_{\aleph_1}(\mathbb{R}^n)$ for (some) $n > 1$? Does BA imply $\mathfrak{p} > \aleph_1$?

We shed some light by looking at a variety of weaker and stronger versions of $\mathsf{BA}_\kappa(X)$ for Polish spaces X and investigating their consequences. For instance we show that many of the important consequences of BA already hold for a weak variation $\mathsf{BA}^-_{\aleph_1}(X)$ for an arbitrary perfect Polish space X while a still weaker variation called $\mathsf{U}_{\kappa,\lambda}(X)$ entails none of these but still fails badly in a variety of models (and comes with a ccc preservation theorem).

Meanwhile strengthening the axioms by adding additional assumptions on the autohomeomorphism leads to much stronger results. For instance it has been known since the 70's that for any uncountable κ it is consistent that $\mathsf{BA}_\kappa(\omega^\omega)$ holds but the covering of the Lebesgue null ideal is $\aleph_1$. In contrast to this we show that a slightly stronger variation obtained by insisting that the function h is Lipschitz is consistent and implies that every cardinal in the Cichoń diagram is larger than $\aleph_1$.

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Countable models of a theory that interprets an infinite discrete linear order[3]

We will sketch the reasons why a complete countable first-order theory that interprets an infinite discrete linear order T has $2^{\aleph_0}$ countable models. These will allow us to conclude that T is Borel complete, provided that the answer to both of the following two (open) questions is positive.

1. If $T_{\bar{a}}$ is Borel complete for some $\bar{a} \in M^n$, must T be Borel complete?
2. If the relativization T_θ is Borel complete for some formula $\theta(x)$, must T be Borel complete?

[3]This research was supported by the Science Fund of the Republic of Serbia, Program IDEAS, Grant No. 7750027: Set-theoretic, model- theoretic and Ramsey-theoretic phenomena in mathematical structures: similarity and diversity–SMART.

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Towards new metrizability criteria for topological groups[4]

We shall analize metrizability criteria for topological groups using the language of Tukey reductions.

[4]This research was supported by the Science Fund of the Republic of Serbia, Program IDEAS, Grant No. 7750027: Set-theoretic, model- theoretic and Ramsey-theoretic phenomena in mathematical structures: similarity and diversity–SMART.

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Higher dimensional chain conditions

Joint work with Stevo Todorčević

For a given poset, the usual chain condition takes the form that given a "large" set of conditions, there is a "non-trivial" subset in which conditions are mutually compatible. The n-dimensional chain conditions deal with conditions indexed by n-tuples of ordinals and the "largeness"/"nontriviality" conditions are replaced with suitable n-dimensional variations. Just like 1-dimensional chain conditions being useful in establishing pigeonhole principles in the forcing extension, higher dimensional ones are useful in deriving Ramsey properties. We will discuss the possibility of deriving higher dimensional chain conditions of a given ideal from its strong 1-dimensional saturation property. We will also discuss some recent applications in the Ramsey-type theorems relevant to problems in homological algebra.