

THE WORD PROBLEM FOR ONE-RELATOR INVERSE MONOIDS: NEW DEVELOPMENTS

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Abstract

In early 1930s, W. Magnus proved his classical result that the word problem is decidable for all one-relator groups (groups given by one defining relation). This result is based on another important theorem proved earlier by Magnus, the *Freiheitssatz*, which, roughly speaking, locates many free subgroups in one-relator groups. This inspired investigations of the word problem for other algebraic structures defined by a single relation. For example, in the 1960s Shirshov proved that the word problem is decidable for all one-relator Lie algebras. Surprisingly, the problem whether the word problem is decidable for all one-relator monoids is still open, although several important cases have been resolved by Adjan in 1966, and Adyan and Oganessyan in 1987.

An important intermediate class of algebraic structures lying between groups and monoids are that of inverse monoids. In 2001 Ivanov, Margolis and Meakin highlighted the importance of investigating one-relator inverse monoids by showing that the (conjectured) decidability of the word problem for one-relator special inverse monoids (in fact, for a particular class of these inverse monoids) would imply a positive solution of the word problem for all one-relator monoids. In this talk, I will present two major recent contributions to this topic:

- (1) A result of R.D. Gray showing that the word problem for one-relator special inverse monoids is undecidable in its full generality; furthermore, there exists a one-relator group with undecidable submonoid membership problem.
- (2) The joint results of the speaker and R.D. Gray pertaining to the so-called prefix membership problem for one-relator groups, immediately implying decidability of the word problem for wide classes of one-relator special inverse monoids.

Along the way, I will explain the relation of the word problem for one-relator inverse monoids to the word problem of one-relator monoids, and also to several problems in group theory such as the prefix membership problem for one-relator groups and the role of embeddability of right-angled Artin groups.

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